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DISCRETE BILINEAR HARDY TYPE INEQUALITIY

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Let $0 < p, s, q < +\infty$; let $u = \{u_n\}, v = \{v_n\}, w = \{w_n\}, n \in N$, be positive sequences of real numbers. Let $f = \{f_n\}_{n=1}^{\infty}$, $g = \{g_n\}_{n=1}^{\infty}$ be arbitrary sequences of non-negative numbers.

In this work we study the characterization problem for the bilinear discreate Hardy type operators of the following form

$$\left(\sum_{n=1}^{\infty} u_n^q \left(\sum_{i=1}^{n} a_{ni} f_i\right)^q \left(\sum_{i=1}^{n} g_i\right)^q\right)^{\frac{1}{q}} \le C\left(\sum_{i=1}^{\infty} |v_i f_i|^p\right)^{\frac{1}{p}} \left(\sum_{i=1}^{\infty} |w_i g_i|^s\right)^{\frac{1}{s}}, \quad (1)$$

where the C is the best constant in (1) and does not depend on f and g; (a_{ij}) be non-negative matrix, with elements $a_{ij} \ge 0$, when $i \ge j \ge 1$, $a_{ij} = 0$, when $i < j$ and which satisfy the Oinarov condition: there are exist $d \ge 1$ such that

$$\frac{1}{d}(a_{ik} + a_{kj}) \le a_{ij} \le d(a_{ik} + a_{kj}) \quad (2)$$

for all $i \ge k \ge j \ge 1$.

When $a_{ij} = 1$, $i \ge j \ge 1$ the inequality (1) coincides with discrete bilinear Hardy inequality which investigated in [1] ,[2] for various combinations of the parameters *p*,*s* and *q*. The integral bilinear Hardy inequalities were studied in [3] -[5] and references therein.

To study discrete bilinear Hardy type inequality (1) we can apply a reduction method reducing the required inequality to linear Hardy inequality and to linear Hardy inequality with kernel (a_{ij}), criteria for which are given in [6]-[8].

Our main result read as follows:

Theorem 1. Let $0 < s \le 1 < p \le q < +\infty$ and the elements of matrix (a_{ij}) satisfy condition (2). Then the inequality (1) holds, if and only if $D = \max\{D_1, D_2\} < \infty$, where

$$D_1 = \sup_{m \ge 1} \left(\sum_{i=m}^{\infty} u_i^q\right)^{\frac{1}{q}} \left(\sum_{j=1}^{m} a_{mj}^{p'} v_j^{-p'}\right)^{\frac{1}{p'}} \sup_{n \le m} w_n^{-\frac{1}{s}},$$

$$D_2 = \sup_{m \ge 1} \left(\sum_{i=m}^{\infty} a_{im}^q u_i^q\right)^{\frac{1}{q}} \left(\sum_{i=1}^{m} v_i^{-p'}\right)^{\frac{1}{p'}} \sup_{n \le m} w_n^{-\frac{1}{s}}.$$

Moreover, $D \approx C$, where C is best constant in (1).

Theorem 2. Let $0 < p \le 1 < s \le q < +\infty$ and the elements of matrix (a_{ij}) satisfy condition (2). Then the inequality (1) holds, if and only if $D_3 < \infty$, where

$$D_3 = \sup_{m \ge 1} \left(\sum_{i=m}^{\infty} a_{im}^q u_i^q \right)^{\frac{1}{q}} v_m^{-1} \left(\sum_{j=1}^{m} w_j^{-s'} \right)^{\frac{1}{s'}}.$$

Moreover, $D_3 \approx C$, where C is best constant in (1).

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