Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

Dark and bright solitons for the two-dimensional complex modified Korteweg-de Vries and Maxwell-Bloch system with time-dependent coefficient

To cite this article: G. Shaikhova *et al* 2018 *J. Phys.: Conf. Ser.* **965** 012035

Dark and bright solitons for the two-dimensional complex modified Korteweg-de Vries and Maxwell-Bloch system with time-dependent coefficient

G. Shaikhova, N. Ozat, K. Yesmakhanova, G. Bekova

Eurasian International Center for Theoretical Physics and Department of General Theoretical Physics, Eurasian National University, Astana, 010008, Kazakhstan.

E-mail: g.shaikhova@gmail.com

Abstract. In this work, we present Lax pair for two-dimensional complex modified Korteweg-de Vries and Maxwell-Bloch (cmKdV-MB) system with the time-dependent coefficient. Dark and bright soliton solutions for the cmKdV-MB system with variable coefficient are received by Darboux transformation. Moreover, the determinant representation of the one-fold and two-fold Darboux transformation for the cmKdV-MB system with time-dependent coefficient is presented.

1. Introduction

The nonlinear phenomena play a important rule in a variety of scientific fields such as optical fibers, fluid mechanics, chemical kinetics, solid state physics. It is modeled by nonlinear partial differential equations (NPDEs) [1-4]. Finding exact solutions of NPDEs is crucial, because, exact solutions help one to well understand the mechanism of the complicated physical phenomena and dynamical processes modeled by these NPDEs. Moreover, the real physical space-time being in higher-dimensional and more complex, research generalized integrable equations in higher dimensions is still open and attractive [5-15]. One of the well-known NPDE is the complex modified Korteweg-de Vries equation. This equation and its generalization have been derived and studied for the dynamical evolution of nonlinear lattices, fluid dynamics, ultra-short pulses in nonlinear optics, plasma physics, nonlinear transmission lines [16-17] .

In this paper, we present the two-dimensional cmKdV-MB system with time-dependent coefficient as the following form

$$q_t + q_{xxy} + wq_x + w_x q + iqv + \gamma(t)p = 0, \tag{1}$$
$$v_x - 2iqr_{xy} + 2irq_{xy} = 0, \tag{2}$$
$$w_x - 2rq_y - 2qr_y = 0, \tag{3}$$
$$\eta_x + qk + rp = 0, \tag{4}$$
$$p_x - 2ip\omega - 2q\eta = 0, \tag{5}$$

where q, v, w, η, p are unknown potentials and $r = q*, k = p*$. The symbol $*$ denotes the complex conjugate. The time-dependent coefficient $\gamma(t)$ describes the averaging with respect

IOP Conf. Series: Journal of Physics: Conf. Series **965** (2018) 012035 doi:10.1088/1742-6596/965/1/012035

to inhomogeneous broadening of the resonant frequency. The real parameter ω is a constant corresponding to the frequency. In the equations (1)-(5), x and y represent the normalized distance and t represented normalized time respectively.

The equations (1)-(5) admit the following integrable reduction, if $p = \eta = 0$:

$$
\begin{aligned}
q_t + q_{xxy} + wq_x + w_x q + iqv &= 0, & (6)\\
v_x - 2iqr_{xy} + 2irq_{xy} &= 0, & (7)\\
w_x - 2rq_y - 2qr_y &= 0, & (8)
\end{aligned}
$$

it is the usual two-dimensional cmKdV equations [18].

If we let $\gamma = -2$ the equations (1)-(5) will be reduced to two-dimensional cmKdV-MB equations as the following form [19]

$$
\begin{aligned}
q_t + q_{xxy} + wq_x + w_x q + iqv - 2p &= 0, & (9)\\
v_x - 2iqr_{xy} + 2irq_{xy} &= 0, & (10)\\
w_x - 2rq_y - 2qr_y &= 0, & (11)\\
\eta_x + qk + rp &= 0, & (12)\\
p_x - 2ip\omega - 2q\eta &= 0. & (13)
\end{aligned}
$$

The paper is organized as follows. In section 2, Lax pair for the two-dimensional cmKdV-MB system with time-dependent coefficient is presented. We derive the one-fold Darboux transformation of the cmKdV-MB system with time-dependent coefficient in section 3. The determinant-formed generalization of one-fold and two-fold Darboux transformation for the two-dimensional cmKdV-MB system with time-dependent coefficient will be given in section 4. In section 5, we receive one-soliton, two-soliton solutions by assuming trivial seed solutions. Section 6 is devoted to conclusion.

