5) $u(x,t) = \left(-\frac{3}{4}+\frac{3}{4}\cdot\left(\tanh\left(-\frac{1}{4}(x-ct)\right)\right)\right)^{-1}$;

6) $u(x,t) = \left(-\frac{3}{4}-\frac{3}{4}\cdot\left(\tanh\left(-\frac{1}{4}(x-ct)\right)\right)\right)^{-1}$;

7) $u(x,t) = \left(-\frac{3}{4}+\frac{3}{4}\cdot\left(\tanh\left(-\frac{1}{4}(x-ct)\right)\right)^{-1}\right)^{-1}$;

8) $u(x,t) = \left(-\frac{3}{4}-\frac{3}{4}\cdot\left(\tanh\left(-\frac{1}{4}(x-ct)\right)\right)^{-1}\right)^{-1}$;

9) $u(x,t) = \left(-\frac{3}{4}+\frac{3}{8}\cdot\left(\tanh\left(\frac{1}{8}(x-ct)\right)\right)+\frac{3}{8}\cdot\left(\tanh\left(\frac{1}{8}(x-ct)\right)\right)^{-1}\right)^{-1}$;

10) $u(x,t) = \left(-\frac{3}{4}-\frac{3}{8}\cdot\left(\tanh\left(\frac{1}{8}(x-ct)\right)\right)-\frac{3}{8}\cdot\left(\tanh\left(\frac{1}{8}(x-ct)\right)\right)^{-1}\right)^{-1}$;

11) $u(x,t) = \left(-\frac{3}{4}+\frac{3}{8}\cdot\left(\tanh\left(-\frac{1}{8}(x-ct)\right)\right)+\frac{3}{8}\cdot\left(\tanh\left(-\frac{1}{8}(x-ct)\right)\right)^{-1}\right)^{-1}$;

12) $u(x,t) = \left(-\frac{3}{4}-\frac{3}{8}\cdot\left(\tanh\left(-\frac{1}{8}(x-ct)\right)\right)-\frac{3}{8}\cdot\left(\tanh\left(-\frac{1}{8}(x-ct)\right)\right)^{-1}\right)^{-1}$.

In this paper, we studied the equations of the family of Camassa – Holm, particularly the Fornberg – Whitham equation. Using the tanh-coth method, we have constructed various exact wave solutions for this equation.

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BIANCHI TYPE I COSMOLOGY OF F-ESSENCE IN F(R) GRAVITY

[1]Darigozova Azhar, [2]Myrzakul Zhnbotaa

darigozova@gmail.com, zhanbota.myrzakul@gmail.com

[1]LN Gumilyov ENU. master student of department of «General and theoretical physics», Nur-Sultan, Kazakhstan

[2]LN Gumilyov ENU.Researcher of Eurasian international center for theoretical physics, Nur-Sultan, Kazakhstan

Scientific supervisor – N. Myrzakulov

Latest observation indicated that expansion of universe is accelerated. For explanation this late time accelerating, from scientists has been proposed two remarkable approaches. One is to assume contents of matter as scalar field in the right-hand side of the Einstein equation in the framework of general relativity, i.e. as scalar field may be consider phantom, quintessence, fermion, tachyon and etc. Another is to make modify the left-hand side of the Einstein equation [1-5], i.e. gravitational part.

The action of f-essence with $F(R)$ gravity reads as

$$S = \int d^4x\sqrt{-g}[F(R) + 2K(Y,\psi,\overline{\psi})], \qquad (1)$$

where Lagrange multiplyer

$$\lambda = F_R \qquad (2)$$

We can rewrite action

$$S = \int d^4x\sqrt{-g}\left[F(R) - \lambda\left(R - \frac{2}{N^2}\left(\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} - \frac{\dot{a}\dot{N}}{aN} - \frac{\dot{b}\dot{N}}{bN} - \frac{\dot{c}\dot{N}}{cN}\right)\right) + 2K(Y,\psi,\overline{\psi})\right] \qquad (3)$$

where K is some function of its arguments, $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ is a fermionic function and $\overline{\psi} = \psi^+\gamma^0$ is its adjoint function. Here

$$Y = 0.5i[\overline{\psi}\Gamma^\mu D_\mu\psi - (D_\mu\overline{\psi})\Gamma^\mu\psi] \qquad (4)$$

is the canonical kinetic term for the fermionic field and D_μ is covariant derivative

$$D_\mu\psi = \partial_\mu\psi + \Omega_\mu\psi, \quad D_\mu\overline{\psi} = \partial_\mu\overline{\psi} - \overline{\psi}\Omega_\mu. \qquad (5)$$

