



On the decay widths of radially excited scalar meson $K_0^*(1430)$ in view of new experimental data

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Abstract Decays of scalar mesons $K_0^*(800) \rightarrow K\pi$ and $K_0^*(1430) \rightarrow K\pi, K\eta, K\eta', K_1\pi$ are described in the extended $U(3) \times U(3)$ Nambu–Jona–Lasinio chiral quark model. The obtained results are in satisfactory agreement with the new experimental data obtained by the BaBar collaboration, which markedly differ from the existing values in the PDG.

1 Introduction

The description of scalar mesons in the ground and excited states is of great interest. In view of recent experimental data the study of the strange scalar meson K_0^* in the ground and first radially excited states is especially interesting.

The scalar meson $K_0^*(800)$ decays into a $K\pi$ pair with 100% probability [1]. At the same time, for the decay of the radially excited $K_0^*(1430)$ meson, the main channels are $K\pi$, $K\eta$ and $K\eta'$ [1]. The decay $K_0^*(1430) \rightarrow K\pi$ was first measured with sufficient accuracy in the study of the reaction $Kp \rightarrow K\pi n$ [2]. In a recent paper by the BaBar collaboration the decays of the charmed pseudoscalar meson η_c were measured, where a detailed analysis of the data showed the decisive role of channels with scalar mesons [3]. This made it possible for the first time to measure the ratio of the branching fractions for the $B(K_0^*(1430) \rightarrow \eta'K)/B(K_0^*(1430) \rightarrow \pi K)$ decays and to estimate the coupling constants $g_{\eta'K}$, $g_{\pi K}$. In this case, the $K_0^*(1430) \rightarrow K\pi$ decay widths obtained using the latter constant turns out to be noticeably smaller than the existing data in PDG [1]. Such discrepancy makes the issue of a the-

oretical estimation of the $K_0^*(1430)$ meson decay widths of topical.

In the present paper, we calculate the decay widths of radially excited scalar meson $K_0^*(1430) \rightarrow K\pi$, $K_0^*(1430) \rightarrow K\eta$, $K_0^*(1430) \rightarrow K\eta'$ and $K_0^*(1430) \rightarrow K_1\pi$ within the extended $U(3) \times U(3)$ NJL chiral quark model [4–9]. Regarding the last decay, in [10] it was shown that the existing PDG data on the width of $K_1(1270) \rightarrow K_0^*(1430)\pi$ imply $K_0^*(1430) \rightarrow K_1(1270)\pi$. We agree with this statement and give an independent estimation for the decay width of $K_0^*(1430) \rightarrow K_1(1270)\pi$. However, the width $\Gamma(K_0^*(1430) \rightarrow K_1(1270)\pi) = 40$ keV obtained by us turns out to be noticeably smaller than the width $\Gamma(K_0^*(1430) \rightarrow K_1(1270)\pi) = 2075(+4100, -1100)$ MeV obtained in [10]. Our result confirms the fact that the $K_0^*(1430)$ meson width is practically exhausted by the $K\pi$, $K\eta$ and $K\eta'$ channels.

For the calculation of the $K_0^*(1430) \rightarrow [K\pi, K\eta, K\eta', K_1\pi]$ decays, we use the extended NJL model [4–7, 9]. The NJL chiral quark model successfully describes interactions of four meson nonets of scalar, pseudoscalar, vector, and axial-vector types in the ground and first radially excited states using a limited number of fixed parameters. In the model, the effective chiral quark-meson Lagrangians are obtained, and the processes of meson production in τ decays and e^+e^- annihilations, as well as numerous decays of radially excited mesons, are successfully described.

