ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ

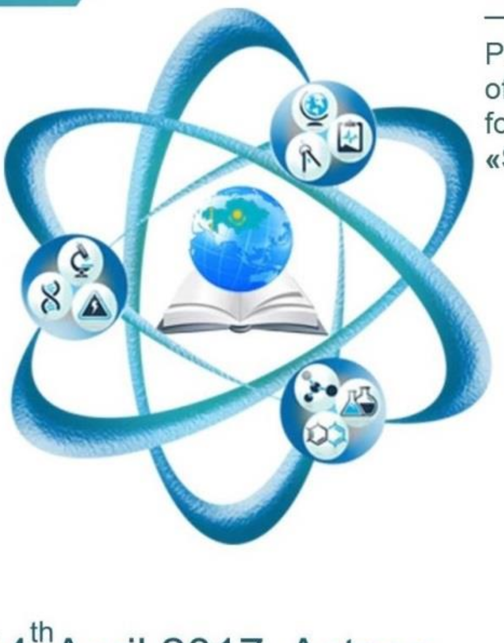
ҚАЗАҚСТАН РЕСПУБЛИКАСЫ ТҰҢҒЫШ ПРЕЗИДЕНТІ - ЕЛБАСЫНЫҢ ҚОРЫ

«ҒЫЛЫМ ЖӘНЕ БІЛІМ – 2017»

студенттер мен жас ғалымдардың
XII Халықаралық ғылыми конференциясының
БАЯНДАМАЛАР ЖИНАҒЫ

СБОРНИК МАТЕРИАЛОВ
XII Международной научной конференции
студентов и молодых ученых
«НАУКА И ОБРАЗОВАНИЕ – 2017»

PROCEEDINGS
of the XII International Scientific Conference
for students and young scholars
«SCIENCE AND EDUCATION - 2017»



14th April 2017, Astana

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ БІЛІМ ЖӘНЕ ҒЫЛЫМ МИНИСТРЛІГІ
Л.Н. ГУМИЛЕВ АТЫНДАҒЫ ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ

«Ғылым және білім - 2017»
студенттер мен жас ғалымдардың
XII Халықаралық ғылыми конференциясының
БАЯНДАМАЛАР ЖИНАҒЫ

СБОРНИК МАТЕРИАЛОВ
XII Международной научной конференции
студентов и молодых ученых
«Наука и образование - 2017»

PROCEEDINGS
of the XII International Scientific Conference
for students and young scholars
«Science and education - 2017»

2017 жыл 14 сәуір

Астана

УДК 378

ББК 74.58
Ғ 96

Ғ 96
«Ғылым және білім – 2017» студенттер мен жас ғалымдардың XII Халықаралық ғылыми конференциясы = The XII International Scientific Conference for students and young scholars «Science and education - 2017» = XII Международная научная конференция студентов и молодых ученых «Наука и образование - 2017». – Астана: http://www.enu.kz/ru/nauka/nauka-i-obrazovanie/, 2017. – 7466 стр. (қазақша, орысша, ағылшынша).

ISBN 978-9965-31-827-6

Жинаққа студенттердің, магистранттардың, докторанттардың және жас ғалымдардың жаратылыстану-техникалық және гуманитарлық ғылымдардың өзекті мәселелері бойынша баяндамалары енгізілген.

The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

УДК 378

ББК 74.58

ISBN 978-9965-31-827-6

СЕКЦИЯ 4
МАТЕМАТИКА, МЕХАНИКА И МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ

Подсекция 4.1 Математика

UDK 517

SEMIFINITE TRACIAL SUBALGEBRAS

Oshanova Azhar
azhar_oshanova@mail.ru
Master of Mechanics and Mathematics, specialty: Scientific Mathematics of ENU. L.N. Gumilev, Astana, Kazakhstan
Supervisor - TURDEBEK N. BEKJAN

