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Nonlocal Schrödinger-Maxwell-Bloch Equations

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Abstract. In this paper we research the (1+1)-dimensional system of Schrödinger-Maxwell-Bloch equations (NLS-MBE), which describes the optical pulse propagation in an erbium doped fiber and find PT-symmetric and reverse space-time Schrödinger-Maxwell-Bloch equations, i.e. the kinds of nonlocal Schrödinger-Maxwell-Bloch equations. In particular case, the system of Schrödinger-Maxwell-Bloch equations is integrable by the Inverse Scattering Method as shown in the work of M.Ablowitz and Z.Muslimani. Following this method we prove the integrability of the nonlocal system of Schrödinger-Maxwell-Bloch equations by Lax pairs. Also the explicit and different seed solutions are constructed by using Darboux transformation.

1. Introduction

Soliton theory is an important field of research of applied mathematics and physics which has applications in hydrodynamics, nonlinear optics, classical and quantum fields theories and etc.. It studies a special class of nonlinear partial differential equations having solutions that are solitons (waves) which behave like particles. Investigations of solitons have become one of the interesting and highly active areas for research in modern science and technology during the last several decades. In particular, many of the completely integrable nonlinear partial differential equations have already been established and investigated [1-7].

Among such integrable nonlinear systems, the system of NLS-MBE have a essential role. This coupled system of equations describes optical soliton propagation in fibers with resonant impurities and erbium-doped nonlinear systems and have a (1+1) dimension [7]. The coupled NLS-MBE system was proposed for the first time by Maimistov and Manykin [4] to treat ultra-short-pulse propagation in a light pipe with a two-level resonant medium with Kerr nonlinearity. Using the Darboux transformation, the (1+1)-dimensional Schrödinger- Maxwell-Bloch equations were analyzed in [7], where soliton and periodic solutions were constructed from various "seeds".

Recently, there has been a growing interest in the nonlocal integrable nonlinear partial differential equations in mathematical physics and soliton theory. Ablowitz and Musslimani first proposed the nonlocal nonlinear Schrödinger equation [8] and then many other nonlocal integrable equations such as nonlocal modified Korteweg-de Vries equation, nonlocal Davey-Stewartson equation, nonlocal sine-Gordon equation and so on have been found [8-10].



In order to solve the nonlinear equations many scientists use the symmetry theory [11] which is useful in both integrable and non-integrable systems. In 1980, Vinogradov and Krasilshchik first proposed the nonlocal symmetry, and then many scientists apply the nonlocal symmetry method to established a relation between the local equations and their corresponding nonlocal equations by selecting appropriate symmetry.

In this paper, we will focus on the nonlocal form of the Schrödinger- Maxwell-Bloch system of equations. The forms of the NLS-MBE system are

$$q_t(x, t) = i \left[\frac{1}{2} q_{xx}(x, t) + r(x, t) q^2(x, t) \right] + 2p(x, t), \quad (1)$$

$$r_t(x, t) = -i\delta \left[\frac{1}{2} r_{xx}(x, t) + q(x, t) r^2(x, t) \right] + 2\delta k(x, t), \quad (2)$$

$$p_x(x, t) = 2i\omega p(x, t) + 2\eta(x, t) q(x, t), \quad (3)$$

$$k_x(x, t) = -2i\delta\omega k(x, t) + 2\delta\eta(x, t) r(x, t), \quad (4)$$

$$\eta_x(x, t) = -q(x, t) k(x, t) - p(x, t) r(x, t), \quad (5)$$

where x and t represent the normalized distance and time respectively, $q(x, t)$ is the slowly varying envelope axial field, $p(x, t)$ is the measure of the polarization of the resonant medium, $\eta(x, t)$ represents the extent of the population inversion, the real parameter ω is a constant corresponding to the frequency. $q(x, t), p(x, t)$ are complex functions, $r(x, t) = \delta\bar{q}(x, t)$, $k(x, t) = \delta\bar{p}(x, t)$ and $\eta(x, t)$ is a real variable. This (1+1)-dimensional NLS-MBE is integrable. Its Lax representations are

$$\Psi_x = U\Psi, \quad (6)$$

$$\Psi_t = V\Psi, \quad (7)$$

where U and V have the form

$$U = -i\lambda\sigma_3 + U_0, \quad V = -2i\sigma_3\lambda^2 + 2\lambda U_0 + V_0 + \frac{i}{\lambda + \omega} V_{-1} \quad (8)$$

where

$$U_0 = \begin{pmatrix} 0 & q \\ -r & 0 \end{pmatrix}, \quad V_0 = i \begin{pmatrix} qr & q_x \\ -r_x & -qr \end{pmatrix} \equiv iqr\sigma_3 + i \begin{pmatrix} 0 & q_x \\ -r_x & 0 \end{pmatrix} \quad (9)$$

$$V_{-1} = \begin{pmatrix} \eta & -p \\ -k & -\eta \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (10)$$

