Quintessential Inflation and curvaton reheating

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In this paper we consider a model of quintessential inflation based upon inverse hyperbolic potential. We employ curvaton mechanism for reheating which is more efficient than gravitational particle production; the mechanism complies with nucleosynthesis constraint due to relic gravity waves. We obtain a lower bound on the coupling constant g that governs the interaction of curvaton with matter fields. We study curvaton decay before domination and decay after domination and plot the allowed region in the parameter space in both cases.

I. INTRODUCTION

The standard model of the Universe has many problems which need to be addressed both at early and at late times. A few examples of the most pertinent problems are the flatness problem, the monopole problem, the horizon problem and the age crisis of the Universe.These problems can be solved by introducing an early phase of accelerated expansion, called inflation, together with late time cosmic acceleration. Recent data obtained from type Ia supernovae, CMB background and galaxy clustering suggest late time cosmic acceleration. [1–3].

Both inflation and late time acceleration are often studied separately but if we build a theory in which both the above phenomena are implemented by using a single scalar field then a unique cause could drive both. This is known as ”quintessential inflation” and its advantage is that it is efficient as it provides a common theoretical framework[4–7]. In order to achieve this unification the field must first evolve very slowly in order to drive inflation, then it should roll very quickly in order to exit from inflation. Then comes a time when the scalar field kind of disappears from the scene and its evolution no longer plays a prominent role in the history of the Universe. It then reemerges to play a key role in driving late time cosmic acceleration. Thus inflation is reborn as late time cosmic acceleration.

In order to model the desired behaviour of the scalar field, we usually describe the dynamics of the scalar field with the help of a potential $V(\phi)$ which is unknown so far and whose functional form is highly debatable [8]. But the potential should have certain characteristics in order to achieve the desired result. It should be shallow at early times in order for slow roll to commence. For inflation to end the shallow behaviour should be followed by the potential falling off steeply. A scaling solution is thus obtained where the energy density of field changes in exactly the same way as that of the background. The potential should then have certain late time characteristics giving rise to scaling behaviour. Most generic potentials do not change their shape frequently enough to be shallow early on, then steep and then again shallow at late times. They are either shallow at early times and steep thereafter or vice versa. The first class of models could give rise to a viable scenario of quintessential inflation if a suitable mechanism was constructed in order to exit from the scaling regime in order to obtain late time acceleration. The second class requires extra damping at early times in order to facilitate inflation but fails as it predicts a high tensor to scalar ratio of perturbations and hence these models do not satisfy observational constraints.

We stick to the first case thus. In this case we exit from the scaling regime and obtain late time acceleration by coupling the field non minimally to matter, generally neutrinos as they become non relativistic at late times. This creates a minimum in the field potential at late times without destroying the matter phase.

Such a model with inverse hyperbolic type potential was recently studied and it was shown that coupling to massive neutrino matter led to late time acceleration [1] . The instant preheating mechanism with Yukawa interaction was considered and bounds on the coupling constants were obtained using the nucleosynthesis constraint on relic gravity waves produced during inflation. Bounds on the reheating temperature were also obtained.

In this paper the curvaton mechanism is used to reheat the Universe and generate the large -scale curvature perturbations in our Universe, which is the dominant cause of structure in the Universe.

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In the curvaton mechanism, the inflaton field only drives inflation. During the inflationary era the energy density of the curvaton field is sub-dominant and is not diluted by the expansion. After horizon exit, the quantum fluctuations of the curvaton field get converted into classical perturbations with a flat spectrum. This corresponds to isocurvature perturbations. The moment inflation ends the energy density of the curvaton field becomes (pre) dominant. Then the isocurvature perturbations are converted into adiabatic yielding curvature perturbations which are fairly large in size. The curvaton then decays into conventional matter forcing the perturbations to stay adiabatic. The curvaton is thus responsible for all the present material in the Universe and also for the Large Scale Structure of the Universe [9–19].

The advantages of curvaton reheating is that it is efficient unlike gravitational particle production and it solves many problems that are associated with various methods of reheating, like instant preheating, fermion preheating or those based on inflaton decay [20–27]. One such problem in literature is the famous η - problem [28, 29]. Introducing the curvaton makes it easier to construct sensible models of slow - roll inflation. With curvaton reheating a higher reheating temperature can be obtained. Curvaton models predict a small value of tensor-to-scalar ratio [30]. Also in this scenario gravitational waves of smaller wavelength do not have a larger amplitude than desired and hence such waves do not dominate. Curvaton models can also predict the duration of the inflationary period [31].

