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OSCILLATION AND NON-OSCILLATION CONDITIONS OF THE SECOND ORDER HALF-LINEAR DIFFERENTIAL EQUATIONS

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Let, $\mu, \gamma \in R, \alpha > 0$

$$t^{\mu}\left(\left|y'(t)\right|^{p-2} y'(t)\right)' + \alpha t^{\gamma}\left|y(t)\right|^{p-2} y(t) = 0,\ t > 0 \tag{1}$$

$$\int_0^{\infty}\left(t^{\mu}\left|y'(t)\right|^{p} - \alpha t^{\gamma}\left|y(t)\right|^{p}\right)dt \geq 0,\ y \in \overset{0}{W}{}^{1}_{p}(0,\infty) \tag{2}$$

(2) the inequality is a necessary and sufficient condition for the disconjugacy of equation (1) on the interval $(0,\infty)$.

Let, $\mu < p-1$, then for $\forall c > 0$ it will be

$$\int_0^c t^{(1-p')\mu} dt < \infty,$$

$$\int_c^\infty t^{(1-p')\mu} dt = \infty$$

Then by Theorem A from [1]

$$\overset{0}{W}{}^1_p(0,\infty) = \overset{0}{W}{}^1_{p,l}(0,\infty) = \left\{ f \in W^1_p(0,\infty) : f(0) = 0 \right\}$$

Then from (2)

$$\int_0^\infty t^\mu |y'(t)|^p dt \geq \alpha \int_0^\infty t^\gamma |y(t)|^p dt, \quad y(0) = 0 \tag{3}$$

$$y'(t) = f(t), y(0) = 0 \Rightarrow y(t) = \int_0^t f(s)ds$$

Substituting the indicated values into (3), we obtain

$$\int_0^\infty t^\gamma \left| \int_0^t f(s)ds \right|^p dt \leq \frac{1}{\alpha} \int_0^\infty t^\mu |f(t)|^p dt . \tag{4}$$

Now consider Hardy's inequality

$$\int_0^\infty t^\gamma \left| \int_0^t f(s)ds \right|^p dt \leq C \int_0^\infty t^\mu |f(t)|^p dt$$

Let, $\gamma = \mu - p$. Then

$$\int_0^\infty \left| \frac{1}{t} \int_0^t f(s)ds \right|^p t^\mu dt \leq C \int_0^\infty t^\mu |f(t)|^p dt \tag{5}$$

By Hardy's theorem [2] the least constant

$$C = \left(\frac{p}{p-\mu-1} \right)^p .$$

For $\mu < 1-p$ and $\gamma = \mu - p$, the (4) inequality holds, if and only (4) the inequality holds, if and only if $\frac{1}{\alpha} \geq \left(\frac{p}{p-\mu-1}\right)^p$, that is, if $\alpha \leq \left(\frac{p-\mu-1}{p}\right)^p$ performed.

If $\alpha > \left(\frac{p-\mu-1}{p}\right)^p$, then it does not hold this (4) $\Longrightarrow$ (3) $\Longrightarrow$ (2). Therefore, $\alpha \leq \left(\frac{p-\mu-1}{p}\right)^p$ and for $\gamma = \mu - p$, $\mu < 1-p$ in the inequality (2), hence equation (1) is disconjugacy on the interval $(0,\infty)$, and then equation (1) will be non-oscillatory.

If, for $\alpha > \left(\frac{p-\mu-1}{p}\right)^p$, $\gamma = \mu - p$, $\mu < 1-p$ then equation (1) will be oscillatory.

For any $a > 0$, by Hardy's inequality, the inequality

$$\int_0^\infty \left|\frac{1}{t}\int_0^t f(s)ds\right|^p t^\mu dt \leq \left(\frac{p}{p-\mu-1}\right)^p \int_0^\infty t^\mu \left|f(t)\right|^p dt \tag{6}$$

is executed with the smallest constant $\left(\frac{p}{p-\mu-1}\right)^p$. If condition (6) is satisfied, equation (1) for any $a > 0$, on the interval (a,∞) is conjugate, which means that equation (1) is oscillatory. Hence it follows that under condition (6), equation (1) is oscillatory.

Literature

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