Here Ω_μ are spin connections, Γ^μ are the Dirac matrices associated with the space-time metric satisfying the Clifford algebra

$$\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu}. \qquad (6)$$

The Γ^μ are related to the flat Dirac matrices, γ^a, through the tetrads e^a_μ as

$$\Gamma^\mu = e^\mu_a\gamma^a, \quad \Gamma_\mu = e^a_\mu\gamma_a. \qquad (7)$$

At the same time, the spin connections Ω_μ satisfy the relation

$$\Omega_\mu = 0.25g_{nu\lambda}(\partial_\mu e^\lambda_a + \Gamma^\lambda_{\sigma\mu}e^\sigma_a)\gamma^\nu\gamma^a. \qquad (8)$$

The tetrads can be easily obtained from their definition, that is

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}. \qquad (9)$$

Let us now consider the Bianchi type I universe filled with f-essence. These models for the special simple spinors have been discussed previously sah04. The metric is given by

$$ds^2 = -N^2(t)dt^2 + a^2(t)dx^2 + b^2(t)dy^2 + c^2(t)dz^2, \qquad (10)$$

where $a(t), b(t), c(t)$ are scale factors in the x, y, z directions respectively and $N(t)$ is the lapse function. The corresponding scalar curvature takes the form

$$R = \frac{2}{N^2}\left(\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+\frac{\dot{b}\dot{c}}{bc}-\frac{\dot{a}\dot{N}}{aN}-\frac{\dot{b}\dot{N}}{bN}-\frac{\dot{c}\dot{N}}{cN}\right), \quad (11)$$

where a dot represents differentiation with respect to t. For the metric (10) the tetrads take the form

$$e^a_\mu = diag(N,a,b,c), e^\mu_a = diag(1/N,1/a,1/b,1/c). \quad (12)$$

These formulas yield

$$\Omega_0 = 0, \quad \Omega_1 = -\frac{\dot{a}}{2N}\gamma^0\gamma^1, \quad \Omega_2 = -\frac{\dot{b}}{2N}\gamma^0\gamma^2, \Omega_3 = -\frac{\dot{c}}{2N}\gamma^0\gamma^3, \quad (13)$$

where γ^0 and γ^i are the Dirac matrices in Minkowski spacetime and we have adopted the following representation

$$\gamma^0 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \quad (14)$$

Substituting (11) and (13) in (14) and integrating over the spatial dimensions, we are led to an effective Lagrangian in the mini-superspace $\{N,a,b,c,\psi,\overline{\psi}\}$

$$L = Nabc(F - F_R R) - \frac{2}{N}\left(\dot{a}bc + a\dot{b}c + ab\dot{c}\right)F_{RR}\dot{R} - \frac{2}{N}(\dot{a}\dot{b}c + \dot{a}b\dot{c} + a\dot{b}\dot{c})F_R + 2NabcK, \quad (15)$$

where

$$L_f = 2NabcK(Y,\psi,\overline{\psi}), \quad Y = \frac{1}{2N}(\overline{\psi}\gamma^0\dot{\psi} - \dot{\overline{\psi}}\gamma^0\psi). \quad (16)$$

Variation of Lagrangian (15) with respect to $N,a,b,c,\overline{\psi}$ and ψ yields the equations of motion of the gravitational and the fermions fields as:

$$\left(\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}\dot{c}}{bc}\right)F_R - \frac{\dot{N}}{N}\left(\frac{\dot{b}}{b}+\frac{\dot{c}}{c}\right)F_R + \left(\frac{\dot{b}}{b}+\frac{\dot{c}}{c}-\frac{\dot{N}}{N}\right)F_{RR}\dot{R} + F_{RRR}\dot{R}^2 + F_{RR}\ddot{R} + \\ + \frac{1}{2}N^2(F - F_R R) + N^2 K = 0, \quad (17)$$

$$\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}+\frac{\dot{c}}{c}\right)F_{RR}\dot{R} + \left(\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+\frac{\dot{b}\dot{c}}{bc}\right)F_R + \frac{1}{2}(F - F_R R) - N^2(YK_Y - K) = 0, \quad (18)$$

$$\left(\frac{\ddot{a}}{a}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{c}}{ac}\right)F_R - \frac{\dot{N}}{N}\left(\frac{\dot{a}}{a}+\frac{\dot{c}}{c}\right)F_R + \left(\frac{\dot{a}}{a}+\frac{\dot{c}}{c}-\frac{\dot{N}}{N}\right)F_{RR}\dot{R} + F_{RRR}\dot{R}^2 + F_{RR}\ddot{R} + \\ + \frac{1}{2}N^2(F - F_R R) + N^2 K = 0, \quad (19)$$