The paper is organized as follows. In section 2, we obtain three kinds of nonlocal SMBE system: S -, T- and ST-symmetric NLS-MBE system. In section 3, we discuss the Darboux transformation of S -symmetric NLS-MBE system, give the concrete forms of the new solutions q' , p' , η' under the one-fold Darboux transformation.

2. The nonlocal NLS-MBE systems

2.1. S-symmetric nonlocal NLS-MBE system

S-symmetric forms are

$$r(x, t) = \delta\bar{q}(-x, t), \quad k(x, t) = \delta\bar{p}(-x, t) \quad (11)$$

Then equations (1)-(5) take form

$$iq_t(x, t) + q_{xx}(x, t) + 2q(x, t)\delta\bar{q}(-x, t) - 2ip(x, t) = 0 \quad (12)$$

$$i\delta\bar{q}_t(-x, t) - \delta\bar{q}_{xx}(-x, t) - 2q(x, t)\bar{q}^2(-x, t) - 2ip(x, t) = 0 \quad (13)$$

$$p_x(x, t) - 2i\omega p(x, t) - 2\eta(x, t)q(x, t) = 0 \quad (14)$$

$$\delta p_x(-x, t) - 2i\omega\delta\bar{p}(-x, t) + 2\eta(x, t)\delta\bar{q}(-x, t) = 0 \quad (15)$$

$$\eta_x(x, t) + q(x, t)\delta\bar{p}(-x, t) + \delta\bar{q}(-x, t)p(x, t) = 0 \quad (16)$$

where $\delta^2 = 1$, This S-symmetric nonlocal NLS-MBE is integrable. Its Lax pairs have the forms

$$U_1 = -i\lambda\sigma_3 + U_{01}, \quad V_1 = -2i\sigma_3\lambda^2 + 2\lambda U_{01} + V_{01} + \frac{i}{\lambda + \omega}V_{-01} \quad (17)$$

where

$$\begin{aligned} U_{01} &= \begin{pmatrix} 0 & q(x, t) \\ -\delta\bar{q}(-x, t) & 0 \end{pmatrix}, \quad V_{01} = i \begin{pmatrix} q(x, t)\delta\bar{q}(-x, t) & q_x(x, t) \\ -\delta\bar{q}_x(-x, t) & -q(x, t)\delta\bar{q}(-x, t) \end{pmatrix} \equiv \\ &\equiv iq(x, t)\delta\bar{q}(-x, t)\sigma_3 + i \begin{pmatrix} 0 & q_x(x, t) \\ -\delta\bar{q}_x(-x, t) & 0 \end{pmatrix}, \quad V_{-01} = \begin{pmatrix} \eta(x, t) & -p(x, t) \\ -\delta\bar{p}(-x, t) & -\eta(x, t) \end{pmatrix} \end{aligned} \quad (18)$$

The couple of equations

$$\Psi_x = U_1\Psi, \quad (19)$$

$$\Psi_t = V_1\Psi, \quad (20)$$

satisfies the next compatibility condition $U_{1t} - V_{1x} + [U_1, V_1] = 0$.