2. Lax representation

The corresponding Lax representation of the two-dimensional cmKdV-MB system with time-dependent coefficient can be expressed as follows:

$$
\begin{aligned}
\Psi_x &= A\Psi, & (14)\\
\Psi_t &= 4\lambda^2\Psi_y + B\Psi, & (15)
\end{aligned}
$$

where

$$
\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \qquad (16)
$$

and

$$
\begin{aligned}
A &= -i\lambda \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 0 & q \\ -r & 0 \end{pmatrix} = -i\lambda\,\sigma_3 + A_0, & (17)\\
B &= \lambda \begin{pmatrix} iw & 2iq_y \\ 2ir_y & -iw \end{pmatrix} + \begin{pmatrix} -\frac{i}{2}v & -q_{xy} - wq \\ r_{xy} + wr & \frac{i}{2}v \end{pmatrix} & (18)\\
&+ V_1 \begin{pmatrix} \eta & -p \\ -k & -\eta \end{pmatrix} & (19)\\
&= \lambda B_1 + B_0 + V_1 B_{-1}, & (20)
\end{aligned}
$$

with

$$B_{-1} = \begin{pmatrix} \eta & -p \\ -k & -\eta \end{pmatrix}, V_1 = -\frac{i\gamma(t)}{2(\lambda+\omega)}. \tag{21}$$

The compatible condition of equations (1)-(5) is

$$A_t - B_x - 4\lambda^2 A_y + [A, B] = 0, \tag{22}$$

where $[A, B] = AB - BA$. By direct calculation of above equation, we can yield the two-dimensional cmKdV-MB system with time-dependent coefficient.

3. Darboux transformation

In this section, we consider the following transformation of equations (14)-(15) based on the Darboux transformation for Ablowitz-Kaup-Newell-Segur system

$$\Psi^{'} = T\Psi = (\lambda I - M)\Psi, \tag{23}$$

where

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{24}$$

New function $\Psi^{'}$ satisfies

$$\Psi^{'}_x = A^{'}\Psi^{'}, \tag{25}$$
$$\Psi^{'}_t = 4\lambda^2\Psi^{'}_y + B^{'}\Psi^{'}, \tag{26}$$

where A' and B' depend on q', v', w', η', p' and λ. In order to hold equation (25)-(26), the T must satisfies the system

$$T_x + TA = A^{'}T, \tag{27}$$
$$T_t + TB = 4\lambda^2 T_y + B^{'}T. \tag{28}$$

The relation between q, p, v, w, η and $q^{[1]}, p^{[1]}, v^{[1]}, w^{[1]}, \eta^{[1]}$ can be obtained from (27)-(28), which is the Darboux transformation of cmKdV-MB system with time-dependent coefficient. Combining coefficients of the different powers of λ^i of the equation (27) and doing some calculations we obtain

$$q^{'} = q - 2im_{12}, \tag{29}$$
$$r^{'} = r - 2im_{21}, \tag{30}$$

and $m_{12}^* = -m_{21}$. At the same time, from the equation (28) we can get

$$v^{'} = v - 4(m_{12}r_y - m_{21}q_y + 2im_{11}m_{11y} + m_{21}m_{12y}), \tag{31}$$
$$w^{'} = w - 4im_{11y} = w + 4im_{22y}, \tag{32}$$
$$B^{'}_{-1} = (M+\omega)B_{-1}(M+\omega)^{-1} \tag{33}$$

and we additionally have $m_{11}^* = m_{22}$. The equations (29)-(33) gives one-fold Darboux transformation of the two-dimensional cmKdV-MB system with time-dependent coefficient.

We now assume that

$$M = H\Lambda H^{-1}, \tag{34}$$

where $\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, H = \begin{pmatrix} \psi_1(\lambda_1;t,x,y) & \psi_1(\lambda_2;t,x,y) \\ \psi_2(\lambda_2;t,x,y) & \psi_2(\lambda_2;t,x,y) \end{pmatrix} := \begin{pmatrix} \psi_{1,1} & \psi_{1,2} \\ \psi_{2,1} & \psi_{2,2} \end{pmatrix}$. In order to satisfy the constraints of M and B'_{-1} as mentioned above, we first notes that

$$\lambda_2 = \lambda_1^*, m_{11}^* = m_{22}, \tag{35}$$

$$H = \begin{pmatrix} \psi_1(\lambda_1,t,x,y) & -\psi_2^*(\lambda_1,t,x,y) \\ \psi_2(\lambda_1,t,x,y) & \psi_1^*(\lambda_1,t,x,y) \end{pmatrix}. \tag{36}$$

In next part, the determinant form of one-fold Darboux transformation of cmKdV-MB system with time-dependent coefficient will be given.

4. Determinant representation of Darboux transformation

In this section, we will consider the first two Darboux transformation of the cmKdV-MB system with time-dependent coefficient. Firstly, we introduce n eigenfunctions $\begin{pmatrix} \psi_{1,i} \\ \psi_{2,i} \end{pmatrix} = \psi(\lambda = \lambda_i), i = 1,2$ with the constraints on eigenvalues as $\lambda_{2i-1} = \lambda_{2i}^*$ and reduction conditions as $\psi_{2,2i} = \psi_{1,2i-1}^*, \psi_{2,2i-1} = -\psi_{1,2i}^*$ [20-23].