$$\left(\frac{\ddot{b}}{b}+\frac{\ddot{a}}{a}+\frac{\dot{b}\dot{a}}{ba}\right)F_R-\frac{\dot{N}}{N}(\frac{\dot{b}}{b}+\frac{\dot{a}}{a})F_R+(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}-\frac{\dot{N}}{N})F_{RR}\dot{R}+F_{RRR}\dot{R}^2+F_{RR}\ddot{R}+ \\ +\frac{1}{2}N^2(F-F_RR)+N^2K=0, \tag{20}$$

$$R-\frac{2}{N^2}(\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+\frac{\dot{b}\dot{c}}{bc}-\frac{\dot{a}\dot{N}}{aN}-\frac{\dot{b}\dot{N}}{bN}-\frac{\dot{c}\dot{N}}{cN})=0, \tag{21}$$

$$K_Y\dot{\psi}+0.5[(\ln(abc))_tK_Y+\dot{K}_Y]\psi+N\gamma^0K_{\bar{\psi}}=0, \tag{22}$$

$$K_Y\dot{\bar{\psi}}+0.5[(\ln(abc))_tK_Y+\dot{K}_Y]\bar{\psi}-NK_{\psi}\gamma^0=0, \tag{23}$$

$$\dot{\rho}+3H(\rho+p)=0, \tag{24}$$

where energy density and pressure take the form

$$\rho=N^2(YK_Y-K)+\frac{1}{2}(F-F_RR), \tag{25}$$

$$p=N^2\left(\frac{1}{2}F-\frac{1}{2}F_RR+K\right). \tag{26}$$

Now we examine the Bianchi type I cosmology for $K(Y,\psi,\bar{\psi})=Y-V(\bar{\psi}\psi)$. In this case we have

$$\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}+\frac{\dot{c}}{c}\right)F_{RR}\dot{R}+\left(\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+\frac{\dot{b}\dot{c}}{bc}\right)F_R+\frac{1}{2}(F-F_RR)-N^2V(\bar{\psi}\psi)=0, \tag{27}$$

$$\left(\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}\dot{c}}{bc}\right)F_R-\frac{\dot{N}}{N}(\frac{\dot{b}}{b}+\frac{\dot{c}}{c})F_R+(\frac{\dot{b}}{b}+\frac{\dot{c}}{c}-\frac{\dot{N}}{N})F_{RR}\dot{R}+F_{RRR}\dot{R}^2+F_{RR}\ddot{R} \\ +\frac{1}{2}N^2(F-F_RR)+N^2K=0, \tag{28}$$

$$\left(\frac{\ddot{a}}{a}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{c}}{ac}\right)F_R-\frac{\dot{N}}{N}(\frac{\dot{a}}{a}+\frac{\dot{c}}{c})F_R+(\frac{\dot{a}}{a}+\frac{\dot{c}}{c}-\frac{\dot{N}}{N})F_{RR}\dot{R}+F_{RRR}\dot{R}^2+F_{RR}\ddot{R} \\ +\frac{1}{2}N^2(F-F_RR)+N^2K=0, \tag{29}$$

$$\left(\frac{\ddot{b}}{b}+\frac{\ddot{a}}{a}+\frac{\dot{b}\dot{a}}{ba}\right)F_R-\frac{\dot{N}}{N}(\frac{\dot{b}}{b}+\frac{\dot{a}}{a})F_R+(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}-\frac{\dot{N}}{N})F_{RR}\dot{R}+F_{RRR}\dot{R}^2+F_{RR}\ddot{R} \\ +\frac{1}{2}N^2(F-F_RR)+N^2K=0, \tag{30}$$

$$R-\frac{2}{N^2}(\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+\frac{\dot{b}\dot{c}}{bc}-\frac{\dot{a}\dot{N}}{aN}-\frac{\dot{b}\dot{N}}{bN}-\frac{\dot{c}\dot{N}}{cN})=0, \tag{31}$$

$$\dot{\psi}+0.5(\ln(abc))_{,t}\psi-N\gamma^{0}V_{\overline{\psi}}=0, \quad (32)$$

$$\dot{\overline{\psi}}+0.5(\ln(abc))_{,t}\overline{\psi}+NV_{\psi}\gamma^{0}=0, \quad (33)$$

$$\dot{\rho}+3H(\rho+p)=0. \quad (34)$$

We take $V(\overline{\psi}\psi)=2\overline{\psi}\psi$. Thus we obtain

$$\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}+\frac{\dot{c}}{c}\right)F_{RR}\dot{R}+\left(\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+\frac{\dot{b}\dot{c}}{bc}\right)F_{R}+\frac{1}{2}(F-F_{R}R)-2N^{2}\overline{\psi}\psi=0, \quad (35)$$