2.2. T-symmetric nonlocal NLS-MBE system

T-symmetric forms are $r(x, t) = \delta\bar{q}(x, -t)$, $k(x, t) = \delta\bar{p}(x, -t)$. The equations (1)-(5) take form

$$iq_t(x, t) + q_{xx}(x, t) + 2q(x, t)\delta\bar{q}(x, -t) - 2ip(x, t) = 0 \quad (21)$$

$$i\delta\bar{q}_t(x, -t) + \delta\bar{q}_{xx}(x, -t) + 2q(x, t)\bar{q}^2(x, -t) + 2ip(x, t) = 0 \quad (22)$$

$$p_x(x, t) - 2i\omega p(x, t) - 2\eta(x, t)q(x, t) = 0 \quad (23)$$

$$\delta p_x(x, -t) + 2i\omega\delta\bar{p}(x, -t) - 2\eta(x, t)\delta\bar{q}(x, -t) = 0 \quad (24)$$

$$\eta_x(x, t) + q(x, t)\delta\bar{p}(x, -t) + \delta\bar{q}(x, -t)p(x, t) = 0 \quad (25)$$

T-symmetric nonlocal NLS-MBE is integrable. Its Lax pairs have the forms

$$U_2 = -i\lambda\sigma_3 + U_{02}, \quad V_2 = -2i\sigma_3\lambda^2 + 2\lambda U_{02} + V_{02} + \frac{i}{\lambda + \omega}V_{-02} \quad (26)$$

where

$$\begin{aligned} U_{02} &= \begin{pmatrix} 0 & q(x, t) \\ -\delta\bar{q}(x, -t) & 0 \end{pmatrix}, \quad V_{02} = i \begin{pmatrix} \delta\bar{q}(x, -t) & q_x(x, t) \\ \delta\bar{q}_x(x, -t) & -q(x, t)\delta\bar{q}(x, -t) \end{pmatrix} \equiv \\ &\equiv iq(x, t)\delta\bar{q}(x, -t)\sigma_3 + i \begin{pmatrix} 0 & q_x(x, t) \\ \delta\bar{q}_x(x, -t) & 0 \end{pmatrix} \end{aligned} \quad (27)$$

$$V_{-02} = \begin{pmatrix} \eta(x, t) & -p(x, t) \\ -\delta\bar{p}(x, -t) & -\eta(x, t) \end{pmatrix} \quad (28)$$

Eqs (26) satisfy $U_{2t} - V_{2x} + [U_2, V_2] = 0$.

2.3. ST-symmetric nonlocal NLS-MBE system

ST-symmetric forms are $r(x, t) = \delta\bar{q}(-x, -t)$, $k(x, t) = \delta\bar{p}(-x, -t)$. The equations (1)-(5) take form

$$iq_t(x, t) + q_{xx}(x, t) + 2q(x, t)\delta\bar{q}(-x, -t) - 2ip(x, t) = 0 \quad (29)$$

$$i\delta\bar{q}_t(-x, -t) + \delta\bar{q}_{xx}(-x, -t) + 2q(x, t)\bar{q}^2(-x, -t) + 2ip(x, t) = 0 \quad (30)$$

$$p_x(x, t) - 2i\omega p(x, t) - 2\eta(x, t)q(x, t) = 0 \quad (31)$$

$$\delta p_x(-x, -t) + 2i\omega\delta\bar{p}(-x, -t) - 2\eta(x, t)\delta\bar{q}(x, -t) = 0 \quad (32)$$

$$\eta_x(x, t) + q(x, t)\delta\bar{p}(-x, -t) + \delta\bar{q}(-x, -t)p(x, t) = 0 \quad (33)$$

ST-symmetric nonlocal NLS-MBE is integrable. Its Lax pairs have the forms

$$U_3 = -i\lambda\sigma_3 + U_{03}, \quad V_3 = -2i\sigma_3\lambda^2 + 2\lambda U_{03} + V_{03} + \frac{i}{\lambda + \omega}V_{-03} \quad (34)$$

where

$$\begin{aligned} U_{03} &= \begin{pmatrix} 0 & q(x, t) \\ -\delta\bar{q}(-x, -t) & 0 \end{pmatrix}, \quad V_{03} = i \begin{pmatrix} \delta\bar{q}(-x, -t) & q_x(x, t) \\ -\delta\bar{q}_x(-x, -t) & -q(x, t)\delta\bar{q}(-x, -t) \end{pmatrix} \equiv \\ &\equiv iq(x, t)\delta\bar{q}(-x, -t)\sigma_3 + i \begin{pmatrix} 0 & q_x(x, t) \\ -\delta\bar{q}_x(-x, -t) & 0 \end{pmatrix} \end{aligned} \quad (35)$$

$$V_{-03} = \begin{pmatrix} \eta(x, t) & -p(x, t) \\ -\delta\bar{p}(-x, -t) & -\eta(x, t) \end{pmatrix} \quad (36)$$

The compatibility condition is $U_{3t} - V_{3x} + [U_3, V_3] = 0$.