Abstract. Let $\mathcal{M}$ be a semifinite von Neumann algebra, and let $\mathcal{A}$ be a tracial subalgebra of $\mathcal{M}$. We show that $\mathcal{A}$ is a subdiagonal algebra of $\mathcal{M}$ if and only if it has the unique normal state extension property and is a τ-maximal tracial subalgebra, which is also equivalent to $\mathcal{A}$ having the unique normal state extension property and satisfying L_2-density

1. Introduction

Noncommutative Hardy space theory has received considerable progress since the seminal paper by Arveson [1] in 1967. He introduced the notion of finite, maximal, subdiagonal algebras $\mathcal{A}$ of $\mathcal{M}$, as non-commutative analogues of weak* Dirichlet algebras. Many classical results of Hardy space have been successfully transferred to the noncommutative setting (cf. e.g.[3, 4]). In [4], among other things, Blecher and Labuschagne transfered a large part of the circle of theorems characterizing weak* Dirichlet algebras, to Arvesons noncommutative setting of subalgebras of finite von Neumann algebras. In [3], the first author and Ospanov proved that if a tracial subalgebra $\mathcal{A}$ has L_E -factorization, then $\mathcal{A}$ is a subdiagonal algebra, where E is a symmetric quasi Banach space on $[0,1]$.

We continue this line of investigation. The aim of this paper is to prove some characterizations of subdiagonal algebras of semifinite von Neumann algebras. We will define the semifinite version of tracial subalgebras of semifinite von Neumann algebras.

This paper is organized as follows. Section 1 contains some preliminary definitions. In section 2, we prove that if a tracial subalgebra $\mathcal{A}$ has the unique normal state extension property and τ -maximal or satisfies L_2-density, then $\mathcal{A}$ is a subdiagonal algebra.

2. Preliminaries

We use standard notation and notions from noncommutative L_P-spaces theory (see e.g. [5, 8]).Throughout this paper, we denote by $\mathcal{M}$ a semifinite von Neumann algebra on the Hilbert space $\mathcal{H}$ with a normal faithful semifinite trace t. A closed densely defined linear operator x in $\mathcal{H}$ with domain $D(x)$ is said to be affiliated with $\mathcal{M}$ if and only if $u^*xu = x$ for all unitary operators u which belong to the commutant $\mathcal{M}'$ of $\mathcal{M}$. If x is affiliated with $\mathcal{M}$, then x is said to be τ –measurable if for every $\varepsilon > 0$ there exists a projection $\varepsilon \in \mathcal{M}$ such that $e(\mathcal{H}) \sqsubseteq D(x)$ and $\tau(e^{\perp}) < \varepsilon$ (where for any projection e we let $e^{\perp} = 1 - e$). The set of all τ -measurable operators will be denoted by $L_0(\mathcal{M})$. The set $L_0(\mathcal{M})$ is a *-algebra with sum and product being the respective closure of the algebraic sum and product. For a positive self-adjoint operator $x = \int_0^\infty \lambda de_\lambda$ (the spectral decomposition) affiliated with $\mathcal{M}$, we set

$$\tau(x) = \sup_n \tau\,(\int_0^n \lambda de_\lambda = \int_0^\infty \lambda \tau(e_\lambda)$$

For $0 < p < \infty$, $L_p(\mathcal{M})$ is defined as the set of all τ -measurable operators x affiliated with $\mathcal{M}$

such that

$$\|x\|_p = \tau(|x|^p)^{1/p} < \infty.$$

In addition, we put $L_\infty(\mathcal{M}) = \mathcal{M}$ and denote by $\|\cdot\|_\infty (= \|\cdot\|)$, the usual operator norm. It is well-known that $L_p(\mathcal{M})$ is a Banach space under $\|\cdot\|_p$ $(1 < p < \infty)$ satisfying all the expected properties such as duality.

In this paper, $[K]_p$ denotes the closed linear span of K in $L_p(\mathcal{M})$ (relative to the w*-topology for $p = \infty$) and $J(K)$ is the family ofthe adjoints ofthe elements of K .

