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Practical output tracking for a class of uncertain nonlinear time-delay systems via state feedback

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Abstract. In this paper, the problem of global practical output tracking is investigated by state feedback for a class of uncertain nonlinear time-delay systems. Under mild conditions on the system nonlinearities involving time delay, we construct a homogeneous state feedback controller with an adjustable scaling gain. By a homogeneous Lyapunov-Krasovskii functional, the scaling gain is adjusted to dominate the time-delay nonlinearities bounded by homogeneous growth conditions and render the tracking error can be made arbitrarily small while all the states of the closed-loop system remain to be bounded.

1 Introduction

Consider the following uncertain nonlinear time-delay system

$$
\begin{aligned}
\dot{x}_1(t) &= x_2(t)^{p_1} + \varphi_1(t, x(t), x(t-d), u(t)),\\
&\vdots\\
\dot{x}_{n-1}(t) &= x_n(t)^{p_{n-1}} + \varphi_{n-1}(t, x(t), x(t-d), u(t)),\\
\dot{x}_n(t) &= u + \varphi_n(t, x(t), x(t-d), u(t)),\\
y(t) &= x_1(t),
\end{aligned}
\tag{1}
$$

where $x(t) := (x_1(t), \ldots, x_n(t))^{\mathrm{T}} \in R^n$, $u \in R$, and $y(t) \in R$ are the system state, control input and output, respectively. The constant $d \geq 0$ is a given time-delay of the system, for $i = 1, \ldots, n$, and the system initial condition is $x(\theta) = \varphi_0(\theta)$, $\theta \in [0, d]$. The terms $\varphi_i(\cdot)$ represent nonlinear perturbations that are unknown continuous functions and $p_i \in R_{odd}^{\geq 1} := \{p/q \in [0, \infty) : p$ and q are odd integers, $p \geq q\}$ $(i = 1, \ldots, n-1)$ are said to be the high orders of the system.

Global practical output tracking problem of nonlinear systems is one of the most important and challenging problems in the field of nonlinear control and has received a great deal of attention. By posed some conditions on system growth and power order, the practical output tracking problem of system (1) has been well-studied and a number of

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interesting results have been achieved over the past years, see [1-8], as well as the references therein.

However, the aforementioned results have not considered the time-delay effect. It is well known that time-delay phenomena exist in many practical systems such as electrical networks, microwave oscillator, and hydraulic systems, etc., due to the presence of time delay in systems, it often significant effect on system performance. Therefore, the study the problem of output tracking and stabilization of time-delay nonlinear systems has important practical significance and has received much attention in recent years. In recent years, by employing the Lyapunov-Krasovskii method to deal with the time-delay, control theory, and techniques for stabilization problem of time-delay nonlinear systems were greatly developed and advanced methods have been made; see, for instance, [9 -13] and reference therein. Compared with study the stabilization problem contain time-delay, the theory of output tracking control developed slower. In the case when the nonlinearities contain time-delay, for the output tracking problems, some interesting results have been obtained [14-16]. However, in [14 -16] only considered special case for the system (1), i.e., $p_i = 1$ case. When the system under consideration is inherently time-delay non-linear, the problem becomes more complicated and difficult to solve. To the best of our knowlege, many interesting output tracking control problems for time delay inherently nonlinear systems unsolved yet. In this paper, we deal with such as the tracking problems via state feedback domination method in [17,18].

2 Mathematical preliminaries

We collect the definition of homogeneous function and several useful lemmas.

Definition1 ([19]). For a set of coordinates $x = (x_1, \ldots, x_n) \in R^n$ and an -tuple $r = (r_1, \ldots, r_n)$ of positive real numbers we introduce the following definitions.

(i) *A dilation* $\Delta_s(x)$ is a mapping defined by $\Delta_s^r(x) = \left(s^{r_1} x_1, \cdots, s^{r_n} x_n\right), \ \forall x = (x_1, \ldots, x_n) \in R^n, \ \forall s > 0$, where r_i are called *the weights of the coordinate*. For simplicity of notation, the dilation weight is denoted by $\Delta = (r_1, \ldots, r_n)$.

(ii) A function $V \in C(R^n, R)$ is said to be *homogeneous of degree* τ if there is a real number $\tau \in R$, such that $V(\Delta_s^r(x)) = s^\tau V(x_1, \cdots, x_n), \quad \forall x \in R^n - \{0\}$.

(iii) A vector field $f \in C(R^n, R^n)$ is said to be *homogeneous of degree* τ if the component f_i is *homogeneous of degree* $\tau + r_i$ for each *i*, that is, $f_i(\Delta_s^r(x)) = s^{\tau + r_i} f_i(x_1, \cdots, x_n), \ \forall x \in R^n, \ \forall s > 0$, for $i = 1, \ldots, n$.

(iv) *A homogeneous* p *-norm* is defined as $\|x\|_{\Delta,p} = \left(\sum_{i=1}^n |x_i|^{p/r_i}\right)^{1/p}, \forall x \in \mathbb{R}^n, p \geq 1$.

For the simplicity, write $\|x\|_\Delta$ for $\|x\|_{\Delta,2}$.

Next, we introduce several technical lemmas which will play an important role and be frequently used in the later control design.

Lemma1[19]. Denote $\Delta = (r_1, \ldots, r_n)$ as dilation weight, and suppose $V_1(x)$ and $V_2(x)$ are homogeneous functions with degree τ_1 and τ_2, respectively. Then, $V_1(x)V_2(x)$ is also homogeneous function with degree of $\tau_1 + \tau_2$ with respect to the same dilation Δ.

Lemma2[19]. Suppose $V: R^n \to R$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then, the following (i) and (ii) hold:

(i) $\partial V/\partial x_i$ is also homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i.