3. Darboux transformation (DT)

It is well-known that the DT has been proved to be an efficient way to find the exact solutions like solitons, dromions, positons, breathers, rogue wave solutions for integrable equations in 1+1 and 2+1 dimensions. In this section, considering the particularity of the Lax representation, we construct the DT of S-symmetric nonlocal NLS-MBE (??)-(??). Furthermore, we will find some solutions of S-symmetric nonlocal NLS-MBE using its DT.

We consider the following transformation of Eq.(??)

$$\Psi' = T\Psi = (\lambda I - M)\Psi \quad (37)$$

such that

$$\Psi'_x = U'_1\Psi', \quad \Psi'_t = V'_1\Psi', \quad (38)$$

where U'_1 and V'_1 depend on q' , p' , η' and λ . Here

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (39)$$

The relation between q' , p' , η' , λ and $U'_1 - V'_1$ is the same as the relation between q , p , η , λ and $U_1 - V_1$. In order to hold Eqs.(38), the T must satisfies the following equations

$$T_x + TU_1 = U'_1T, \quad (40)$$

$$T_t + TV_1 = V'_1T. \quad (41)$$

Then the relation between q, p, η and q', p', η' can be reduced from these equations, which is in fact the DT of the NLS-MBE. Comparing the coefficients of λ^i of the two sides of the equation (40), we get

$$\lambda^0 : M_x = U'_{01}M - MU_{01}, \quad (42)$$

$$\lambda^1 : U'_{01} = U_{01} + i[M, \sigma_3], \quad (43)$$

$$\lambda^2 : iI\sigma_3 = i\sigma_3I. \quad (44)$$

From (43) we obtain

$$q^{[1]}(x, t) = q(x, t) - 2im_{12}, \quad (45)$$

$$\delta\bar{q}^{[1]}(-x, t) = \delta\bar{q}^{[1]}(-x, t) - 2im_{21}. \quad (46)$$

Hence we get $m_{21} = -\bar{m}_{12}$. Then comparing the coefficients of λ^i of the two sides of the equation (41) gives us

$$\lambda^0 : -M_t = iB'_{-01} - V'_{01}M - iV_{-1} + MV_{01}, \quad (47)$$

$$\lambda^1 : U'_{01}M - MU_{01} = V'_{01} - V_{01}, \quad (48)$$

$$\lambda^2 : U'_{01} = U_{01} - i[\sigma_3, M], \quad (49)$$

$$(\lambda + \omega)^{-1} : 0 = -i\omega V'_{-01} - iV'_{-01}M + i\omega V_{-1} + iMV_{-01}. \quad (50)$$

The last equation of this system gives

$$V'_{-1} = (M + \omega I)V_{-1}(M + \omega I)^{-1}. \quad (51)$$

At the same, from Eq.(49) we get

$$q'(x, t) = q(x, t) - 2im_{12}, \quad \delta\bar{q}(-x, t) = \delta\bar{q}(-x, t) - 2im_{21} \quad (52)$$

and hence we additionally have $m_{22} = \bar{m}_{11}$. So the matrix M has the form

$$M = \begin{pmatrix} m_{11} & m_{12} \\ -\bar{m}_{12} & \bar{m}_{11} \end{pmatrix}, \quad M^{-1} = \frac{1}{|m_{11}|^2 + |m_{12}|^2} \begin{pmatrix} \bar{m}_{11} & -m_{12} \\ \bar{m}_{12} & m_{11} \end{pmatrix}, \quad (53)$$

$$M + \omega I = \begin{pmatrix} m_{11} + \omega & m_{12} \\ -\bar{m}_{12} & \omega + \bar{m}_{11} \end{pmatrix}, \quad (M + \omega I)^{-1} = \frac{1}{\Delta_1} \begin{pmatrix} \bar{m}_{11} + \omega & -m_{12} \\ \bar{m}_{12} & \omega + m_{11} \end{pmatrix}. \quad (54)$$

Here

$$\Delta_1 = \det(M + \omega I) = \omega^2 + \omega(m_{11} + \bar{m}_{11}) + |m_{11}|^2 + |m_{12}|^2. \quad (55)$$