Henceforth we will assume that $\mathcal{D}$ is a von Neumann subalgebra of M such that the restriction of t to $\mathcal{D}$ is still semifinite. Let $\mathcal{E}$ be the (unique) normal positive faithful conditional expectation of $\mathcal{M}$ with respect to $\mathcal{D}$ such that $\tau \circ \mathcal{E} = \tau$.

Definition 2.1. $\mathcal{A}$ w*-closed subalgebra $\mathcal{A}$ of $\mathcal{M}$ is called a subdiagonal algebra of $\mathcal{M}$ with respect to $\mathcal{E}$ (or $\mathcal{D}$) if

(i) $\mathcal{A} + \mathcal{J}(\mathcal{A})$ is w*-dense in $\mathcal{M}$,

(ii) $\mathcal{E}(xy) = \mathcal{E}x)\mathcal{E}(y), \forall x, y \in \mathcal{A}$.

(iii) $\mathcal{A} \cap \mathcal{J}(\mathcal{A}) = \mathcal{D}$.

$\mathcal{D}$ is then called the diagonal of $\mathcal{A}$.

It is proved by Ji [6] that a semifinite subdiagonal algebra $\mathcal{A}$ is automatically maximal, i.e., A is not properly contained in any other subalgebra of $\mathcal{M}$ which is subdiagonal algebra respect to $\mathcal{E}$.

Since $\mathcal{D}$ is semifinite, we can choose an increasing family of $\{e_i\}_{i\in I}$ of τ-finite projections in $\mathcal{D}$ such that $e_i \to 1$ strongly, where 1 is identity of $\mathcal{M}$ (see Theorem 2.5.6 in [9]). Throughout, the $\{e_i\}_{i\in I}$ will be used to indicate this net.

Let $\mathcal{B}$ be a von Neumann subalgebra of $\mathcal{M}$ such that the restriction of T to $\mathcal{B}$ is still semifinite, and let $\mathcal{N}$ be a subset of $\mathcal{M}$ containing $\mathcal{B}$. We call subset $\mathcal{N}$ is $\mathcal{B}$ -invariant, if $\mathcal{B}\mathcal{N}\mathcal{B} \sqsubseteq \mathcal{N}$. We call $\Phi: \mathcal{N} \to \mathcal{B}$ is 'conditional expectation', if $\Phi(\mathrm{asb}) = \mathrm{a}\Phi(\mathrm{s})\mathrm{b}$ for all *a.b* G ***B***.*s* G $a, b \in \mathcal{B}, s \in \mathcal{N}$. We say that $\Phi: \mathcal{N} \to \mathcal{B}$ is normal if for any net $\{x_\propto\}_{\propto\in\Lambda} \subset \mathcal{N}$ with $\sup_{\propto\in\Lambda} \Phi(\mathrm{x}_\propto)$, the following equality holds: $\Phi(\sup_{\propto\in\Lambda}\mathrm{x}_\propto) = \sup_{\propto\in\Lambda}\Phi(\mathrm{x}_{\propto)}$.

Lemma 2.2. *Let $\mathcal{N}$ be a weak*-closed $\mathcal{B}$ -invariant subset of $\mathcal{M}$, and let $\Phi: \mathcal{N} \to \mathcal{B}$ be a normal 'conditional expectation', which is preserved by τ. Then $\Phi(a) = a$ for all* a G $a \in \mathcal{B}$, *and* $\Phi \circ \Phi = \Phi$.