4.1. One-fold Darboux transformation

As the simplest Darboux transformation, the determinant of one-fold Darboux transformation of cmKdV-MB system with time-dependent coefficient given in the following form [20-23]

$$T_1(\lambda;\lambda_1,\lambda_2) = \frac{1}{\Delta_1}\begin{pmatrix} (T_1)_{11} & (T_1)_{12} \\ (T_1)_{21} & (T_1)_{22} \end{pmatrix}, \tag{37}$$

where

$$(T_1)_{11} = \begin{vmatrix} 1 & 0 & \lambda \\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} \\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} \end{vmatrix}, (T_1)_{12} = \begin{vmatrix} 0 & 1 & 0 \\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} \\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} \end{vmatrix}, \tag{38}$$

$$(T_1)_{21} = \begin{vmatrix} 1 & 0 & 0 \\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{2,1} \\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{2,2} \end{vmatrix}, (T_1)_{22} = \begin{vmatrix} 0 & 1 & \lambda \\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{2,1} \\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{2,2} \end{vmatrix}, \tag{39}$$

and

$$A^{[1]} = A - [T_1, \sigma_3], \quad B_{-1}^{[1]} = T_1|_{\lambda=-\omega} B_{-1} T_1^{-1}|_{\lambda=-\omega}, \tag{40}$$

Then one-fold DT can be presented by next form

$$q^{[1]} = q - 2i\frac{(T_1)_{12}}{\Delta_1}, \tag{41}$$

$$v^{[1]} = v + 4\left(q_y^*\frac{(T_1)_{12}}{\Delta_1} - q_y\frac{(T_1)_{21}}{\Delta_1} + 2i\frac{(T_1)_{11}}{\Delta_1}\left(\frac{(T_1)_{11}}{\Delta_1}\right)_y + 2i\frac{(T_1)_{21}}{\Delta_1}\left(\frac{(T_1)_{21}}{\Delta_1}\right)_y\right), \tag{42}$$

$$w^{[1]} = w - 4i\left(\frac{(T_1)_{11}}{\Delta_1}\right)_y, \tag{43}$$

$$p^{[1]} = \frac{2\eta(T_1)_{11}(T_1)_{12} - p^*(T_1)_{12}(T_1)_{12} + p(T_1)_{11}(T_1)_{11}}{(T_1)_{11}(T_1)_{22} - (T_1)_{12}(T_1)_{21}}|_{\lambda=-\omega}, \tag{44}$$

$$\eta^{[1]} = \frac{\eta\left((T_1)_{11}(T_1)_{12} + (T_1)_{21}(T_1)_{12}\right) + p(T_1)_{11}(T_1)_{21} - p^*(T_1)_{12}(T_1)_{22}}{(T_1)_{11}(T_1)_{22} - (T_1)_{12}(T_1)_{21}}|_{\lambda=-\omega}. \tag{45}$$

We can find the transformation T_1 has following property

$$T_1(\lambda;\lambda_1,\lambda_2)|_{\lambda=\lambda_i}\begin{pmatrix}\psi_{1,i}\\ \psi_{2,i}\end{pmatrix}=0, \tag{46}$$

where $i=1,2$.

4.2. Two-fold Darboux transformation

The two-fold Darboux transformation of cmKdV-MB system with time-dependent coefficient is as following [20-23]

$$T_2(\lambda;\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\frac{1}{\Delta_2}\begin{pmatrix}(T_2)_{11} & (T_2)_{12}\\ (T_2)_{21} & (T_2)_{22}\end{pmatrix}, \tag{47}$$

where

$$\Delta_2=\begin{vmatrix}\psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} & \lambda_1\psi_{2,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} & \lambda_2\psi_{2,2}\\ \psi_{1,3} & \psi_{2,3} & \lambda_3\psi_{1,3} & \lambda_3\psi_{2,3}\\ \psi_{1,4} & \psi_{2,4} & \lambda_4\psi_{1,4} & \lambda_4\psi_{2,4}\end{vmatrix}, \tag{48}$$

$$(T_2)_{11}=\begin{vmatrix}1 & 0 & \lambda & 0 & \lambda^2\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} & \lambda_1\psi_{2,1} & \lambda_1^2\psi_{1,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} & \lambda_2\psi_{2,2} & \lambda_2^2\psi_{1,2}\\ \psi_{1,3} & \psi_{2,3} & \lambda_3\psi_{1,3} & \lambda_3\psi_{2,3} & \lambda_3^2\psi_{1,3}\\ \psi_{1,4} & \psi_{2,4} & \lambda_4\psi_{1,4} & \lambda_4\psi_{2,4} & \lambda_4^2\psi_{1,4}\end{vmatrix},(T_2)_{12}=\begin{vmatrix}0 & 1 & 0 & \lambda & 0\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} & \lambda_1\psi_{2,1} & \lambda_1^2\psi_{1,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} & \lambda_2\psi_{2,2} & \lambda_2^2\psi_{1,2}\\ \psi_{1,3} & \psi_{2,3} & \lambda_3\psi_{1,3} & \lambda_3\psi_{2,3} & \lambda_3^2\psi_{1,3}\\ \psi_{1,4} & \psi_{2,4} & \lambda_4\psi_{1,4} & \lambda_4\psi_{2,4} & \lambda_4^2\psi_{1,4}\end{vmatrix},$$