In this paper the same model with inverse hyperbolic type potential as discussed above is considered with curvaton as reheating mechanism.

II. THE INFLATON FIELD

In this section we describe the inflaton field, a real scalar field denoted by ϕ. The field does not couple much to other constituents of the Universe - matter, radiation and neutrinos. Its dynamics is governed by the potential,

$$V(\phi) = \frac{V_0}{\cosh\left(\frac{\phi^n}{\lambda^n}\right)} = \frac{V_0}{\cosh\left[\beta^n\left(\frac{\phi}{M_{pl}}\right)^n\right]} \tag{1}$$

,where V_0, λ are free parameters. We consider a flat FRW background, with a metric,

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j. \tag{2}$$

Here $\lambda = \alpha M_{pl}$ and $\beta = \frac{1}{\alpha}$, where α and β are dimensionless parameters and n is an integer. The following action is considered for the resulting dynamical system,

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right] + S_m + S_\nu + S_R. \tag{3}$$

S_m, S_ν, S_R represent the actions for standard matter, massive neutrino matter and radiation respectively and all three play a pivotal role in the post inflationary era. The Friedmann equations for the action (3) in flat FRW geometry reduce to,

$$3H^2M_{pl}^2 = \rho_m + \rho_r + \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{4}$$

and

$$\left(2\dot{H} + 3H^2\right)M_{pl}^2 = -\frac{1}{3}\rho_r - \frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{5}$$

The equation of motion for the scalar field is,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0. \tag{6}$$

The slow roll parameters for a potential $V(\phi)$ are defined as usual [32, 33],

$$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{1}{V}\frac{dV}{d\phi}\right)^2, \tag{7}$$

and

$$\eta = \frac{M_{pl}^2}{V}\frac{d^2V}{d\phi^2}. \tag{8}$$

The end of inflation is marked by,

$$\epsilon|_{\phi=\phi_{end}} = 1, \tag{9}$$

where "end" represents the value at the end of inflation. Let us consider a period which begins when the modes cross the horizon and ends with the end of inflation. Then the number of e-foldings during this period is given by [32, 33],

$$N(k) = M_{pl}^{-1}\int_{\phi_{end}}^{\phi}\frac{d\phi}{\sqrt{2\epsilon(\phi)}} \tag{10}$$

$$= \frac{1}{M_{pl}^2}\int_{\phi_{end}}^{\phi}\frac{V(\phi')}{V'(\phi')}d\phi'. \tag{11}$$

The tensor to scalar ratio r is given by,

$$r = 16\epsilon, \tag{12}$$

and the scalar spectral index n_s, which is defined as,

$$n_s - 1 = \frac{d(logP_R)}{d(logk)}, \tag{13}$$

where P_R is the spectrum of curvature perturbations, is reduced to the form,

$$n_s = 2\eta - 6\epsilon + 1. \tag{14}$$

Throughout the rest of the paper we use $n = 6$ and $\beta = 1$. For these values of n and β, the theoretical value of the spectral index n_s is in agreement with the Planck 2015 data up-to the 1σ confidence level and also the tensor-to-scalar ratio r satisfies the Planck 2015 data, i.e., $r < 0.1$ [34].

After inflation ends, the kinetic epoch begins and during this era the energy density of the inflaton field falls as,

$$\rho_\phi = \rho_\phi^{(kin)} \left(\frac{a_{kin}}{a}\right)^6 \tag{15}$$

Thus the inflaton field describes what is known as "stiff matter" during the kinetic epoch. We use 'kin' to label the value of the different quantities at the beginning of the kinetic epoch.