Proof. Let e be a τ -finite projection in $\mathcal{B}$, we let

$\mathcal{M}_e = e\mathcal{M}e, \mathcal{N}_e = e\mathcal{N}e, \mathcal{B}_e = e\mathcal{B}e,$

and Φ_e be the restriction of Φ to $\mathcal{N}_e$. Then $\mathcal{N}_e$ is a weak*-closed $\mathcal{B}_e$-invariant subset of $\mathcal{M}_e$, and Φ_e is a normal 'conditional expectation'. Hence, we have that

$$\begin{aligned}\tau(|\Phi_e(\mathrm{e}) - \mathrm{e}|^2) &= \tau((\Phi_e(\mathrm{e}) - \mathrm{e})^*(\Phi_e(\mathrm{e}) - \mathrm{e})) = \tau((\Phi_e(\mathrm{e})^* - \mathrm{e})(\Phi_e(\mathrm{e}) - \mathrm{e}))\\ &= \tau(\Phi_e(\mathrm{e})^*\Phi_e(\mathrm{e})) - \tau(\Phi_e(\mathrm{e})^*\mathrm{e}) - \tau(\mathrm{e}\Phi_e(\mathrm{e}) + \tau(\mathrm{e})\\ &= \tau(\Phi_e(\Phi_e(\mathrm{e})^*\mathrm{e})) - \tau(\Phi_e(\mathrm{e})^*\mathrm{e}) - \tau(\Phi_e(\mathrm{e})) + \tau(e)\\ &= \tau(\Phi_e(\mathrm{e})^*\mathrm{e}) - \tau(\Phi_e(\mathrm{e})^*\mathrm{e}) - \tau(e) + \tau(e) = 0\end{aligned}$$

From faithfulness of τ, it follows that $\Phi_e(e) = e$. Since $\mathcal{B}$ is semifinite, we can choose an increasing family of $\{e_\propto\}_{\propto\in\Lambda}$ of τ-finite projections in $\mathcal{B}$ such that $e_i \to 1$ strongly. Therefore,

$$\Phi(1) = \Phi\left(\sup_{\propto\in\Lambda} e_\propto\right) = \sup_{\propto\in\Lambda} \Phi(e_\propto) = \sup_{\propto\in\Lambda} \Phi_{e_\propto}(e_\propto) = \sup_{\propto\in\Lambda} e_\propto = 1$$

From this follows that

$\Phi(\mathrm{a}) = \Phi(\mathrm{a}1) = a\Phi(1) = a$, for all $\propto\in \mathcal{B}$

and

$\Phi(\Phi(\mathrm{x})) = \Phi(\Phi(\mathrm{x})1) = \Phi(\mathrm{x})\Phi(1) = \Phi(\mathrm{x})$, for all $x \in \mathcal{N}$.

□

Lemma 2.3. *There is at most one normal 'conditional expectation' from any weak*-closed $\mathcal{B}$ -invariant subset $\mathcal{N}$ of $\mathcal{M}$ containing $\mathcal{B}$ onto $\mathcal{B}$, which is preserved by τ.*

Proof. Suppose that Φ, Ψ, T are normal conditional expectations of $\mathcal{N}$ onto $\mathcal{B}$, which is preserved by τ. Let $\{e_\propto\}_{\propto\in\Lambda}$ be an increasing family of τ-finite projections in $\mathcal{B}$ such that $e_\alpha \to 1$ strongly. Then using the conditional expectation property we have for $x \in \mathcal{N}$ and $\alpha \in \mathcal{A}$ that

$$\begin{aligned}\tau(|\Phi(e_\alpha x e_\alpha) - \Psi(e_\alpha x e_\alpha)|^2) &= \tau(\big(\Phi(e_\alpha x e_\alpha) - \Psi(e_\alpha x e_\alpha)\big)^*\big(\Phi(e_\alpha x e_\alpha) - \Psi(e_\alpha x e_\alpha)\big)\big) \\ &= \tau\big(\Phi(e_\alpha x e_\alpha)^*\Phi(e_\alpha x e_\alpha)\big) - \tau\big(\Psi(e_\alpha x e_\alpha))^*\Phi(e_\alpha x e_\alpha)\big) - \tau\big(\Phi(e_\alpha x e_\alpha)^*\Psi(e_\alpha x e_\alpha)\big) \\ &\quad + \tau\big(\Psi(e_\alpha x e_\alpha)^*\Psi(e_\alpha x e_\alpha)\big) \\ &= \tau(\Phi(e_\alpha x e_\alpha)^* e_\alpha x e_\alpha) - \tau(\Psi(e_\alpha x e_\alpha))^* e_\alpha x e_\alpha) - \tau(\Phi(e_\alpha x e_\alpha)^* e_\alpha x e_\alpha) \\ &\quad - \tau(\Psi(e_\alpha x e_\alpha))^* e_\alpha x e_\alpha) = 0\end{aligned}$$