$$(T_2)_{21}=\begin{vmatrix}1 & 0 & \lambda & 0 & 0\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} & \lambda_1\psi_{2,1} & \lambda_1^2\psi_{2,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} & \lambda_2\psi_{2,2} & \lambda_2^2\psi_{2,2}\\ \psi_{1,3} & \psi_{2,3} & \lambda_3\psi_{1,3} & \lambda_3\psi_{2,3} & \lambda_3^2\psi_{2,3}\\ \psi_{1,4} & \psi_{2,4} & \lambda_4\psi_{1,4} & \lambda_4\psi_{2,4} & \lambda_4^2\psi_{2,4}\end{vmatrix},(T_2)_{22}=\begin{vmatrix}0 & 1 & 0 & \lambda & \lambda^2\\ \psi_{1,1} & \psi_{2,1} & \lambda_1\psi_{1,1} & \lambda_1\psi_{2,1} & \lambda_1^2\psi_{2,1}\\ \psi_{1,2} & \psi_{2,2} & \lambda_2\psi_{1,2} & \lambda_2\psi_{2,2} & \lambda_2^2\psi_{2,2}\\ \psi_{1,3} & \psi_{2,3} & \lambda_3\psi_{1,3} & \lambda_3\psi_{2,3} & \lambda_3^2\psi_{2,3}\\ \psi_{1,4} & \psi_{2,4} & \lambda_4\psi_{1,4} & \lambda_4\psi_{2,4} & \lambda_4^2\psi_{2,4}\end{vmatrix}.$$

We can find

$$T_2(\lambda;\lambda_1,\lambda_2,\lambda_3,\lambda_4)|_{\lambda=\lambda_i}\begin{pmatrix}\psi_{1,i}\\ \psi_{2,i}\end{pmatrix}=0,$$

where $i=1,2,3,4$. Similarly, for transformation T_2, following transformation formula holds

$$T_{2x}+T_2A = A^{[2]}T_2, \tag{49}$$

$$T_{2t}+T_2B = 4\lambda^2T_{2y}+B^{[2]}T_2. \tag{50}$$

Then the relation between q,p,v,w,η and $q^{[2]},p^{[2]},v^{[2]},w^{[2]},\eta^{[2]}$ will be got in the following equations

$$A^{[2]} = A-[T_2,\sigma_3], \tag{51}$$

$$B_{-1}^{[2]} = T_2|_{\lambda=-\omega}B_{-1}T_2^{-1}|_{\lambda=-\omega}. \tag{52}$$

Finally, we can get from (49)-(50) the following two-fold Darboux transformation in detail,

$$q^{[2]} = q - 2i\frac{(T_2)_{12}}{\Delta_2}, \tag{53}$$

$$v^{[2]} = v + 4\left(q_y^*\frac{(T_2)_{12}}{\Delta_2} - q_y\frac{(T_2)_{21}}{\Delta_2} + 2i\frac{(T_2)_{11}}{\Delta_2}\left(\frac{(T_2)_{11}}{\Delta_2}\right)_y + 2i\frac{(T_2)_{21}}{\Delta_2}\left(\frac{(T_2)_{21}}{\Delta_2}\right)_y\right), \tag{54}$$

$$w^{[2]} = w - 4i\left(\frac{(T_2)_{11}}{\Delta_2}\right)_y, \tag{55}$$

$$p^{[2]} = \frac{2\eta(T_2)_{11}(T_2)_{12} - p^*(T_2)_{12}(T_2)_{12} + p(T_2)_{11}(T_2)_{11}}{(T_2)_{11}(T_2)_{22} - (T_2)_{12}(T_2)_{21}}|_{\lambda=-\omega}, \tag{56}$$

$$\eta^{[2]} = \frac{\eta\left((T_2)_{11}(T_2)_{12} + (T_2)_{21}(T_2)_{12}\right) + p(T_2)_{11}(T_2)_{21} - p^*(T_2)_{12}(T_2)_{22}}{(T_2)_{11}(T_2)_{22} - (T_2)_{12}(T_2)_{21}}|_{\lambda=-\omega}. \tag{57}$$

The equations (53)-(57) can be used to generate two-soliton solutions of the two-dimensional cmKdV-MB system with time-dependent coefficient later.

5. Dark and bright soliton solutions

Having the explicit form of the DT, we are ready to construct exact solutions of the two-dimensional cmKdV-MB system with time-dependent coefficient (1)-(5). We assume trivial seed solutions as $q = p = w = v = 0, \eta = 1$.

Then the corresponding associated linear system takes the form

$$\Psi_x = A\Psi, \tag{58}$$

$$\Psi_t = 4\lambda^2\Psi_y + B\Psi, \tag{59}$$

where

$$A = \begin{pmatrix} -i\lambda & 0 \\ 0 & i\lambda \end{pmatrix}, \tag{60}$$

$$B = \begin{pmatrix} -\frac{i\gamma(t)}{2(\lambda+\omega)} & 0 \\ 0 & \frac{i\gamma(t)}{2(\lambda+\omega)} \end{pmatrix}. \tag{61}$$