In our version of the $U(3) \times U(3)$ chiral NJL model, the value of the cutoff parameter is $\Lambda_4 = 1250$ MeV [9]. This makes it possible to include in the model, in addition to the 4-meson nonets in the ground states, also their first radially excited states, and in this case, one can hope to obtain satisfactory results at a qualitative level in the framework of partial chiral symmetry conservation. Such an attempt was made in the works [5–7, 11]. Radially excited states were described by

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introducing into the model the simplest form factor quadratic in the transverse momentum of quarks. In this case, the mixing of mesons in the radially excited state with the ground states was also taken into account, which leads to the appearance of off-diagonal terms in the free Lagrangian. These terms are diagonalized using mixing angles or a matrix in the case of $\eta, \eta', \eta(1295)$ and $\eta(1475)$ mesons. It is important to note that the slope parameter d introduced in the extended model is fixed without using experimental data, based on the requirement that the quark condensate remains unchanged after radially excited states are taken into account. In this case, the values of quark masses and the ultraviolet cutoff parameter do not change.

It is interesting to note that within the extended NJL model, the mass spectrum of the ground and excited scalar nonets was satisfactorily described (19 states of scalar mesons, taking into account mixings of five states: 4 scalar and one glueball) [6, 12].

2 Effective quark-meson Lagrangians of the NJL model

The quark-meson Lagrangian for the strong interaction of scalar, pseudoscalar and axial vector mesons necessary for describing the processes considered here in the NJL model takes the form [6, 7, 9, 11]

$$\begin{aligned} \mathcal{L}_{int} = & \bar{q} \left[i\gamma_5 A_\pi \sum_{i=\pm,0} \lambda_i^\pi \pi^i + i\gamma_5 A_K \sum_{i=\pm,0} \lambda_i^K K^i \right. \\ & + \frac{1}{2} \gamma_\mu \gamma_5 A_{K_1} \sum_{i=\pm,0} \lambda_i^K K_{1\mu}^i \\ & + \sum_{i=\pm,0} \lambda_i^K (A_{K_0^*} K_0^{*i} + B_{K_0^*} \hat{K}_0^{*i}) \\ & \left. + i\gamma^5 \sum_{i=u,s} \lambda_i [A_\eta^i \eta + A_{\eta'}^i \eta'] \right] q, \end{aligned} \tag{1}$$

where q and \bar{q} are u, d and s quark fields with constituent quark masses $m_u \approx m_d = 270$ MeV, $m_s = 420$ MeV; excited mesonic states of mesons are marked with a hat and λ are linear combinations of the Gell-Mann matrices [9]

$$\begin{aligned} \lambda_\pm^K &= \frac{\lambda_4 \pm i\lambda_5}{\sqrt{2}}, \quad \lambda_0^K = \frac{\lambda_6 + i\lambda_7}{\sqrt{2}}, \\ \lambda_\pm^\pi &= \frac{\lambda_1 \pm i\lambda_2}{\sqrt{2}}, \quad \lambda_0^\pi = \lambda_3, \\ \lambda_u &= \frac{\sqrt{2}\lambda_0 + \lambda_8}{\sqrt{3}}, \quad \lambda_s = \frac{-\lambda_0 + \sqrt{2}\lambda_8}{\sqrt{3}}. \\ A_M &= A_M^0 [g_M \sin \theta_M^+ + g'_M f_M(k_\perp^2) \sin \theta_M^-], \\ B_M &= -A_M^0 [g_M \cos \theta_M^+ + g'_M f_M(k_\perp^2) \cos \theta_M^-], \end{aligned} \tag{2}$$

Table 1 Mixing parameters of η mesons [6, 9]

	η	$\hat{\eta}$	η'	$\hat{\eta}'$
a_1^u	0.71	0.62	-0.32	0.56
a_2^u	0.11	-0.87	-0.48	-0.54
a_1^s	0.62	0.19	0.56	-0.67
a_2^s	0.06	-0.66	0.3	0.82

where $A_M^0 = 1/\sin(2\theta_M^0)$ and $\theta_M^\pm = \theta_M \pm \theta_M^0$. The subscript M indicates the corresponding meson; $\theta_\pi = 59.48^\circ$, $\theta_\pi^0 = 59.12^\circ$, $\theta_K = 58.11^\circ$, $\theta_K^0 = 55.52^\circ$, $\theta_{K_1} = 85.97^\circ$, $\theta_{K_1}^0 = 59.56^\circ$, $\theta_{K_0^*} = 74.0^\circ$ and $\theta_{K_0^*}^0 = 60.0^\circ$ are the mixing angles [9]. The mixing angles for the K and π mesons $\theta \approx \theta_0$; so for the ground states of these mesons one can use $A_\pi = g_\pi$ and $A_K = g_K$.