Introducing dimensionless scalar field $\chi = \frac{\phi}{M_{pl}}$ in equation 1 one obtains,

$$V = \frac{V_0}{\cosh\left(\beta^n \chi^n\right)} \tag{16}$$

Now using the following values: $\chi_{in} = 0.44$, $\chi_{end} = 0.88$ and $V_0 = 4.64 * 10^{60} GeV^4$. Upon calculation we obtain initially, $V_i = V_{in} = 4.63988 \times 10^{60} GeV^4$ and also finally $V_f = V_{end} = 4.18098 \times 10^{60} GeV^4$. Further using the relation, $H_{in}^2 = \frac{V_{in}}{3M_{pl}^2}$ one obtains,

$$H_{in}^2 = 2.68512 \times 10^{23} GeV^2 \tag{17}$$

and hence,

$$H_{in} = 5.18181 \times 10^{11} GeV \tag{18}$$

Also using, $H_{end}^2 = \frac{V_{end}}{2M_{pl}^2}$, we calculate,

$$H_{end}^2 = 3.62932 \times 10^{23} GeV^2 \tag{19}$$

and hence,

$$H_{end} = 6.02438 \times 10^{11} GeV \tag{20}$$

The amplitude of gravitational waves h_{GW}^2 is given by,

$$h_{GW}^2 = \frac{H_{in}^2}{8\pi M_{pl}^2} = 5.82139 \times 10^{-15} \tag{21}$$

Gravitational waves behave as spin-less fields having no mass and then their amplitude remains constant during inflation. During the kinetic epoch, the energy density of gravitational waves evolves as,

$$\rho_g = \frac{64}{3\pi} h_{GW}^2 \rho_\phi \left(\frac{a}{a_{kin}}\right)^2 \tag{22}$$

During the moment when the energy density of the stiff scalar matter equals the energy density of the radiation the energy density of the gravitational waves is given by, $(\rho_\phi = \rho_r)$ is,

$$\frac{\rho_g}{\rho_r}\bigg|_{a=a_{eq}} = \frac{64}{3\pi} h_{GW}^2 \left[\frac{a_{eq}}{a_{kin}}\right]^2 \tag{23}$$

III. THE CURVATON FIELD

The curvaton, denoted by σ, is a real scalar field whose dynamics is governed by the potential -

$$U(\sigma) = \frac{m_\sigma^2 \sigma^2}{2} \tag{24}$$

and obeys the Klein Gordan equation,

$$\ddot{\sigma} + 3H\dot{\sigma} + m_\sigma^2 \sigma = 0 \tag{25}$$

Here m_σ is the curvaton mass. During the inflationary period the curvaton field stays massless, $m \ll H_f$ and it can be shown mathematically that under these conditions the curvaton field remains constant during the inflationary era, $\sigma_f \simeq \sigma_i$ and also $\dot{\sigma}_f = 0$. Then we have,

$$U_f = \frac{1}{2} m_\sigma^2 \sigma_f^2 = \frac{1}{2} m_\sigma^2 \sigma_i^2 \tag{26}$$

The subscripts i and f denote the values of the quantities at the beginning and end of inflation.

When inflation ends, the kinetic regime begins, and it is during this era that the curvaton acquires a mass. This happens at a time when $m \simeq H$. We then get,

$$\frac{m}{H_{kin}} = \frac{a_{kin}^3}{a_m^3} \tag{27}$$

Quantities labelled by the subscript m indicate that they were evaluated at exactly the same moment when the curvaton acquired a mass. Till this point, the curvaton field stayed massless and then, $\sigma_m \simeq \sigma_i$. We do not desire a period where the curvaton field drives inflation and for this to occur the the scalar stiff-matter should dominate the Universe. Thus the energy of the curvaton field must be much less than that of stiff matter. This leads to the equation,

$$\sigma_i^2 \ll \frac{3}{4\pi} m_{pl}^2 \tag{28}$$

where σ_i is the initial value of the curvaton field.

IV. CONSTRAINING THE REHEATING TEMPERATURE

A. The Curvaton Reheating Mechanism

If the Curvaton field, denoted by σ, decays into two fermions f and $\bar{f}$ during Curvaton Reheating then the Curvaton Reheating is governed by the reaction:

$$\sigma \to f\bar{f}$$

If the Lagrangian that governs this reaction is characterized by the coupling constant g, then the decay width of this reaction is given by,

$$\Gamma = \frac{g^2 m_\sigma}{8\pi} \tag{29}$$

The reheating temperature is thus given by,

$$T_{rh} \approx 0.78\, g_*^{-\frac{1}{4}} \sqrt{M_{pl}\Gamma} \tag{30}$$

Using equations (29) and (30) ,

$$T_{rh} \approx 0.78\, g_*^{-\frac{1}{4}} \sqrt{\frac{m_\sigma M_{pl}}{8\pi}} \times g \tag{31}$$

