ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ ТҰҢҒЫШ ПРЕЗИДЕНТІ - ЕЛБАСЫНЫҢ ҚОРЫ

«ҒЫЛЫМ ЖӘНЕ БІЛІМ – 2017»

студенттер мен жас ғалымдардың
XII Халықаралық ғылыми конференциясының
БАЯНДАМАЛАР ЖИНАҒЫ

СБОРНИК МАТЕРИАЛОВ
XII Международной научной конференции
студентов и молодых ученых
«НАУКА И ОБРАЗОВАНИЕ – 2017»

PROCEEDINGS
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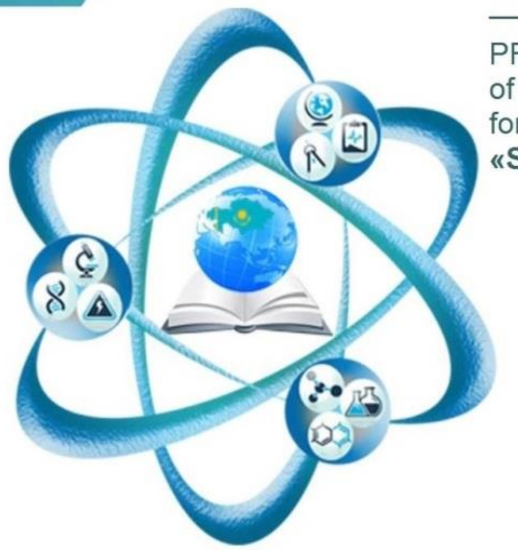
«SCIENCE AND EDUCATION - 2017»



14th April 2017, Astana

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ БІЛІМ ЖӘНЕ ҒЫЛЫМ МИНИСТРЛІГІ
Л.Н. ГУМИЛЕВ АТЫНДАҒЫ ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ

«Ғылым және білім - 2017»
студенттер мен жас ғалымдардың
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The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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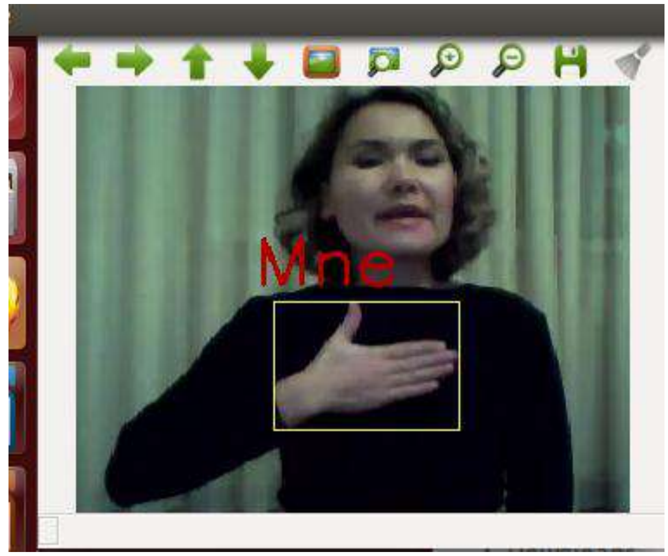
Pic2. Finger-spelling recognition



Pic3. Gesture meaning recognition

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MATHEMATICAL MODEL OF NEURAL NETWORK

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In our talk we discuss how boundary control method can be applied to inverse problem for differential equation on graph-tree. This is a new approach to the analysis of heat equation with memory and a local reconstruction algorithm of source identification.

Differential equations on graphs are used to describe problems which arise in nano-technology, chemistry, biology, and other sciences and engineering.

We consider neural network. The first mathematical model of a neuron was created in 1943 by the American scientist Warren McCulloch and his student Walter Pitts. Neuron is the structural and functional unit of the nervous system. It is an electrically excitable cell that processes and transmits information through electrical and chemical signals. A typical neuron consists of a cell body (soma), dendrites, and an axon. The average healthy human brain contains roughly 100 billion neurons. Neurons can connect to each other to form a biological neural network. Dendrite is a short and highly branched part of a neuron. An axon is a long, slender branch of a nerve cell.