For the η mesons, the factor A takes a slightly different form. This is due to the fact that in the case of the η mesons four states are mixed

$$\begin{aligned} A_M^u &= g_{\eta^u} a_{1M}^u + g'_{\eta^u} a_{2M}^u f_{uu}(k_\perp^2), \\ A_M^s &= g_{\eta^s} a_{1M}^s + g'_{\eta^s} a_{2M}^s f_{ss}(k_\perp^2). \end{aligned} \tag{3}$$

Here $f(k_\perp^2) = (1 + dk_\perp^2) \Theta(\Lambda^2 - k_\perp^2)$ is the form-factor describing the first radially excited meson states. The slope parameters, $d_{uu} = -1.784 \times 10^{-6} \text{MeV}^{-2}$ and $d_{ss} = -1.737 \times 10^{-6} \text{MeV}^{-2}$, are unambiguously fixed from the condition of constancy of the quark condensate after the inclusion of radially excited states and depends only on the quark composition of the corresponding meson [9].

The values of the mixing (A) parameters are shown in Table 1. The η' meson corresponds to the physical state $\eta'(958)$ and the $\hat{\eta}, \hat{\eta}'$ mesons correspond to the first radial excitation mesons η and η' .

The quark-meson coupling constants have the form

$$\begin{aligned} g_\pi &= g_{\eta^u} = \left(\frac{4}{Z_\pi} I_{20} \right)^{-1/2}, \quad g'_\pi = g'_{\eta^u} = \left(4I_{20}^{f^2} \right)^{-1/2}, \\ g_{\eta^s} &= \left(\frac{4}{Z_{\eta^s}} I_{02} \right)^{-1/2}, \quad g'_{\eta^s} = \left(4I_{02}^{f^2} \right)^{-1/2}, \\ g_K &= \left(\frac{4}{Z_K} I_{11} \right)^{-1/2}, \quad g'_K = \left(4I_{11}^{f^2} \right)^{-1/2}, \\ g_{K_1} &= \left(\frac{2}{3} I_{11} \right)^{-1/2}, \\ g'_{K_1} &= \left(\frac{2}{3} I_{11}^{f^2} \right)^{-1/2}, \quad g_{K_0^*} = \left(4I_{11} \right)^{-1/2}, \\ g'_{K_0^*} &= \left(4I_{11}^{f^2} \right)^{-1/2}, \end{aligned} \tag{4}$$

here Z_π and Z_{η^s} are additional renormalization constants appearing in the pseudoscalar and axial-vector transitions [7, 9].

Integrals appearing in the quark loops are

$$I_{n_1 n_2}^{f^m} = -i \frac{N_c}{(2\pi)^4} \int \frac{f^m(k_\perp^2)}{(m_u^2 - k^2)^{n_1} (m_s^2 - k^2)^{n_2}} \Theta(\Lambda_3^2 - k_\perp^2) d^4k, \quad (5)$$

where $\Lambda_3 = 1030$ MeV is the three-dimensional cutoff parameter, the value of four-dimensional cutoff parameter is $\Lambda_4 = 1250$ MeV [7].