$$\left(\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}\dot{c}}{bc}\right)F_{R}-\frac{\dot{N}}{N}(\frac{\dot{b}}{b}+\frac{\dot{c}}{c})F_{R}+(\frac{\dot{b}}{b}+\frac{\dot{c}}{c}-\frac{\dot{N}}{N})F_{RR}\dot{R}+F_{RRR}\dot{R}^{2}+F_{RR}\ddot{R}$$
$$+\frac{1}{2}N^{2}(F-F_{R}R)+N^{2}(\frac{1}{2N}(\overline{\psi}\gamma^{0}\dot{\psi}-\dot{\overline{\psi}}\gamma^{0}\psi)-2\overline{\psi}\psi)=0, \quad (36)$$

$$\left(\frac{\ddot{a}}{a}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{c}}{ac}\right)F_{R}-\frac{\dot{N}}{N}(\frac{\dot{a}}{a}+\frac{\dot{c}}{c})F_{R}+(\frac{\dot{a}}{a}+\frac{\dot{c}}{c}-\frac{\dot{N}}{N})F_{RR}\dot{R}+F_{RRR}\dot{R}^{2}+F_{RR}\ddot{R}$$
$$+\frac{1}{2}N^{2}(F-F_{R}R+N^{2}(\frac{1}{2N}(\overline{\psi}\gamma^{0}\dot{\psi}-\dot{\overline{\psi}}\gamma^{0}\psi)-2\overline{\psi}\psi)=0, \quad (37)$$

$$\left(\frac{\ddot{b}}{b}+\frac{\ddot{a}}{a}+\frac{\dot{b}\dot{a}}{ba}\right)F_{R}-\frac{\dot{N}}{N}(\frac{\dot{b}}{b}+\frac{\dot{a}}{a})F_{R}+(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}-\frac{\dot{N}}{N})F_{RR}\dot{R}+F_{RRR}\dot{R}^{2}+F_{RR}\ddot{R}$$
$$+\frac{1}{2}N^{2}(F-F_{R}R)+N^{2}(\frac{1}{2N}(\overline{\psi}\gamma^{0}\dot{\psi}-\dot{\overline{\psi}}\gamma^{0}\psi)-2\overline{\psi}\psi)=0, \quad (38)$$

$$R-\frac{2}{N^{2}}(\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+\frac{\dot{b}\dot{c}}{bc}-\frac{\dot{a}\dot{N}}{aN}-\frac{\dot{b}\dot{N}}{bN}-\frac{\dot{c}\dot{N}}{cN})=0, \quad (39)$$

$$\dot{\psi}+0.5(\ln(abc))_{,t}\psi-2N\gamma^{0}\psi=0, \quad (40)$$

$$\dot{\overline{\psi}}+0.5(\ln(abc))_{,t}\overline{\psi}+2N\overline{\psi}\gamma^{0}=0, \quad (41)$$

$$\dot{\rho}+3H(\rho+p)=0. \quad (42)$$

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FRW COSMOLOGY OF NONCANONICAL FERMIONIC FIELD IN F(R) GRAVITY

[1]Darigozova Azhar, [2]Myrzakul Zhanbota

darigozova@gmail.com, zhanbota.myrzakul@gmail.com

[1]LN Gumilyov ENU. master student of department of «General and theoretical physics», Nur-Sultan, Kazakhstan

[2]LN Gumilyov ENU.Researcher of Eurasian international center for theoretical physics, Nur-Sultan, Kazakhstan

Scientific supervisor – N. Myrzakulov

In cosmology fermion fields have been studied as possible sources of early and late time expansion without the need of a cosmological constant term or a scalar field. In most of the papers are considered fermions fields are minimally coupled to gravity. But, in recently appears works effects of fermionic fields non - minimally coupling to gravity [1-6]. The fermion fields has been investigated via several approaches, with results including exact solutions, numerical solutions, cyclic cosmologies and anisotropy-to-isotropy scenario, perturbations, dark spinors. The relation between general relativity and the equation for fermion fields is done via the tetrad formalism. The components of the tetrad play the role of the gravitational degrees of freedom

In this section we would like to present the derivation of the equations of motion for FRW metric in the f-essence.

Let us consider the following action of f-essence

$$S = \int d^4x\sqrt{-g}[F(R) + 2K(Y,\psi,\overline{\psi})], \quad (1)$$

$$\lambda = F_R \quad (2)$$

$$S = \int d^4x\sqrt{-g}\left[F(R) - \lambda\left(R - 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\right) + 2K(Y,\psi,\overline{\psi})\right] \quad (3)$$

where R is the scalar curvature, Y is the kinetic term for the fermionic field ψ and K is some function of its arguments. In the case of the FRW metric

$$ds^2 = -dt^2 + a^2(dx^2 + dy^2 + dz^2), \quad (4)$$

R and Y have the form

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right), \quad (5)$$

$$Y = 0.5i(\overline{\psi}\gamma^0\dot{\psi} - \dot{\overline{\psi}}\gamma^0\psi), \quad (6)$$