The curvaton field decays at a time when $\Gamma_\sigma = H$ and then

$$\frac{\Gamma_\sigma}{H_{kin}} = \frac{a_{kin}^3}{a_d^3} \Rightarrow \frac{a_{kin}}{a_d} = \sqrt[3]{\frac{\Gamma_\sigma}{H_{kin}}} \tag{32}$$

B. Obtaining the Constraint

We now derive the bound on the reheating temperature that is consistent with the nucleosynthesis constraint using this mechanism. This reduces to finding those values of the coupling constant g for which the desired reheating temperature is obtained. These values of g are obtained by deriving a bound on g and it gives us the permissible values of the coupling constant g for our model. While deriving the bound in equation (28) only the properties of the Curvaton field was used. The properties of the inflaton field have not been considered so far. Thus apart from the Curvaton model this result is model independent. We can thus apply it to our model of Quintessential Inflation. The above bound combined with the fact that the curvaton energy should also be sub-dominant at the end of inflation compared with the energy of the inflaton field gives us a bound on the curvaton mass m_σ. This then combined with the required reheating temperature gives us a bound on g where we use equation (31).

We have from equation (26),

$$U_f = \frac{1}{2} m_\sigma^2 \sigma_i^2 \tag{33}$$

and from equation (16)

$$V_f = \frac{V_0}{\cosh\left(\beta^n \chi_f^n\right)} \tag{34}$$

Dividing (33) by (34),

$$\frac{U_f}{V_f} = \frac{1}{2}\frac{m_\sigma^2}{V_0} \cosh\left(\beta^n \chi_f^n\right) \sigma_i^2 \tag{35}$$

Now using the bound on σ_i given by equation (28) in equation (35) one can obtain,

$$\frac{U_f}{V_f} \ll \frac{3}{8\pi}\frac{m_\sigma^2 m_{pl}^2}{V_0} \cosh\left(\beta^n \chi_f^n\right) \tag{36}$$

Now $\frac{U_f}{V_f} \ll 1$ implies the strong condition that,

$$\frac{3}{8\pi}\frac{m_\sigma^2 m_{pl}^2}{V_0} \cosh\left(\beta^n \chi_f^n\right) \ll 1 \tag{37}$$

Then,

$$m_\sigma^{\frac{1}{2}} \ll \left[\frac{8\pi}{3}\frac{1}{m_{pl}^2}\frac{V_0}{\cosh\left(\beta^n \chi_f^n\right)}\right]^{\frac{1}{4}} \tag{38}$$

We have now obtained the bound on the mass of the curvaton field. Using equation (38) in equation (31),

$$T_{rh} \ll 0.78 g_*^{-\frac{1}{4}} g \frac{1}{3^{\frac{1}{4}}}\frac{1}{(8\pi)^{\frac{1}{2}}}\left[\frac{V_0}{\cosh\left(\beta^n \chi_f^n\right)}\right]^{\frac{1}{4}} \tag{39}$$

We have,

$$T_{rh} \geq 2.2 \times 10^{12}\mathrm{GeV} \tag{40}$$

Combining equations (39) and (40),

$$2.2 \times 10^{12}\mathrm{GeV} \leq T_{rh} \ll 0.78 g_*^{-\frac{1}{4}} g \frac{1}{3^{\frac{1}{4}}}\frac{1}{(8\pi)^{\frac{1}{2}}}\left[\frac{V_0}{\cosh\left(\beta^n \chi_f^n\right)}\right]^{\frac{1}{4}} \tag{41}$$

Now using the constraints on the inflaton field, $n = 6$, $\beta = 1$, $V_0 = 4.64 \times 10^{60}$GeV, $\chi_f = 0.88$ together with the value of g_*, $g_* = 140$, one obtains

$$2.2 \times 10^{12}\text{GeV} \le T_{rh} \ll 4.91577 \times 10^{13} g \text{ GeV} \quad (42)$$