When describing decays involving K_1 axial vector mesons, we take into account the mixing effect of the K_{1A} and K_{1B} states [13, 14]. The mixing of the axial vector mesons K_{1A} and K_{1B} leads to physical states $K_1(1270)$ and $K_1(1400)$ [1]. This mixing is described as follows:

$$\begin{aligned} K_{1A} &= K_1(1270) \sin \alpha + K_1(1400) \cos \alpha, \\ K_{1B} &= K_1(1270) \cos \alpha - K_1(1400) \sin \alpha, \end{aligned} \quad (6)$$

where $\alpha = 57^\circ$ [9]. This effect was also considered in the works [15–19].

3 Amplitudes and decay widths

We start with considering the decay $K_0^*(800) \rightarrow K\pi$. This process is described by the quark diagram given in Fig. 1. Quark loops are calculated using the methods developed in the NJL model and successfully tested on other physical processes [7, 9]. The loop integrals are expanded in terms of the external fields momentums, and only the logarithmic divergent parts are preserved. Accounting for such terms makes it possible to preserve the chiral symmetry in the model [13]. Model calculations lead to the following formula for the decay width $K_0^*(800) \rightarrow K\pi$:

$$\Gamma(K_0^{*-} \rightarrow K^- \pi^0) = \frac{1}{2J_{K_0^*} + 1} \frac{(8m_s I_{11}^{K_0^* K \pi})^2 \sqrt{E_K^2 - M_{K_0^*}^2}}{2M_{K_0^*} 4\pi M_{K_0^*}}, \quad (7)$$

where

$$J_{K_0^*} = 0, \quad E_K^2 = \frac{M_{K_0^*}^2 + M_K^2 - M_{\pi^0}^2}{2}, \quad (8)$$

where the meson masses are taken from PDG [1].

The integral with mesons vertices $K_0^* K \pi$ takes the form:.

$$I_{11}^{K_0^* K \pi}(m_u, m_s) = -i \frac{N_c}{(2\pi)^4} \int \frac{B_{K_0^*}(k_\perp^2) A_K(k_\perp^2) A_\pi(k_\perp^2)}{(m_u^2 - k^2)(m_s^2 - k^2)} \Theta(\Lambda_3^2 - k_\perp^2) d^4k. \quad (9)$$

Integrals for other decays can be obtained in a similar way with the replacement of the corresponding vertices defined in (2) and (3).

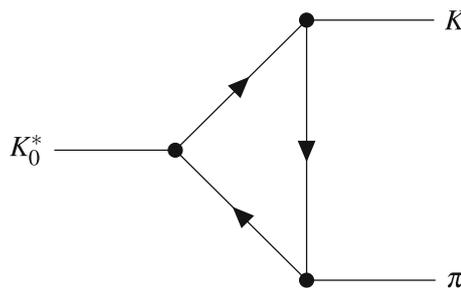


Fig. 1 Triangle quark diagram for the decay of scalar meson $K_0^* \rightarrow K\pi$

The decay amplitude of $K_0^{*-} \rightarrow K^0 \pi^-$ has a similar structure with an additional factor $\sqrt{2}$. As a result, for the width we obtain $\Gamma(K_0^{*-} \rightarrow K\pi) = 430$ MeV. Calculations in the standard NJL model lead to a close result $\Gamma(K_0^{*-} \rightarrow K\pi) = 450$ MeV. The experimental value for the width of this decay is $\Gamma(K_0^* \rightarrow K\pi)_{exp} = 468 \pm 30$ MeV [1].

To calculate the decay of a radially excited meson $K_0^*(1430) \rightarrow K\pi$ in the obtained amplitude (7), we replace the vertex $K_0^*(800) \rightarrow K_0^*(1430)$. As a result, for the width and decay constant $g_{K\pi}$ in the extended NJL model, we obtain

$$\begin{aligned} \Gamma(K_0^*(1430) \rightarrow K\pi)_{NJL} &= 18.543 \text{ MeV}, \\ g_{K\pi}^2 &= 0.515 \text{ GeV}^2. \end{aligned} \quad (10)$$