The system (58)-(59) admits the following exact solutions

$$\psi_1 = \exp(-i\lambda x + \mu y + \int(4\lambda^2\mu - \frac{i\gamma}{2(\lambda+\omega)})dt + \delta_0), \tag{62}$$

$$\psi_2 = \exp(i\lambda x - \mu y - \int(4\lambda^2\mu - \frac{i\gamma}{2(\lambda+\omega)})dt + \delta_0 + i\delta_1), \tag{63}$$

where δ_0 and δ_1 are all arbitrary fixed real constants. Choosing $\lambda = a_1 + b_1 i, \omega = 1.5, \mu = c_1 + d_1 i, \delta_0 = 0, \delta_1 = 0$ and substituting (62)-(63) into the one-fold Darboux transformation (41)-(45) and we can get the one-soliton solutions in the following form:

$$q^{[1]} = 8i\cosh(\frac{\theta_2}{A_1})\exp(\frac{-i\theta_1}{A_1}), \tag{64}$$

$$v^{[1]} = \frac{A_2}{A_1}\cosh^{-2}\left(\frac{\theta_2}{A_1}\right), \tag{65}$$

$$w^{[1]} = \frac{A_3}{A_1}\cosh^{-2}\left(\frac{\theta_2}{A_1}\right), \tag{66}$$

$$\eta^{[1]} = \frac{(\omega^2+2\,\omega\,a_1+{a_1}^2+{b_1}^2)\cosh(\frac{\theta_2}{A_1})+2\,\omega^2+4\,\omega\,a_1+2\,{a_1}^2-6\,{b_1}^2}{\left(\omega^2+2\,\omega\,a_1+{a_1}^2+{b_1}^2\right)\cosh(\frac{\theta_2}{A_1})+2\,\omega^2+4\,\omega\,a_1+2\,{a_1}^2+2\,{b_1}^2}, \tag{67}$$

$$p^{[1]} = \frac{2\sinh(\frac{\theta_3}{A_1})+(2\omega+2a_1)\cosh(\frac{\theta_3}{A_1})}{(2\omega^2+2\omega a_1+2a_1^2+2b_1^2)\cosh(\frac{\theta_2}{A_1})}, \tag{68}$$

where $\theta_1 = 8\,\omega^2 t{a_1}^2 d_1 + 16\,\omega^2 t a_1 b_1 c_1 - 8\,\omega^2 t{b_1}^2 d_1 + 16\,\omega\, t{a_1}^3 d_1 + 32\,\omega\, t{a_1}^2 b_1 c_1 - 16\,\omega\, t a_1{b_1}^2 d_1 + 8\,t{a_1}^4 d_1 + 16\,t{a_1}^3 b_1 c_1 + 16\,t a_1{b_1}^3 c_1 - 8\,t{b_1}^4 d_1 - 2\,\omega^2 x a_1 + 2\,\omega^2 y d_1 - 4\,\omega\, x{a_1}^2 + 4\,\omega\, y a_1 d_1 - 2\,x{a_1}^3 - 2\,x a_1{b_1}^2 + 2\,y{a_1}^2 d_1 + 2\,y{b_1}^2 d_1 - \omega\, t\gamma - t a_1\gamma$,

$\theta_2 = 8\,\omega^2 t{a_1}^2 c_1 - 16\,\omega^2 t a_1 b_1 d_1 - 8\,\omega^2 t{b_1}^2 c_1 + 16\,\omega\, t{a_1}^3 c_1 - 32\,\omega\, t{a_1}^2 b_1 d_1 - 16\,\omega\, t a_1{b_1}^2 c_1 + 8\,t{a_1}^4 c_1 - 16\,t{a_1}^3 b_1 d_1 - 16\,t a_1{b_1}^3 d_1 - 8\,t{b_1}^4 c_1 + 2\,\omega^2 x b_1 + 2\,\omega^2 y c_1 + 4\,\omega\, x a_1 b_1 + 4\,\omega\, y a_1 c_1 + 2\,x{a_1}^2 b_1 + 2\,x{b_1}^3 + 2\,y{a_1}^2 c_1 + 2\,y{b_1}^2 c_1 - t\gamma b_1$,

$\theta_3 = 16\,i\omega^2 t a_1 b_1 c_1 + 32\,i\omega\, t{a_1}^2 b_1 c_1 + 4\,\omega\, x a_1 b_1 + 4\,\omega\, y a_1 c_1 + 8\,\omega^2 t{a_1}^2 c_1 - 8\,\omega^2 t{b_1}^2 c_1 + 16\,\omega\, t{a_1}^3 c_1 - 16\,t{a_1}^3 b_1 d_1 - 16\,t a_1{b_1}^3 d_1 + 2\,iy{b_1}^2 d_1 + 8\,it{a_1}^4 d_1 + 2\,i\omega^2 y d_1 + 2\,iy{a_1}^2 d_1 + 4\,i\omega\, y a_1 d_1 + 16\,it a_1{b_1}^3 c_1 + 16\,it{a_1}^3 b_1 c_1 + 16\,i\omega\, t{a_1}^3 d_1 + 8\,i\omega^2 t{a_1}^2 d_1 - 16\,i\omega\, t a_1{b_1}^2 d_1 - 2\,ix{a_1}^3 - 8\,it{b_1}^4 d_1 - 2\,i\omega^2 x a_1 - 4\,i\omega\, x{a_1}^2 - 2\,ix a_1{b_1}^2 - 8\,i\omega^2 t{b_1}^2 d_1 - 32\,\omega\, t{a_1}^2 b_1 d_1 - 16\,\omega\, t a_1{b_1}^2 c_1 - i\omega\, t\gamma - 16\,\omega^2 t a_1 b_1 d_1 - it\gamma a_1 + 2\,x{b_1}^3 + 2\,y{a_1}^2 c_1 + 2\,x{a_1}^2 b_1 - 8\,t{b_1}^4 c_1 + 2\,\omega^2 y c_1 + 2\,y{b_1}^2 c_1 - t\gamma b_1 + 2\,\omega^2 x b_1 + 8\,t{a_1}^4 c_1$, $A_1 = (ib_1 + a_1 + \omega)\,(ib_1 - \omega - a_1)$, $A_2 = 64\,\left({b_1}^2 + (\omega + a_1)^2\right) b_1\,((\omega + 2\,a_1 - d_1)\,b_1 + c_1\,(\omega + a_1))$, $A_3 = 32\,c_1\left(\omega^2 + 2\,\omega\,a_1 + {a_1}^2 + {b_1}^2\right) b_1$.