Hence $\Phi(e_\alpha x e_\alpha) = \Psi(e_\alpha x e_\alpha)$, so $e_\alpha \Phi(\mathrm{x}) e_\beta = e_\alpha e_\beta \Phi(\mathrm{x}) e_\beta = e_\alpha \Psi(\mathrm{x}) e_\beta e_\alpha e_\beta \Psi(\mathrm{x}) e_\beta$ for any $\alpha \leq \beta$. Therefore, for any $\xi \in \mathcal{H}$, we have that

$e_\alpha \Phi(\mathrm{x})\xi = \lim_{\beta\in\Lambda} e_\alpha \Phi(\mathrm{x}) e_\beta \xi = \lim_{\beta\in\Lambda} e_\alpha \Psi(\mathrm{x}) e_\beta \xi = e_\alpha \Psi(\mathrm{x})\xi.$

It follows that $e_\alpha \Phi(\mathrm{x}) = e_\alpha \Psi(\mathrm{x})$, so $\Phi = \Psi$.

Definition 2.4. A weak*-closed subalgebra $\mathcal{A}$ of $\mathcal{M}$ is called a tracial subalgebra of $\mathcal{M}$ with respect to Φ(or $\Delta = \mathcal{A} \cap \mathcal{J}(\mathcal{A})$)) if

(i) $\Delta(\mathcal{A})$ is semifinite,

(ii) $\Phi : \mathcal{A} \to \Delta(\mathcal{A})$ is a normal homomorphism,

(iii) $\tau(x) = \tau\big(\Phi(\mathrm{x})\big), \forall \mathrm{x} \in \mathcal{A}$.

We claim that if $\mathcal{A}$ is a tracial subalgebra of a von Neumann algebra $\mathcal{M}$, then the map Φ in Definition 2.4 is unique normal homomorphism. Indeed, the conditional expectation $\mathcal{E}$ from $\mathcal{M}$ onto $\Delta(\mathcal{A})$ restricts to a normal 'conditional expectation' from $\mathcal{A}$ onto $\Delta(\mathcal{A})$. Clearly Φ is a normal 'conditional expectation' from $\mathcal{A}$ onto $\Delta(\mathcal{A})$. The claim then follows by Lemma 2.3. Hence we may to write Φ as $\mathcal{E}$ and write $\Delta(\mathcal{A})$ as $\mathcal{D}$. Therefore, A tracial subalgebra $\mathcal{A}$ of a von Neumann algebra $\mathcal{M}$ is a subdiagonal algebra of $\mathcal{M}$ if and only if $\mathcal{A} + \mathcal{J}(\mathcal{A})$ is w*-dense in $\mathcal{M}$.

It is well-known that E extends to a contractive projection from $L_p(\mathcal{M})$ onto $L_p(\mathcal{D})$ for every $1 \leq p \leq \infty$. The extension will still be denoted by $\mathcal{E}$.

Let $\mathcal{A}_0 = \mathcal{A} \cap \ker(\mathcal{E})$. We call $\mathcal{A}$ is τ-maximal, if

$$\mathcal{A} = \{x \in \mathcal{M}: \tau(xy) = 0, \forall y \in \mathcal{A}_0\}.$$

We say that a tracial subalgebra $\mathcal{A}$ of $\mathcal{M}$ satisfies L_2- density, if $\mathcal{A} \cap L_2(\mathcal{M}) + \mathcal{J}(\mathcal{A}) \cap L_2(\mathcal{M})$ is dense in $L_2(\mathcal{M})$ in the usual Hilbert space norm on that space.