We thus obtain the following bound on g,

$$g \gg 0.0447539 \quad (43)$$

V. EXPLORING THE PARAMETER SPACE

We study here two cases -

1. curvaton decays before domination
2. curvaton decays after domination

A. Curvaton Decay before Domination

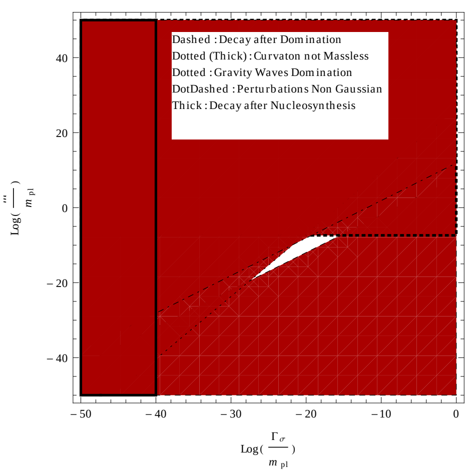

FIG. 1: Curvaton constraints for the case in which the curvaton field decays before domination, using N = 60 in Eqs. (56), (57), (58), (61), (67), (70), (72) The regions excluded by each constraint is shaded, and the allowed region is unshaded.

We now study the case of curvaton decay before domination as applied to the same inverse hyperbolic cosine potential. There comes a time when the energy density of the stiff scalar matter equals the energy density of the curvaton matter. During this moment,

$$\frac{\rho_\sigma}{\rho_\phi}\Big|_{a=a_{eq}} = \frac{4\pi}{3}\frac{m^2\sigma_i^2}{m_{pl}^2 H_{kin}^2}\frac{a_m^3}{a_{kin}^3}\frac{a_{eq}^3}{a_{kin}^3} = 1 \quad (44)$$

This is an equation that is model independent in the sense that it only depends on the properties of the curvaton field and not of the inflaton field. This means that we can use this equation for our potential directly. Using the above equation one obtains,

$$H_{eq} = H_{kin}\,\frac{a_{kin}^3}{a_{eq}^3} = \frac{4\pi}{3}\,\frac{\sigma_i^2}{m_{pl}^2}\,m \quad (45)$$

Now in order that the curvaton field should not dominate the expansion history of the Universe and that it should decay after it becomes massive the constraint $H_{eq} < \Gamma_\sigma < m$ needs to be satisfied and thus from equation (45),

$$\frac{4\pi}{3}\frac{\sigma_i^2}{m_{pl}^2} m < \Gamma_\sigma < m \quad (46)$$

Just like equation (44), equation (46) can also be applied to our potential directly.

The Bardeen parameter in this case is given by,

$$P_\zeta = \frac{r_d^2}{36\pi^2}\frac{H_i^2}{\sigma_i^2} \quad (47)$$

The normalization factor indicates that the dominant component at that time is the scalar stiff matter. Unlike in equation (44) the energy density of the curvaton field does not equal that of stiff scalar matter and r_d represents the ratio of these two densities at the exact moment when the curvaton decays. Here r_d is given by,

$$r_d = \frac{\rho_\sigma}{\rho_\phi}\Big|_{a=a_d} = \frac{4\pi}{3}\frac{m}{\Gamma_\sigma}\frac{\sigma_i^2}{m_{pl}^2} \quad (48)$$

Combining equations (47) and (48),

$$\frac{\sigma_i^2}{m_{pl}^2} = \frac{81}{4}P_\zeta\frac{m_{pl}^2}{H_i^2}\frac{\Gamma_\sigma^2}{m^2} \quad (49)$$

Now substituting equation (49) in (46),

$$\left(27\pi P_\zeta\frac{m_{pl}^2}{H_i^2}\frac{\Gamma_\sigma^2}{m^2}\right)m < \Gamma_\sigma < m \quad (50)$$

Define,

$$A = 27\pi P_\zeta\frac{m_{pl}^2}{H_i^2} = 27\pi P_\zeta\frac{m_{pl}^2}{H_{in}^2} \quad (51)$$

Fitting our model now, $H_{in}^2 = 2.68512 \times 10^{23} GeV^2$, $m_{pl} = 1.220910 \times 10^{19} GeV$ and as observed from COBE $P_\zeta = 2.3 \times 10^{-9}$, one obtains

$$A = 1.08304 \times 10^8 \quad (52)$$

From equation (50), we have three constraints for this model.