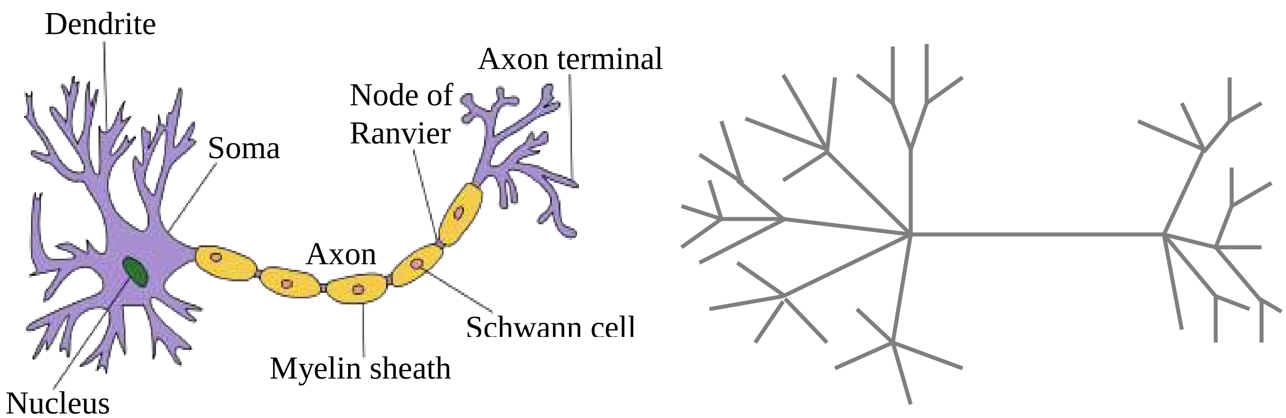


Figure 1: Structure of a typical neuron

First we consider the algorithm on an interval.

It is well known, in the usual heat equation $u_t(x,t) = a^2 u_{xx}(x,t)$, a 'heat signal' spreads with infinite speed. If at the initial moment accept $u(x,0) = \delta(x)$, the 'heat signal' will be spread according to the normal law in the form of a Gaussian curve:

$$u(x,t) = \frac{1}{2a\sqrt{\pi t}}\ e^{\frac{-x^2}{4a^2t}}.$$

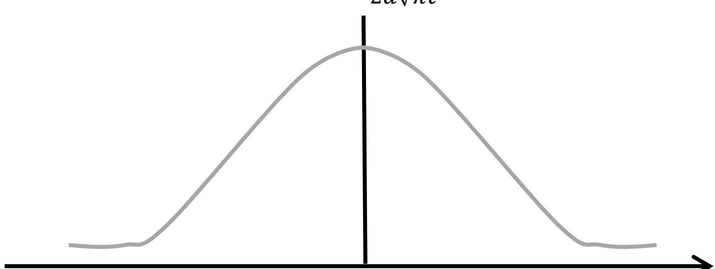
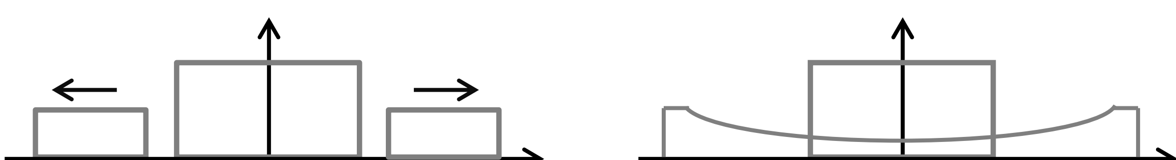


Figure 2: heat kernel

The heat equation with memory was proposed by Cattaneo (1948) [1] and, in more general form, by Gurtin and Pipkin (1968) [2]: $u_t(x,t) = \int_0^t N(t-s)u_{xx}(x,s)ds$.
Differentiating by t, we get a viscoelasticity equation:

$$u_{tt}(x,t) = N(0)u_{xx}(x,t) + \int_0^t N'(t-s)u_{xx}(x,s)ds.$$

The speed of the wave distribution is $\sqrt{N(0)}$.



x x

Figure 3: wave fronts

The usual heat equation does not describe physical phenomena in real situations. In 1949, Landau theoretically proved that at very low temperatures heat is propagated as a wave. Later in the same year this theory was verified experimentally. However, it is not clear for temperature phase transition. Well as the usual heat equation does not describe the diffusion in materials with complex molecular structure (polymers).