Given a projection *e* in $\mathcal{D}$, we let

$$\mathcal{M}_e = e\mathcal{M}e,\ \mathcal{A}_e = e\mathcal{A}e, \mathcal{D}_e = e\mathcal{D}e,$$

and $\mathcal{E}_e$ be the restriction of $\mathcal{E}$ to $\mathcal{M}_e$. Then we have the following results:

Lemma 2.5. Let $\mathcal{A}$ be a tracial subalgebra of $\mathcal{M}$ with respect to $\mathcal{D}$ and let *e* be a projection in $\mathcal{D}$. We have that

(i) $\mathcal{A}_e$ is a tracial subalgebra of $\mathcal{M}$ e with respect to $\mathcal{E}_e$ (or $\mathcal{D}_e$).

(ii) $(\mathcal{A}_e)_0 = e\mathcal{A}_0 e$

(iii) If $\mathcal{A}$ is τ-maximal, then $\mathcal{A}_e$ is τ -maximal.

(iv) If $\mathcal{A}$ satisfies L_2- density, then $\mathcal{A}_e$ satisfies L_2- density.

Proof. Using the methods as in the proof (i) and (ii) of Lemma 3.1 in [2] we obtain (i) and (ii).

(iii) It is clear that $\mathcal{A}_e \subseteq \{x \in \mathcal{M}_e : \tau(xa) = 0, \forall a \in (\mathcal{A}_e)_0\}$. Conversely, let $x \in \mathcal{M}_e$ and $\tau(xa) = 0$ for all $a \in (\mathcal{A}_e)_0\}$. Then

$$\tau(xy) = \tau(exey) = \tau(xeye) = 0, y \in \mathcal{A}_0 .$$

Hence, $x \in \mathcal{A}$, since $\mathcal{A}$ is τ -maximal. So $x \in \mathcal{A}_e$.

(iv) From (i) and (ii) follow that $[(\mathcal{A}_e)_0]_2 = e[\mathcal{A}_0 \cap L_2(\mathcal{M})]_2 e, [(\mathcal{J}(\mathcal{A}_e))_0]_2 = e[\mathcal{J}(\mathcal{A}_0) \cap L_2(\mathcal{M})]_2 e$ and $[\mathcal{D}_e]_2 = e[\mathcal{D}_0 \cap L_2(\mathcal{M})]_2 e$. On the other hand, $L_2(\mathcal{M}_e) = eL_2(\mathcal{M})e$.

Hence, $\mathcal{A}_e + \mathcal{J}(\mathcal{M})$ is dense in $L_2(\mathcal{M}_e)$ in the usual Hilbert space norm on that space.

3. Characterizations of subdiagonal algebra

Proposition 3.1. Let $\mathcal{A}$ be a tracial subalgebra of $\mathcal{M}$. Then the following conditions are equivalent:

(i) $\mathcal{A}$ is a subdiagonal algebra of $\mathcal{M}$.

(ii) For any $i \in I$, $\mathcal{A}_{e_i}$ is a subdiagonal algebra of $\mathcal{M}_{e_i}$.

Proof. (i)⇒(ii) follows from (i) of Lemma 3.1 in [2].