$$\frac{m}{m_{pl}} > A\frac{\Gamma_\sigma}{m_{pl}} \qquad (53)$$

$$\frac{m}{m_{pl}} > \sqrt{A}\frac{\Gamma_\sigma}{m_{pl}} \qquad (54)$$

$$\frac{m}{m_{pl}} > \frac{\Gamma_\sigma}{m_{pl}} \qquad (55)$$

Thus the first three constraints for our model are,

$$\frac{m}{m_{pl}} > \left(1.08304 \times 10^8\right)\frac{\Gamma_\sigma}{m_{pl}} \qquad (56)$$

$$\frac{m}{m_{pl}} > \left(1.04069 \times 10^4\right)\frac{\Gamma_\sigma}{m_{pl}} \qquad (57)$$

$$\frac{m}{m_{pl}} > \frac{\Gamma_\sigma}{m_{pl}} \qquad (58)$$

where we have used equation (52).
Now from equation (36),

$$\frac{U_f}{V_f} \ll \frac{3}{8\pi}\frac{m^2 m_{pl}^2}{V_0}\cosh\left(\beta^n \chi_f^n\right) \ll 1 \qquad (59)$$

which gives us,

$$\frac{m}{m_{pl}} \ll \frac{1}{m_{pl}^2}\left[\frac{8\pi}{3}\left(\frac{V_0}{\cosh\left(\beta^n \chi_f^n\right)}\right)\right]^{\frac{1}{2}} \qquad (60)$$

Fitting our model,

$$\frac{m}{m_{pl}} \ll 3.97037 \times 10^{-8} \qquad (61)$$

This is the fourth constraint for our model.
Radiation equals the stiff scalar matter $\left(\rho_r^{(\sigma)} = \rho_\phi\right)$ at a time given by,

$$\frac{4\pi}{3}\frac{m^2\sigma_i^2}{m_{pl}^2 H_{kin}^2}\frac{a_m^3}{a_{kin}^3}\frac{a_{eq}^2}{a_{kin}^2}\frac{a_d}{a_{kin}} = 1 \qquad (62)$$

Then using equations (23), (27), (32), (62),

$$\frac{\rho_g}{\rho_r}\Big|_{a=a_{eq}} = \frac{16}{\pi^2}h_{GW}^2\sqrt[3]{\frac{\Gamma_\sigma}{H_{kin}}}\frac{m}{H_{kin}}\frac{m_{pl}^2}{\sigma_i^2} \times \frac{H_{kin}^2}{m^2} \qquad (63)$$

We have,

$$\frac{\rho_g}{\rho_r}\Big|_{a=a_{eq}} = \frac{16}{\pi^2}h_{GW}^2\sqrt[3]{\frac{\Gamma_\sigma}{H_{kin}}}\frac{m_{pl}^2}{\sigma_i^2} \times \frac{H_{kin}}{m} \ll 1 \qquad (64)$$

Now upon using equation (49),

$$\frac{64}{81\pi^2 P_\zeta}h_{GW}^2\frac{\Gamma_\sigma^{\frac{1}{3}}}{H_{kin}^{\frac{1}{3}}}\frac{H_i^2}{m_{pl}^2}\frac{m^2}{\Gamma_\sigma^2} \times \frac{H_{kin}}{m} \ll 1 \qquad (65)$$

Simplifying and using $H_{kin} \approx H_{end}$,

$$\frac{m}{m_{pl}} \ll \frac{81\pi^2 P_\zeta}{64 h_{GW}^2}\left(\frac{H_{end}}{H_i^2}\right)^2 \times \left(\frac{m_{pl}}{H_{end}}\right)^{\frac{8}{3}} \times \left(\frac{\Gamma_\sigma}{m_{pl}}\right)^{\frac{5}{3}} \qquad (66)$$

Also using , $P_\zeta = 2.3 \times 10^{-9}$, $h_{GW}^2 = 5.82139 \times 10^{-15}$, $m_{pl} = 1.220910 \times 10^{19} GeV$, $H_{in} = 5.18181 \times 10^{11}$GeV and $H_{end} = 6.02438 \times 10^{11} GeV$ we obtain the 5th constraint for the model,

$$\frac{m}{m_{pl}} \ll 2.03652 \times 10^{26}\left(\frac{\Gamma_\sigma}{m_{pl}}\right)^{\frac{5}{3}} \qquad (67)$$