By substituting eigenfunctions (62) and (63) into Darboux transformation (41)-(45), one-soliton solutions of the system (1)-(5) are obtained as Fig.1,Fig.2 and Fig.3. These present that $q^{[1]}, p^{[1]}, w^{[1]}$ and $v^{[1]}$ are bright solitons, $\eta^{[1]}$ is dark soliton.

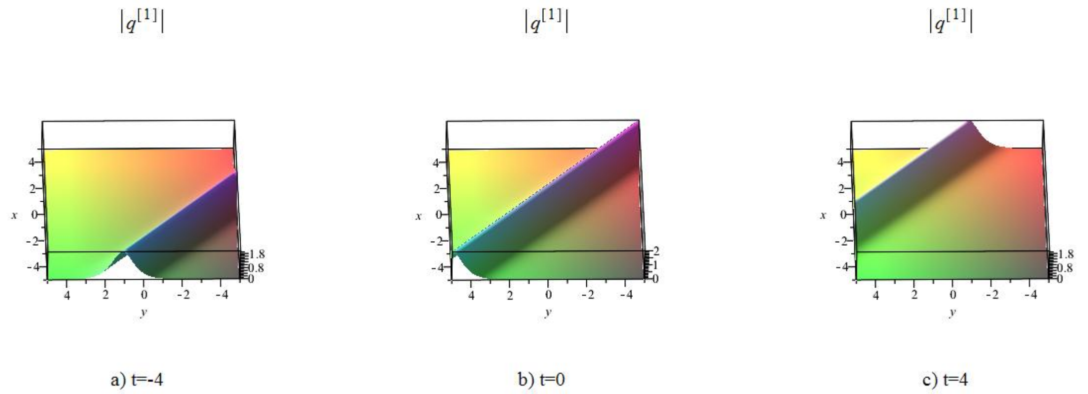

Figure 1. Evolution of bright one-soliton solution $q^{[1]}$ when $a_1 = 1, b_1 = 0.5, c_1 = 1$, $d_1 = -0.5, \omega = 1.5, \gamma(t) = 8t$.

In order to construct the two-soliton solutions of the two-dimensional cmKdV-MB system with time-dependent coefficient we use four spectral parameters $\lambda_1 = a_1 + b_1 i, \lambda_2 = a_2 + b_2 i, \mu_1 = c_1 + d_1 i, \mu_2 = c_2 + d_2 i$. The two-soliton solutions of the two-dimensional cmKdV-MB system with time-dependent coefficient can be obtained by taking $\omega = 1.5, a_1 = 0.5, b_1 = 2, c_1 = 0.5, d_1 = -0.5, a_2 = 1, b_2 = 1.5, c_2 = 0.5, d_2 = 1, \gamma(t) = 8t$ and using two-fold Darboux transformation (53)-(57). The pictorial representation for interaction of two-solitons of the two-dimensional cmKdV-MB system with time-dependent coefficient is shown in Fig.4, Fig.5 and Fig.6.

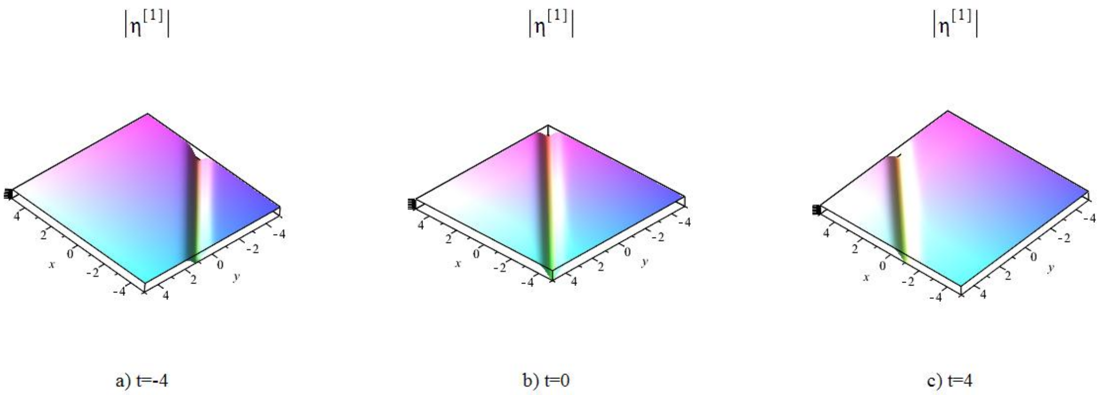

Figure 2. Evolution of dark one-soliton solution $\eta^{[1]}$ when $a_1 = 1, b_1 = 0.5, c_1 = 1,$ $d_1 = -0.5, \omega = 1.5, \gamma(t) = 8t$.