The model predictions for this decay can be compared with the data of the BaBar collaboration derived from the analysis of the processes $\gamma\gamma \rightarrow \eta_c \rightarrow (\pi, \eta') K K$ [3]

$$g_{K\pi}^2 = 0.458 \pm 0.032_{stat} \pm 0.044_{sys} \text{ GeV}^2. \quad (11)$$

As we can see, the results of the calculation in the NJL model for the value of the constant agree satisfactorily with the new experimental data. The decay width obtained using the experimental value of the constant $g_{K\pi}$ is equal to $\Gamma(K_0^*(1430) \rightarrow K\pi) = 16.46 \pm 1.15$ MeV at the mass $M_{K_0^*(1430)} = 1425 \pm 50$ MeV. It turns out to be less than the width $\Gamma(K_0^*(1430) \rightarrow K\pi) = 251.10 \pm 27.0$ MeV [2], which is given in PDG.

Our calculations show that the decay width of the radially excited meson $K_0^*(1430) \rightarrow K\pi$ is noticeably smaller than the width in the ground state $K_0^*(800) \rightarrow K\pi$. Note that a similar situation was observed in the NJL model when describing the decays $\rho \rightarrow 2\pi$ and $\rho' \rightarrow 2\pi$ [6].

Next, we consider the decays of $K_0^*(1430)$ with the production of meson pairs $K\eta$ and $K\eta'$. Here it is necessary to take into account both the u, d and s quark parts of these mesons. As a result, we obtain the following amplitudes

$$\mathcal{M}(K_0^*(1430) \rightarrow K\eta) = 8m_s I_{11}^{\hat{K}_0^* K \eta_u} - 8\sqrt{2}m_u I_{11}^{\hat{K}_0^* K \eta_s}, \quad (12)$$

$$\mathcal{M}(K_0^*(1430) \rightarrow K\eta') = 8m_s I_{11}^{\hat{K}_0^* K\eta'_u} - 8\sqrt{2}m_u I_{11}^{\hat{K}_0^* K\eta'_s}. \quad (13)$$

Numerical estimates lead to the following values for the width and decay constants in the NJL model

$$\begin{aligned} \Gamma(K_0^*(1430) \rightarrow K\eta)_{NJL} &= 0.291 \text{ MeV}, \\ g_{K\eta}^2 &= 0.030 \text{ GeV}^2, \quad g_{K\eta'}^2 = 0.671 \text{ GeV}^2. \end{aligned} \quad (14)$$

In the work, [3] the BaBar collaboration presented

$$\frac{g_{K\eta'}^2}{g_{K\pi}^2} = 1.50 \pm 0.24_{\text{stat}} \pm 0.24_{\text{sys}}. \quad (15)$$

For this ratio in the NJL model we get $g_{K\eta'}^2/g_{K\pi}^2 = 1.30$. These results can be considered satisfactory within the experimental and model accuracies. The model precision is estimated as $\pm 15\%$ based on the statistical analysis of previous numerous calculations and partial axial current conservation (PCAC) [9]. For the decay widths ratio $R = \Gamma(K_0^*(1430) \rightarrow K\eta)/\Gamma(K_0^*(1430) \rightarrow K\pi)$ taking into account the model error we can obtain the estimate $R = 0.016 \pm 0.005$. This is much lower than the values obtained by the BaBar collaboration $R = 0.092 \pm 0.025$ [20]. Here we can claim only a qualitative description of the scalar meson decay $K_0^*(1430) \rightarrow K\eta$. The discrepancies between the results for the ratio R obtained in the NJL model with experimental data BaBar are possibly a consequence of the lack of the tetraquark part consideration of the $K_0^*(1430)$ meson in our model. Note that a similar situation takes place in the case of the decay $f_0(980) \rightarrow \pi\pi$, where, without taking into account the tetraquark component, an underestimated value for the decay width was obtained [21,22].