(ii) ⇒ (i) Since $e_i \longrightarrow 1$ strongly, we get $\lim_i \|xe_i - x\|_1 = 0$ and $\lim_i \|e_i x - x\|_1 = 0$ for any $x \in L_1(\mathcal{M})$ (cf. Lemma 2.3 in [7]). Hence, for any $y \in \mathcal{M}$, we have that $\lim_i |\tau((y - e_i y e_i)x)| \le \lim_i \left|\tau\big((y - ye_i)x\big)\right| + \lim_i \left|\tau\big((ye_i - e_i y e_i)x\big)\right| \le \|y\|_\infty (\lim_i \|x - e_i x\|_1 + \lim_i \|e_i(x - xe_i)\|_1 = 0.$

Thus $\cup_{i\in I} \mathcal{M}_{e_i}$ is weak*-dense in $\mathcal{M}$. On the other hand, $\mathcal{A}_{e_i} + \mathcal{J}(\mathcal{A}_{e_i})$ is weak*- dense in $\mathcal{M}_{e_i} (\forall i \in I)$ So $\cup_{i\in I} (\mathcal{A}_{e_i} + \mathcal{J}(\mathcal{A}_{e_i})$ is weak*-dense in $\mathcal{M}$. Therefore $\mathcal{A} + \mathcal{J}(\mathcal{A})$ is weak*-dense in $\mathcal{M}$, i.e., $\mathcal{A}$ is a subdiagonal algebra of $\mathcal{M}$.

Definition 3.2. Let $\mathcal{A}$ be a tracial subalgebra of $\mathcal{M}$ with respect to $\mathcal{D}$. We say that $\mathcal{A}$ has the unique normal state extension property if it satisfies:

If $x \in L_1(\mathcal{M})_+$ and $\tau(xa) = 0$ for all $a \in \mathcal{A}_0$, then $x \in L_1(\mathcal{D})$.

Remark 3.3. In [4], for a tracial subalgebra $\mathcal{A}$ of finite von Neumann algebra $\mathcal{M}$, the unique normal state extension property is defined by the following condition:

If $x \in L_1(\mathcal{M})_+$ and $\tau(xa) = \tau(a)$ for all $a \in \mathcal{A}$, then $x \in 1$.

By Lemma 4.1 in [4], this definition is equivalent to our definition of the unique normal state extension property.

Lemma 3.4. *Let $\mathcal{A}$ be a tracial subalgebra of $\mathcal{M}$ with respect to $\mathcal{D}$. Then the following conditions are equivalent:*

(i) *$\mathcal{A}$ has the unique normal state extension property.*

(ii) *For any $i \in I$, $\mathcal{A}_{e_i}$ has the unique normal state extension property.*

Proof. (i) ⇒ (ii) Let $i \in I$. If $x \in L_1\big(\mathcal{M}_{e_i}\big)_+$ and $\tau(xa) = 0$ for all $a \in (\mathcal{A}_{e_i})_0$. By (ii) of Lemma 2.5, we have that $\tau(e_i x e_i a) = 0$ for all $a \in (\mathcal{A})_0$. Hence $x \in L_1(\mathcal{D})$, so $x \in L_1(\mathcal{D}_{e_i})$.

(ii) ⇒(i) If $x \in L_1(\mathcal{M})_+$ and $\tau(xa) = 0$ for all $a \in \mathcal{A}_0$, then $\tau(xe_i a e_i) = 0$ for all $a \in \mathcal{A}_0$ and $i \in I$. It follows that $\tau(e_i x e_i a) = 0$ for all $a \in (\mathcal{A}_{e_i})_0$ and $i \in I$. Hence $e_i x e_i \in L_1(\mathcal{D}_{e_i})$, for all $i \in I$. Since $e_i x e_i \longrightarrow x$ in norm in $L_1(\mathcal{M})$, we conclude that $x \in L_1(\mathcal{D})$,. □

Theorem 3.5. *Let $\mathcal{A}$ be a tracial subalgebra of $\mathcal{M}$ with respect to* D. *Then the following conditions are equivalent:*

(i) *$\mathcal{A}$ is a subdiagonal algebra of $\mathcal{M}$.*

(ii) *$\mathcal{A}$ is a τ -maximal tracial subalgebra of $\mathcal{M}$ satisfying the unique normal state extension property.*

(iii) *$\mathcal{A}$ satisfies* L_2*-density and the unique normal state extension property.*

Proof. (i) ⇒ (ii), (iii) are trivial.