The curvaton perturbations should satisfy the Gaussianity condition. This condition does not hold automatically. By imposing this condition we restrict the amplitude of the perturbations which are then negligible compared to the mean value of the curvaton field,

$$\sigma_i^2 \gg \frac{H_i^2}{4\pi^2} \qquad (68)$$

We now divide by m_{pl}^2, use equation (49) and simplify to obtain,

$$\frac{m}{m_{pl}} \ll 9\pi\sqrt{P_\zeta} \times \frac{m_{pl}^2}{H_i^2}\left(\frac{\Gamma_\sigma}{m_{pl}}\right) \qquad (69)$$

Now $P_\zeta = 2.3 \times 10^{-9}$, $m_{pl} = 1.220910 \times 10^{19} GeV$ and $H_{in} = 5.18181 \times 10^{11} GeV$

$$\frac{m}{m_{pl}} \ll \left(7.52766 \times 10^{11}\right)\frac{\Gamma_\sigma}{m_{pl}} \qquad (70)$$

We thus obtain the sixth constraint for our model. At nucleosynthesis things should now proceed according to the standard Big Bang scenario and hence we also have a seventh constraint for our model,

$$\Gamma_\sigma > 10^{-40} m_{pl} \qquad (71)$$

or

$$\frac{\Gamma_\sigma}{m_{pl}} > 10^{-40} \qquad (72)$$

We plot the allowed region in parameter space as permitted by these constraints in Fig.1. While plotting this figure we take the number of e foldings, $N = 60$, $n = 6$ $\beta = 1$, $V_0 = 4.64 \times 10^{60}$GeV and $\chi_f = 0.88$, which are constraints on the inflaton field. Out of the constraints given by equations. $(56 - 58)$ only the strongest one is shown in the figure whereas the remaining are automatically satisfied. We then plot the constraints given by equations. (61), (67), (70) and (72).

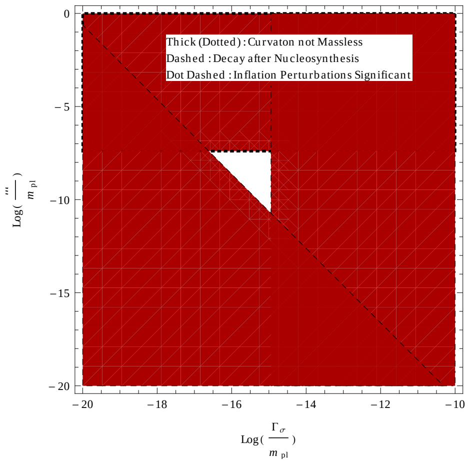

FIG. 2: Curvaton constraints for the case in which the curvaton field decays after domination, using N = 60 in Eqs. (73), (74), (76), (80), (85) The regions excluded by each constraint is shaded, and the allowed region is unshaded.

B. Curvaton Decay after Domination

Equation (28) gives our first constraint,

$$\frac{\sigma_i}{m_{pl}} \ll 0.488603 \tag{73}$$

Equation (61),

$$\frac{m}{m_{pl}} \ll 3.97037 \times 10^{-8} \tag{74}$$

gives us our second constraint.

We desire that the curvaton field should decay before nucleosynthesis occurs. Then the following inequality is satisfied, $H_{\text{nucl}} = 10^{-40} m_{pl} < \Gamma_\sigma$. The curvaton should also decay after it dominates the expansion of the Universe, and then we have the following inequality $\Gamma_\sigma < H_{eq} = \frac{4\pi}{3}\frac{\sigma_i^2}{m_{pl}^2} m$. Thus we obtain,

$$10^{-40} m_{pl} < \Gamma_\sigma < \frac{4\pi}{3}\frac{\sigma_i^2}{m_{pl}^2} m \tag{75}$$

This gives us our third constraint,

$$\frac{m}{m_{pl}}\frac{\sigma_i^2}{m_{pl}^2} \gg \frac{3}{4\pi} \times 10^{-40} \tag{76}$$

We now require that gravity waves do not dominate,

$$\frac{\rho_g}{\rho_\sigma}\Big|_{a=a_{eq}} = \frac{64}{3\pi} h_{GW}^2 \left(\frac{3}{4\pi}\frac{m_{pl}^2}{\sigma_i^2}\frac{H_{kin}}{m}\right)^{\frac{2}{3}} \ll 1 \tag{77}$$

When decay occurs after curvaton domination, the produced perturbation is,

$$P_\zeta = \frac{1}{9\pi^2}\frac{H_{in}^2}{\sigma_i^2} \tag{78}$$

Here P_ζ is the spectrum of the Bardeen parameter.