The equation (51) gives

$$\begin{aligned} \eta' &= \frac{(|\omega + m_{11}|^2 - |m_{12}|^2)\eta - p\bar{m}_{12}(\omega + m_{11}) - \delta\bar{p}(-x, t)m_{12}(\omega + \bar{m}_{11})}{\Delta_1}, \\ p' &= \frac{p(\omega + m_{11})^2 - \delta\bar{p}(-x, t)m_{12}^2 + 2\eta m_{12}(\omega + m_{11})}{\Delta_1}. \end{aligned} \quad (56)$$

3.1. One-fold DT

We now assume that

$$M = H\Lambda H^{-1}, \quad (57)$$

where

$$H = \begin{pmatrix} \psi_1(\lambda_1; x, t) & \psi_1(\lambda_2; x, t) \\ \psi_2(\lambda_1; x, t) & \psi_2(\lambda_2; x, t) \end{pmatrix} \equiv \begin{pmatrix} \psi_{1,1}(x, t) & \psi_{1,2}(x, t) \\ \psi_{2,1}(x, t) & \psi_{2,2}(x, t) \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad (58)$$

and $\det H \neq 0$, where λ_1 and λ_2 are complex constants. Taking account S-symmetric forms (11) and relations $m_{12} = -\bar{m}_{21}$ and $m_{11} = \bar{m}_{22}$ we get $\lambda_2 = \bar{\lambda}_1$. So for the matrix M we have

$$M = \frac{1}{\Delta} \begin{pmatrix} \lambda_1 \psi_{1,1} \bar{\psi}_{1,1}(-x, t) + \delta \bar{\lambda}_1 \psi_{2,1} \bar{\psi}_{2,1}(-x, t) & \delta(\lambda_1 - \bar{\lambda}_1) \psi_{1,1} \bar{\psi}_{2,1}(-x, t) \\ (\lambda_1 - \bar{\lambda}_1) \psi_{1,1}(-x, t) \psi_{2,1} & \bar{\lambda}_1 \psi_{1,1} \psi_{1,1}(-x, t) + \delta \lambda_1 \psi_{2,1} \bar{\psi}_{2,1}(-x, t) \end{pmatrix}, \quad (59)$$

where

$$\Delta = \psi_{1,1}(x, t) \bar{\psi}_{1,1}(-x, t) + \delta \psi_{2,1}(x, t) \bar{\psi}_{2,1}(-x, t) \quad (60)$$

Substituting (59) into (52) and (56) we get the following one-fold DT of S-symmetric nonlocal NLS-MBE system:

$$q'(x, t) = q(x, t) + \frac{2\delta i(\lambda_1 - \bar{\lambda}_1) \psi_{1,1}(x, t) \bar{\psi}_{2,1}(-x, t)}{\Delta}, \quad (61)$$

$$\begin{aligned} \eta'(x, t) &= \frac{\eta}{\Delta^2} [(\psi_{1,1}(x, t) \bar{\psi}_{1,1}(-x, t) - \delta \psi_{2,1}(x, t) \bar{\psi}_{2,1}(-x, t))^2 + \\ &+ 2\delta \left(\frac{\tilde{\lambda}_1}{\bar{\lambda}_1} + \frac{\bar{\lambda}_1}{\tilde{\lambda}_1} \right) \psi_{1,1}(x, t) \bar{\psi}_{1,1}(-x, t) \psi_{2,1}(x, t) \bar{\psi}_{2,1}(-x, t)] + \\ &+ \frac{p}{\Delta^2} \left[\left(1 - \frac{\tilde{\lambda}_1}{\bar{\lambda}_1} \right) \psi_{1,1}(x, t) \bar{\psi}_{1,1}(-x, t) - \delta \left(1 - \frac{\tilde{\lambda}_1}{\bar{\lambda}_1} \right) \psi_{2,1}(x, t) \bar{\psi}_{2,1}(-x, t) \right] \bar{\psi}_{1,1}(-x, t) \psi_{2,1}(x, t) - \\ &- \frac{\delta \bar{p}(-x, t)}{\Delta^2} \left[\delta \left(1 - \frac{\tilde{\lambda}_1}{\bar{\lambda}_1} \right) \psi_{1,1}(x, t) \bar{\psi}_{1,1}(-x, t) - \left(1 - \frac{\tilde{\lambda}_1}{\bar{\lambda}_1} \right) \psi_{2,1}(x, t) \bar{\psi}_{2,1}(-x, t) \right] \psi_{1,1}(x, t) \bar{\psi}_{2,1}(-x, t) \\ p'(x, t) &= \frac{2\eta}{\Delta^2} \left[\delta \left(\frac{\tilde{\lambda}_1}{\bar{\lambda}_1} - 1 \right) \psi_{1,1}(x, t) \bar{\psi}_{1,1}(-x, t) + \left(1 - \frac{\tilde{\lambda}_1}{\bar{\lambda}_1} \right) \psi_{2,1}(x, t) \bar{\psi}_{2,1}(-x, t) \right] \psi_{1,1}(x, t) \bar{\psi}_{2,1}(-x, t) + \\ &+ \frac{p}{\Delta^2} \left[\frac{(\tilde{\lambda}_1 \psi_{1,1}(x, t) \bar{\psi}_{1,1}(-x, t) + \delta \bar{\lambda}_1 \psi_{2,1}(x, t) \bar{\psi}_{2,1}(-x, t))^2}{\tilde{\lambda}_1 \bar{\lambda}_1} \right] - \\ &- \frac{\delta \bar{p}(-x, t)}{\Delta^2} \frac{(\tilde{\lambda}_1 - \bar{\lambda}_1)^2}{\tilde{\lambda}_1 \bar{\lambda}_1} \psi_{1,1}^2(x, t) \bar{\psi}_{2,1}^2(-x, t), \end{aligned} \quad (62)$$