We consider the equation

$$u_t(x,t) - \int_0^t N(t-s)u_{xx}(x,s)ds = f(t)g(x), \qquad (1)$$

with boundary and initial conditions

$$u(0,t) = u(l,t) = 0,\ u(x,0) = 0;\ \ 0 < x < l,\ t > 0 \qquad (2)$$

Here $N, f \in H^1(0,T)$ are known function, and we assume that $f(0) \neq 0$, $N(0) > 0$. The function $g \in L^2(0,1)$ is unknown and can be recovered from the source of the surveillance $\mu(t) := u_x(0,t)$, $t \in [0,T]$ (Neumann`s condition). It is proved that this problem solvable for $T \geq \frac{l}{\sqrt{N(0)}}$. For simplicity of the presentation we assume that $N(0) = 1$. Then the speed of the wave distribution in the equation (1) is one, and the identification problem is solvable for $T \geq l$.

Solution of the initial-boundary value problem (1)-(2) can be presented in the form:

$$u(x,t) = \sum_{n=1}^{\infty} a_n(t)\,\varphi_n(x), \qquad \varphi_n(x) = \sqrt{\frac{2}{l}}\sin\frac{\pi n x}{l}$$

Substituting to (1) - (2) leads to the following equations for $a_n(t)$:

$$a'_n(t) + \omega_n^2 \int_0^t N(t-s)a_n(s)ds = f(t)\int_0^l g(x)\varphi_n(x)dx, \quad a_n(0) = 0.$$

It follows that the coefficients a_n can be written as

$$a_n(t) = \gamma_n \int_0^t h(\tau)\frac{s_n(t-\tau)}{\omega_n}d\tau,$$

where $\gamma_n = \int_0^l g(x)\varphi_n(x)dx$, $\quad \omega_n = \frac{\pi n}{l}$, $\quad h(t) = f(t) + \int_0^t N(t-\tau)f(\tau)d\tau$,

and $s_n(t)$ satisfies to the initial value problem

$$s''_n(t) + \omega_n^2 s_n(t) + \omega_n^2\int_0^t N(t-\tau)s_n(\tau)d\tau = 0, \quad s_n(0) = 0, \quad s'_n(0) = \omega_n.$$

In [3] and [4] it was proved that the functions $s_n(t)$ are asymptotically close to $\sin\omega_n t$ and form a Riesz sequence in $L^2(0,T)$ for $T \geq l$. For the surveillance we have

$$\mu(t) = u_x(0,t) = \int_0^l [\sum_{n=1}^{\infty} \varphi'_n(0) \int_0^t h(\tau)\frac{s_n(t-\tau)}{\omega_n}d\tau\,\varphi_n(x)]\,g(x)dx == \int_0^l w(x,t)g(x)dx = \int_0^t w(x,t)g(x)dx \qquad (3)$$

We have changed the upper limit in the last integral since $w(x,t) = 0$ for $x > t$ as a solution to the initial-boundary value problem

$$w_t(x,t) - \int_0^t N(t-s)w_{xx}(x,s)ds = 0; \quad 0 < x < l,\ t > 0, \tag{4}$$

$$w(0,t) = h(t), w(l,t) = 0, w(x,0) = 0. \tag{5}$$

The function *w* can be presented in the form

$$w(x,t) = h(t-x) + \int_0^t k(x,s)h(t-s)ds,$$

where the kernel k is determined by N and $k(x,s) = 0$ for $x \geq s$. Substituting this presentation into (3) we get

$$\mu(t) = \int_0^t [h(t-x) + \int_0^t k(x,s)h(t-s)ds]g(x)dx$$

and integrating by parts,

$$\mu(t) = h(0)G(t) + \int_0^t [h'(t-x) - \int_0^t k_x(x,s)h(t-s)ds]G(x)\,dx,$$

where $G(x) = \int_0^x g(\xi)d\xi$. This is a second kind Volterra equation for *G(x)*. Solving it, we find *G(x)* and then, *g(x)*.