Now using equations (77), (78), $h_{GW}^2 = \frac{H_{in}^2}{8\pi M_{pl}^2}$ and $H_{kin} \approx H_{end}$, we obtain,

$$\frac{m}{m_{pl}} \gg \left[\frac{(192\pi P_\zeta)^{\frac{3}{2}} \times 3 \times H_{end}}{4\pi m_{pl}}\right]\frac{\sigma_i}{m_{pl}} \tag{79}$$

giving us our fourth constraint,

$$\frac{m}{m_{pl}} \gg \left(1.9249 \times 10^{-17}\right)\frac{\sigma_i}{m_{pl}} \tag{80}$$

The fluctuations of the inflaton field are measured by a quantity called the power spectrum,

$$P_\phi = \left(\frac{H}{2\pi}\right)^2 \tag{81}$$

Now the inflaton field fluctuations should be insignificant compared to those of the curvaton field. Then the power spectrum P_ϕ should be less than unity,

$$P_\phi \ll 1 \tag{82}$$

Thus,

$$\left(P_\phi\right)_i \ll 1 \tag{83}$$

This gives us,

$$\frac{\sigma_i}{m_{pl}} \ll \frac{2}{3 m_{pl}\sqrt{P_\zeta}} \tag{84}$$

upon using equation (78).

Now using, $m_{pl} = 1.220910 \times 10^{19} GeV/c^2$ and $P_\zeta = 2.3 \times 10^{-9}$ we obtain our 5th constraint,

$$\left(\frac{\sigma_i}{m_{pl}}\right) \ll 1.13857 \times 10^{-15} \tag{85}$$

We plot the allowed region in parameter space as permitted by these constraints in figure 2. While plotting this figure we take the number of e foldings, $N = 60$, $n = 6$ $\beta = 1$, $V_0 = 4.64 \times 10^{60}$GeV and $\chi_f = 0.88$, which are constraints on the inflaton field. We then show the constraints given by eqns. (73), (74), (76), (80) and (85). Out of these, two are redundant and we plot the other three.

VI. CONCLUSION

We study a single scalar field model of quintessential inflation with inverse cosh hyperbolic potential where the reheating temperature is obtained using the curvaton mechanism. The temperature thus obtained is consistent with the nucleosynthesis constraint,$T_{rh} \gtrsim 2.2 \times 10^{12}$GeV due to relic gravity waves. We find that upon implementing this mechanism, $2.2 \times 10^{12}\text{GeV} \leq T_{rh} \ll 4.91577 \times 10^{13} g$ GeV. The upper bound on temperature translates into a bound on the coupling constant g, namely, $g \gg 0.0447539$. The nucleosynthesis constraint is thus satisfied.

We have studied the cases of curvaton decay before domination and curvaton decay after domination and plotted the allowed region in parameter space. The case for decay before domination is given in Figure 1 and that for decay after domination is given in Figure 2. While plotting these figures we take the number of e-foldings, $N = 60$, $n = 6$ $\beta = 1$, $V_0 = 4.64 \times 10^{60}$GeV and $\chi_f = 0.88$, which represent constraints on the inflaton field. Out of the constraints given by equations. $(56 - 58)$ only the strongest one is shown in the Fig.1 whereas the remaining two are automatically satisfied. We then plot the constraints given by Eqs. (61), (67), (70) and (72). While plotting Fig.2 we show the constraints given by eqns. (73), (74), (76), (80) and (85). Out of these, two are redundant and we plot the other three. While plotting these figures the regions excluded by each constraint is shaded, and the allowed region is unshaded. From Fig. 1 and Fig. 2 it is clear that the allowed region in parameter space is very small and that real situations arising in both the cases are severely restricted by many constraints.

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