Next, consider the decay $K_1(1270) \rightarrow K_0^*(1430)\pi$. This process is possible in the case of $M_{K_1(1270)} > M_{K_0^*(1430)} + M_\pi$, which is unlikely and can be due to the large width of the mesons $K_1(1270)$ and $K_0^*(1430)$ and uncertainties in mass definitions $M_{K_0^*(1430)} = 1425 \pm 50$ MeV and $M_{K_1(1270)} = 1253 \pm 7$ MeV [1]. In [10], the decay $K_0^*(1430) \rightarrow K_1\pi$ is described, which is a reversal reaction of $K_1(1270) \rightarrow K_0^*(1430)\pi$, and a relatively large width $\Gamma = 2075(+4100, -1100)$ MeV is obtained. This width turns out to be wider than the meson width $\Gamma_{K_0^*(1430)} = 270 \pm 80$ MeV and does not correspond to the fact that the $K_0^*(1430)$ meson predominantly decays into $K\pi$ and $K\eta$ [1]. However, the authors did not attempt to give an exact value for the width but showed a discrepancy of the process $K_1(1270) \rightarrow K_0^*(1430)\pi$ given in the PDG. This decay in the NJL model is described by the amplitude

$$\mathcal{M}(K_0^*(1430) \rightarrow K_1\pi) = 2 \sin \alpha I_{11}^{\hat{K}_0^* K_1\pi} (p_{K_0^*} + p_\pi)_\mu \epsilon_\mu(p_{K_1}), \quad (16)$$

where $\epsilon_\mu(p_{K_1})$ is the polarization vector of the K_1 meson with the momentum p_{K_1} ; $p_{K_0^*}$ and p_π are the momenta of the pion and scalar meson $K_0^*(1430)$. Accordingly, in the model, we obtain the decay width

$$\Gamma(K_0^*(1430) \rightarrow K_1\pi)_{NJL} = 40 \text{ keV}. \quad (17)$$

4 Conclusions

In this paper, we have described the decay channels of scalar mesons in the ground $K_0^*(800)$ and first radially excited state $K_0^*(1430)$. Our results for the decays of the excited meson $K_0^*(1430)$ are in satisfactory agreement with the recent experimental data of the BaBar collaboration [3] and at the same time deviate noticeably from the PDG data [1].

At the present time, in describing scalar mesons, an important role is played by the assumption of the tetraquark structure of scalar mesons [23]. This especially concerns isovector scalar mesons a_0 , where it is impossible to correctly describe the mass $M_{a_0(980)} = 980 \pm 20$ MeV without the assumption of a tetraquark structure. At the same time, the masses of the isoscalar mesons $f_0(980)$, as well as strange mesons K_0^* , both in the ground and first radially excited states, are quite satisfactorily described based on the quark-antiquark structure [6]. This allows us to assume in this paper that the strange mesons $K_0^*(800)$ and $K_0^*(1430)$ mainly have a quark-antiquark structure. We also admit the possibility of the existence of a tetraquark structure in the definitions of $K_0^*(800)$. However, in the case of strange mesons, this part does not play a leading role. And here we confirm the assumption about the predominant role of $\bar{q}q$ states made in [22], where the relative roles of tetraquark and $\bar{q}q$ structures of scalar mesons were studied. The presence of quark-antiquark structures in other scalar mesons also follows from the existence of chiral symmetry. There are a number of works that evaluate the relative role of quark-antiquark and tetraquark states in determining the $a_0(980)$ structure [22,24,25].

As regards the $K_1(1270) \rightarrow K_0^*(1430)\pi$ decay, we agree with the statement of the authors of [10] that the existing PDG data on the partial width of the $K_1(1270) \rightarrow K_0^*(1430)\pi$ imply the process $K_0^*(1430) \rightarrow K_1(1270)\pi$. In addition, we give an independent estimate of the $K_0^*(1430) \rightarrow K_1(1270)\pi$ decay width, which turns out to be much smaller than the width obtained in [10] and gives a small contribution to the total width of $\Gamma_{K_0^*(1430)}$.

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