Now we consider a star graph *Γ* consisting of *N* edges e_j identified with intervals $[0,l_j]$, $j=1,\dots,N$, connected at an inner vertex γ_0 which we identify with the set of the left end points of the intervals. The boundary vertices γ_j are identified with the right end points of the corresponding intervals.

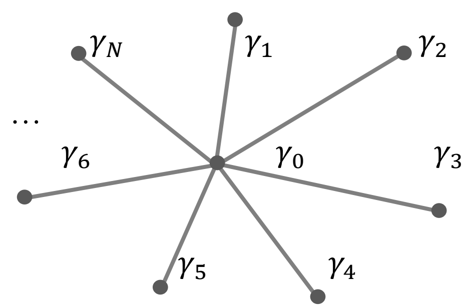


Figure 4: A Star graph

The following initial boundary value problem is considered on each interval:

$$\begin{cases} u_t^j(x,t) - \int_0^t N(t-s)u_{xx}^j(x,s)ds = f_j(t)g_j(x), & 0 < x < l_j,\ 0 < t < T, \\ u^j(l_j,t) = 0, & 0 < t < T, \\ u^j(x,0) = u_t^j(x,0) = 0, & 0 < x < l_j. \end{cases} \tag{6}$$

At the inner vertex we impose the standard Kirchhoff – Neumann`s matching conditions

$$\begin{cases} u^1(0,t) = \ldots = u^N(0,t), & 0 < t < T, \\ \sum_{j=1}^N u_x^j(0,t) = 0, & 0 < t < T. \end{cases} \tag{7}$$

In mechanical systems, the first condition expresses the continuity of the solution, the second is the balance of forces (Newton`s second law).

The problem is to recover unknown functions $g_j, j = 1, \dots, N$, from the surveillances

$$\mu_j(t) := u_x^j(l_j,t), \qquad j=1,\dots,N-1, t\in[0,T].$$

Note, that we use the surveillances at all but one boundary vertices.

On the first step we recover the functions $g_j, j=1,\dots,N-1$, using correspondingly surveillances $\mu_j(t), j=1,\dots,N-1$. It can be done in the case of one interval for $T \geq \max_{j=1,\dots,N-1}\{l_j\}$.

Then, since the functions $g_j, j=1,\dots,N-1$ are known, we can consider the initial boundary value problem

$$\begin{cases} v_t^j(x,t) - \int_0^t N(t-s)v_{xx}^j(x,s)ds = f_j(t)g_j(x), & j \neq N, \\ v_t^N(x,t) - \int_0^t N(t-s)v_{xx}^N(x,s)ds = 0. \end{cases}$$

Subtracting solution of this problem from solution of (6), we reduce our identification problem to the case where all g_j except g_N are equal to zero:

$$\begin{cases} u_t^j - \int_0^t N(t-s)u_{xx}^j(x,s)ds = 0, j \neq N, & 0<x<l_j, \quad 0<t<T, \\ u_t^N - \int_0^t N(t-s)u_{xx}^N(x,s)ds = f_N(t)g_N(x), & 0<x<l_N, \quad 0<t<T, \\ u^j(l_j,t) = 0, & 0<t<T, \\ u^j(x,0) = u_t^j(x,0) = 0, & 0<x<l_j. \end{cases}$$