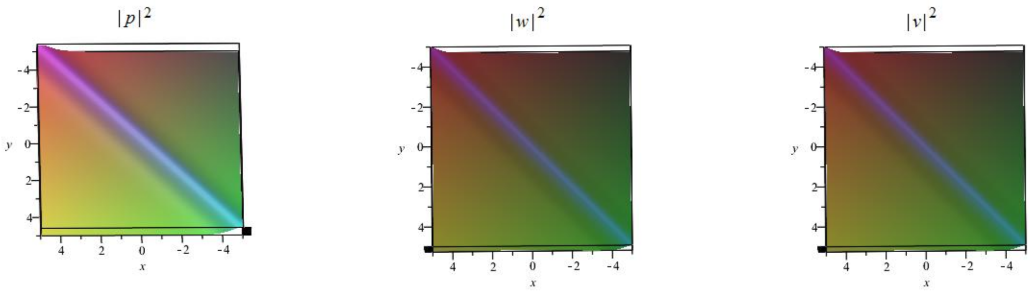

Figure 3. Bright one-soliton solutions $p^{[1]}, w^{[1]}, v^{[1]}$ when $a_1 = 1, b_1 = 0.5, c_1 = 1,$ $d_1 = -0.5, \omega = 1.5, \gamma(t) = 8t$.

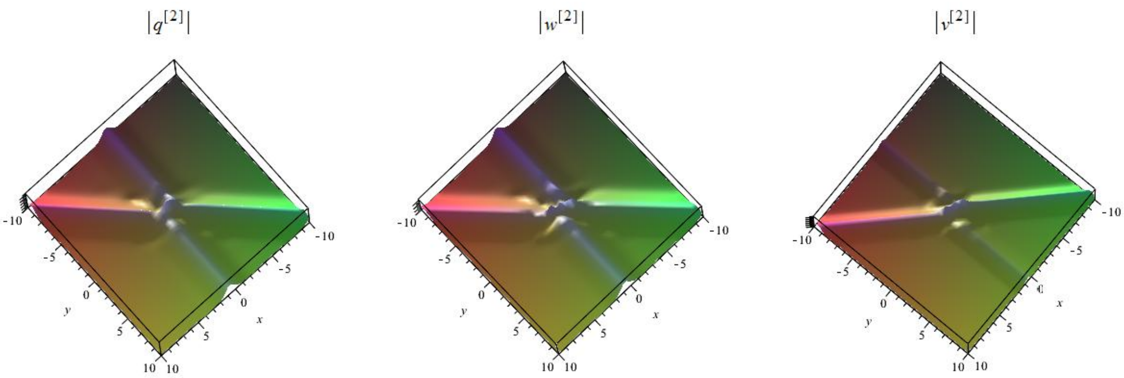

Figure 4. Bright two-soliton solutions $q^{[2]}, w^{[2]}, v^{[2]}$ when $a_1 = 1, b_1 = 0.5, \omega = 1.5, c_1 = 1,$ $d_1 = -0.5, a_2 = 1, b_2 = 1.5, c_2 = 0.5, d_2 = 1, \gamma(t) = cos(t)$.

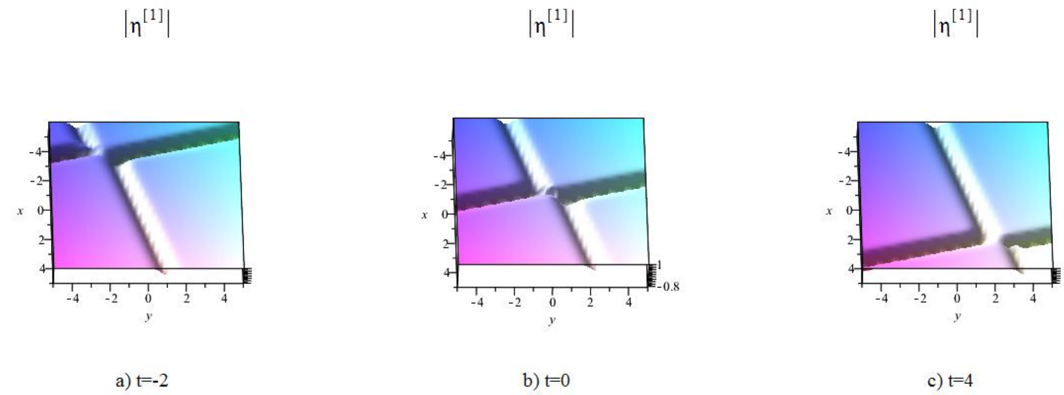

Figure 5. Evolution of dark two-soliton solutions $\eta^{[2]}$ when $a_1 = 1, b_1 = 0.5, \omega = 1.5,$ $c_1 = 1, d_1 = -0.5, a_2 = 1, b_2 = 1.5, c_2 = 0.5, d_2 = 1, \gamma(t) = cos(t)$.