(ii) ⇒ (i) Let $i \in I$. By Lemma 2.5 and 3.4, we know that $\mathcal{A}_{e_i}$ is a τ-maximal tracial subalgebra of $\mathcal{M}_{e_i}$ satisfying the unique normal state extension property. Using Theorem 1.1 in [4] we obtain that $\mathcal{A}_{e_i}$ is a subdiagonal algebra of $\mathcal{M}_{e_i}$. So by Proposition 3.1, it follows that $\mathcal{A}$ is a subdiagonal algebra of $\mathcal{M}$.

(iii) ⇒ (i) Similar to the above, we use Theorem 1.1 in [4], Lemma 2.5 and 3.4 and Proposition 3.1 to obtain the desired result.

Acknowledgement. The authors are grateful to the referee for his/her valuable comments. This work is supported by pro ject 3606/GF4 of Science Committee of Ministry of Education and Science of Republic of Kazakhstan.

Literature

1. W.B. Arveson, *Analyticity in operator algebras,* Amer. J. Math. **89** (1967), 578-642.
2. T. N. Bekjan, *Noncommutative Hardy space associated with semi-finite subdiagonal algebras*, J. Math. Anal. Appl. **429**(2015), 1347-1369.
3. T. N. Bekjan and K.N. Ospanov, *Factorization properties of subdiagonal algebras*, Funkt-sional. Anal. i Prilozhen. **50** (2016),no.2, 77-81; English transl. in Functional Anal. Appl. **50**(2016), no.2, 146-149.
4. D. P. Blecher and L. E. Labuschagne, *Characterization of noncommutaive H/'"*, Integr. Equ. Oper. Theory **56** (2006), 301-321.
5. T. Fack and H. Kosaki, *Generalized s-numbers of τ-measurable operators*, Pac. J. Math **123** (1986), 269-300.
6. G. Ji, *Maximality of semifinite subdiagonal algebras, Journal of Shaanxi normal university (natural science edition)* **28** (2000), no.1, 15-17.
7. M. Junge, *Doob's inequality for noncommutative martingales*, J. Reine. Angew. Math. **549** (2002), 149-190.
8. G. Pisier and Q. Xu, *Noncommutative L^p-spaces, In Handbook of the geometry of Banach spaces*, **Vol. 2** (2003), 1459-1517.
9. S. Sakai, *C*-algebras and W*-algebras*, Springer-Verlag, New York, 1971.

[1] L. N. Gumilyov Eurasian national University, Astana 010008, Kazakhstan. *E-mail address*: bekjant@yahoo.com [2] L. N. Gumilyov Eurasian national University, Astana 010008, Kazakhstan. *E-mail address*: azhar oshanova@mail.ru

УДК 517.51

ВЕСОВАЯ АДДИТИВНАЯ ОЦЕНКА ОДНОГО КЛАССА ИНТЕГРАЛЬНЫХ ОПЕРАТОРОВ

Айманбетова Бибинур Дауренкызы
b.aimanbetova@mail.ru
Магистрант 2-го курса по специальности «6М060100-Математика»
маханико-математического факультета
ЕНУ им. Л.Н.Гумилева, Астана, Казахстан
Научный руководитель – Темирханова А.М.

Пусть $R=(0;+\infty)$ и $u(\cdot), v(\cdot), \rho(\cdot)$ и $k_i(\cdot), i=1,2,..,n-1$ – весовые функции на R_+, т.е. неотрицательные локально интегрируемые на R_+функции. Пусть K_n интегральный оператор вида

$$K_n f(x)=\int_0^x K_{n-1,1}(x,t)f(t)dt \tag{1}$$

где функция $K_{n-1,1}(x,t)$ является элементом следующего семейства:

$$K_{j,i}(x,t)=\int_t^x k_j(t_j)\int_{t_j}^x k_j(t_{j-1})\dots\int_{t_{i+1}}^x k_i(t_i)dt_i dt_{i+1}\dots dt_j,$$