Now we can recover g_N using any of the boundary surveillances $\mu_j, j=1,\dots,N-1$, say, μ_1. Using the spectral representation of the solution (similar to the case of one interval), we obtain

$$\mu_1(t) = u_x^1(l_1,t) = \int_0^{t-l_1} g_N(x)w_N^1(x,t)dx. \tag{8}$$

In this formula $w_N^1(x,t)$ is the restriction to the edge e_N of the solution of the heat equation with memory on the graph Γ with the Dirichlet boundary control f_N applied to the boundary vertex γ_1:

$$\begin{cases} w_t^j(x,t) - \int_0^t N(t-s)w_{xx}^j(x,s)ds = 0, & i=1,\dots,N, \\ w^j(l_1,t) = f_N(t), w^j(l_i,t) = 0, i\neq 1, & 0<t<T, \\ w^j(x,0) = w_t^j(x,0) = 0, & 0<x<l_i, i=1,\dots,N. \end{cases}$$

This solution satisfies also the Kirchhoff – Neumann`s matching conditions.

Based on this formula we can recover the function g_N in the time interval $l_1 \leq t \leq l_1 + l_N$ using the method described for the case of one interval.

This method allows us to solve the source identification problems on arbitrary tree (a graph without cycles). Since any tree consists of a finite number of star graphs, we need just to repeat the steps described below. We use surveillances at all but one boundary vertices. It can be proved that we have surveillances at less than all but one boundary vertices, the source identification problem does not generally have a unique solution.

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УДК 621.39

ОБ ОДНОЙ ОРТОНОРМИРОВАННОЙ СИСТЕМЕ ФУНКЦИЙ

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Введение

В данной работе, система Хаара [1], в определенном смысле, обобщается, а именно, вводится некоторый параметр $\alpha \geq 0$, доказывается ортонормированность введенной системы и исследуется влияние данного параметра в разложении некоторой (степенной) функции в ряд по определенной системе [1,2], причем устанавливается условие на введенный параметр. Дальнейшим продолжением теоретических исследований, предполагается исследования по разложению сигналов не степенного вида, а функции из класса L^p.

Об ортонормированности системы типа Хаара

Определение. Пусть даны числа $\alpha > log_2\left(1+\sqrt{5}\right)-1$ и целое $m \geq 1$. Систему функций $\chi_m^{(k)}(x,\alpha)$ $k=1,2,...,2^m$, *определленную в виде*

$$\chi_m^k(x,\alpha)=\begin{cases} \sqrt{\dfrac{2^{(m+1)(1+\alpha)}}{2\left[(k-1)^{1+\alpha}\left(1-2^{1+\alpha}\right)+k^{1+\alpha}\right]}} & x\in\left(\dfrac{(k-1)^{1+\alpha}}{2^{m(1+\alpha)}},\dfrac{(k-1)^{1+\alpha}+k^{1+\alpha}}{2^{(m+1)(1+\alpha)}}\right) \\ -\sqrt{\dfrac{2^{(m+1)(1+\alpha)}}{2\left[k^{1+\alpha}\left(2^{1+\alpha}-1\right)-(k-1)^{1+\alpha}\right]}} & x\in\left(\dfrac{(k-1)^{1+\alpha}+k^{1+\alpha}}{2^{(m+1)(1+\alpha)}},\dfrac{k^{1+\alpha}}{2^{m(1+\alpha)}}\right) \\ 0, & x=0, x\in\left(\dfrac{(l-1)^{1+\alpha}}{2^{m(1+\alpha)}},\dfrac{l^{1+\alpha}}{2^{m(1+\alpha)}}\right),\ l\neq k \end{cases}$$

назовем системой типа Хаара. [1]

Справедлива

Теорема 1. Пусть даны число $\alpha > log_2\left(1+\sqrt{5}\right)-1$ *и целое* $m \geq 1$. *Тогда определенная на сегменте* $[0,1]$ *система типа* $\chi_m^{(k)}(x,\alpha)$ $k=1,2,...,2^m$ *вида (*$m>1$, $k=1,2,...,2^m$ *и* $(0,1)=\bigcup\limits_{k=1}^{m}\left(\dfrac{(k-1)^{1+\alpha}}{2^{m(1+\alpha)}},\dfrac{k^{(1+\alpha)}}{2^{m(1+\alpha)}}\right)$*)*