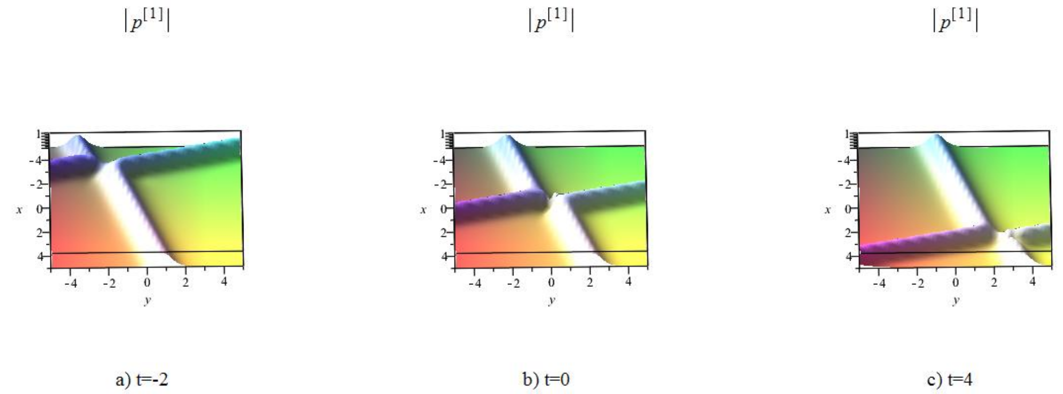

Figure 6. Evolution of bright two-soliton solutions $p^{[2]}$ when $a_1 = 1, b_1 = 0.5, \omega = 1.5,$ $c_1 = 1, d_1 = -0.5, a_2 = 1, b_2 = 1.5, c_2 = 0.5, d_2 = 1, \gamma(t) = cos(t)$.

6. Conclusion

In this work, we presented the Lax representation and Darboux transformaion of the two-dimensional complex modified Korteweg-de Vries and Maxwell-Bloch system with time-dependent coefficient. Also, dark and bright soliton solutions of the cmKdV-MB system with time-dependent coefficient have been constructed explicity by assuming trivial seed solutions. Further, the figures of soliton solutions for the two-dimensional cmKdV-MB system with time-dependent coefficient are given. We note that using the above presented Darboux transformation, one can also construct other type exact solutions of the two-dimensional cmKdV-MB system with time-dependent coefficient.

The work of G.Sh., O.N., K.Y.and G.B. is supported by the Ministry of Education and Science of the Republic of Kazakhstan (the base part of the state task No.0888/GF4)

References

[1] Debnath L 1997 *Nonlinear partial differential equations for scientist and engineers* (Boston: Birkhauser)
[2] Wadati M 2001 *Pramana: Journal of Physics* **57** 841-847
[3] Wadati M 1972 *J. Phys Soc. Jpn* **32** 1681-1687
[4] Wazwaz A M 2009 *Partial Differential Equations and Solitary Waves Theory* (Berlin: Springer)
[5] Peng Y Z 2010 *Commun. Theor. Phys.* **54** 863-865
[6] Lu X, Ma W X, Khalique C M 2015 *Appl. Math. Lett.* **50** 37-42

[7] Wazwaz A M 2011 *Phys. Scr.* **84** 035010
[8] Grses M, Pekcan A 2011 *J. Math. Phys.* **52** 083516
[9] Hereman W, Nuseir A 1997 *Math. Comput. Simul.* **43** 1327
[10] Lou S 1997 *Commun. Theor. Phys* **28** 4150
[11] Biswas A 2009 *Nonlinear Dyn.* **58** 345-348
[12] Ma W X, Zhu Z 2012 *Appl. Math. and Comput.* **218** 11871-11879
[13] Ma W X, Abdeljabbar A and Assad M G 2011 *Appl. Math. Comput.* **217** 10016-10023
[14] El-Tantawy S A, Moslem W M 2014 *Phys. Plasma* **21** 052112
[15] Yesmakanova K R, Shaikhova G N, Bekova G T 2016 *AIP Conf. Proceedings* **1759** 020147
[16] He J, Wang L,Li L, Porseizian K and Erdely R *Preprint* 1405.7845
[17] Bekova G T, Shaikhova G N, Yesmakanova K R, Myrzakulov R 2017 *J. Phys.: Conf. Series* **804** 012004
[18] Myrzakulov R, Mamyrbekova G K, Nugmanova G N, Lakshmanan M 2015 *Symmetry* **7** 1352-1375
[19] Yesmakanova K R, Shaikhova G N, Bekova G T, Myrzakulov R 2016 *J. Phys.: Conf. Series* **738** 012018
[20] Yesmakanova K R, Shaikhova G N , Bekova G T, Myrzakulova Zh R 2016 *Advances in Intelligent Systems and Computing* **441** 183-198
[21] Li C, He J *Preprint* 1210.2501v1
[22] Yang J, Li Ch,Li T, Cheng Zh 2013 *Chinese Physics Letters* **30** 104201
[23] Li Ch, He J, Porsezian K 2013 *Physical Review E* **87** 012913