where $\tilde{\lambda}_1 = \lambda_1 + \omega$.

3.2. Explicit solutions of S-symmetric nonlocal NLS-MBE

We can find various types of solutions of S-symmetric nonlocal NLS-MBE (12)-(16) by taking various "seed" solutions, for example, $q(x, t) = 0$, $p(x, t) = 0$ and $\eta(x, t) = 1$. From (19)-(20) the seed become

$$\psi = \begin{pmatrix} \psi_{1,1} \\ \psi_{2,1} \end{pmatrix} = \begin{pmatrix} \exp(-i\lambda x + (-2i\lambda^2 + \frac{i}{\lambda+\omega})t) \\ \exp(i\lambda x + (2i\lambda^2 - \frac{i}{\lambda+\omega})t) \end{pmatrix}, \quad (64)$$

We take $\lambda = a + ib$ ($a, b \in R$), then we obtained the one soliton solution of nonlocal NLS-MBE for $\delta = 1$ (defocusing case)

$$q'(x, t) = \frac{2be^{F_1}}{sh(F_2)}, \quad (65)$$

$$\eta'(x, t) = \frac{1}{sh^2(F_2)}[ch^2(F_2) - \frac{(a + \omega)^2 - b^2}{a^2 + b^2}] \quad (66)$$

$$p'(x, t) = \frac{2ib}{(a^2 + b^2)sh(F_2)}[a + \omega + ibcth(F_2)], \quad (67)$$

and for $\delta = -1$ (focusing case)

$$q'(x, t) = -\frac{2be^{F_1}}{ch(F_2)}, \quad (68)$$

$$\eta'(x, t) = \frac{1}{ch^2(F_2)}[sh^2(F_2) + \frac{(a + \omega)^2 - b^2}{a^2 + b^2}] \quad (69)$$

$$p'(x, t) = \frac{2ib}{(a^2 + b^2)ch(F_2)}[a + \omega + ibth(F_2)], \quad (70)$$

where

$$F_1 = 2bx + 2i(-2(a^2 - b^2) + \frac{a + \omega}{(a + \omega)^2 + b^2})t \quad (71)$$

$$F_2 = -2iax + 2(4ab + \frac{b}{(a + \omega)^2 + b^2})t. \quad (72)$$

4. Conclusion

In this paper we research the (1+1)-dimensional system of Schrödinger-Maxwell-Bloch equations (NLS-MBE), which describes the optical pulse propagation in an erbium doped fiber and find S-, T- and ST-symmetric nonlocal Schrödinger-Maxwell-Bloch equations. These equations are integrable by the Inverse Scattering Method. Following this method we prove the integrability of the nonlocal system of Schrödinger-Maxwell-Bloch equations by Lax pairs. Also we obtain explicit solutions by Darboux transformation.

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