(ii) There is a constant $\sigma > 0$ such that $V(x) \le \sigma \|x\|_{\Delta}^{\tau}$. Moreover, if $V(x)$ is positive definite, there is a constant $\rho > 0$ such that $\rho \|x\|_{\Delta}^{\tau} \le V(x)$.

Lemma3[17]. For all $x, y \in R$ and a constant $p \ge 1$ the following inequalities hold:

(i) $|x+y|^p \le 2^{p-1} |x^p + y^p|$, $(|x|+|y|)^{1/p} \le |x|^{1/p} + |y|^{1/p} \le 2^{(p-1)/p} (|x|+|y|)^{1/p}$

If $p \in R_{odd}^{\ge 1}$, then

(II) $|x-y|^p \le 2^{p-1} |x^p - y^p|$ and $|x^{1/p} - y^{1/p}| \le 2^{(p-1)/p} |x-y|^{1/p}$.

Lemma4[18]. Let c, d be positive constants. Then, for any real-valued function $\gamma(x,y) > 0$, the following inequality holds:

$$|x|^c |y|^d \le \frac{c}{c+d} \gamma(x,y) |x|^{c+d} + \frac{d}{c+d} \gamma^{-c/d}(x,y) |y|^{c+d}.$$

This paper deals with the practical output tracking problem by state feedback for time-delay high-order nonlinear systems (1). Here, we give a precise definition of the problem.

The problem of global practical tracking by a state feedback: Consider system (1) and assume that the reference signal $y_r(t)$ is a time-varying C^1-bounded function on $[0, \infty)$. For any given $\varepsilon > 0$, design a state feedback controller having the following structure

$$u(t) = g(x(t), y_r(t)), \tag{2}$$

such that

(i) All the state of the closed-loop system (1) with state controller (2) is well-defined and globally bounded on $[0, \infty)$.

(ii) For any initial condition, there is a finite time $T > 0$, such that

$$|y(t) - y_r(t)| < \varepsilon, \quad \forall t \ge T > 0. \tag{3}$$

In order to solve the global practical output tracking problem, we made the following two assumptions:

Assumption1. There are constants C_1, C_2 and $\tau \ge 0$ such that

$$\begin{aligned} |\varphi_i(t, x(t), x(t-d), u(t))| \le C_1 \Big(& |x_1(t)|^{(r_i+\tau)/r_1} + |x_2(t)|^{(r_i+\tau)/r_2} + \cdots + |x_i(t)|^{(r_i+\tau)/r_i} \\ & + |x_1(t-d)|^{(r_i+\tau)/r_1} + |x_2(t-d)|^{(r_i+\tau)/r_2} + \cdots + |x_i(t-d)|^{(r_i+\tau)/r_i} \Big) + C_2 \end{aligned} \tag{4}$$

where $r_1 = 1, \quad r_{i+1} p_i = r_i + \tau > 0, \quad i = 1, \ldots, n$ and $p_n = 1$.

Assumption2. The reference signal $y_r(t)$ is continuously differentiable. Moreover, there is a known constant $D > 0$, such that

$$|y_r(t)| + |\dot{y}_r(t)| \le D, \quad \forall t \in [0, \infty) \tag{5}$$

3 State feedback tracking control design

In this paper, we deals with the practical output tracking problem by delay-independent state feedback for high-order time-delay nonlinear systems (1) under Assumptions 1-2. To this end, we first introduce the following coordinate transformation:

$$z_1 := x_1 - y_r, \quad z_i := \frac{x_i}{L^{\kappa_i}}, \quad i = 2,\ldots,n, \quad v := \frac{u}{L^{\kappa_n+1}} \tag{6}$$

where $\kappa_1 = 0$, $\kappa_i = (\kappa_{i-1}+1)/p_{i-1}$, $i = 2,\ldots,n$ and $L \ge 1$ is a scaling gain to be determined later. Then, the system (1) can be described in the new coordinates z_i as

$$\begin{aligned} \dot z_i &= Lz_{i+1}^{p_i} + \psi_i(t, z(t), z(t-d), v), \quad i = 1,\ldots,n-1, \\ \dot z_n &= Lv + \psi_n(t, z(t), z(t-d), v), \\ y &= z_1 \end{aligned} \tag{7}$$

where

$$\begin{aligned} \psi_1(t, z(t), z(t-d), v) &= \varphi_1(t, z(t), z(t-d), v) - \dot y_r, \\ \psi_i(t, z(t), z(t-d), v) &= \varphi_i(t, z(t), z(t-d), v)/L^{\kappa_i}, \quad i = 2,\ldots,n. \end{aligned} \tag{8}$$

Now, using Assumption 1, Lemma 3, the fact that $L \ge 1$ and the boundedness of y_r and $\dot y_r$ guaranteed by Assumption 2, ensures the existence of constants $\bar C_i$, $i = 1, 2$ only depending on constants C_1, C_2, τ, κ_i and L, under which (4) becomes

$$\begin{aligned} |\psi_1(t, z(t), z(t-d), v)| &\le \bar C_1\left(|z_1(t)|^{(r_1+\tau)/r_1} + |z_1(t-d)|^{(r_1+\tau)/r_1}\right) + \bar C_2 \\ |\psi_i(t, z(t), z(t-d), v)| &\le \bar C_1 L^{1-\nu_i}\sum_{j=1}^{i}\left(|z_j(t)|^{(r_i+\tau)/r_j} + |z_j(t-d)|^{(r_i+\tau)/r_j}\right) + \frac{\bar C_2}{L^{\kappa_i}}, \quad i = 2,\ldots,n \end{aligned} \tag{9}$$

where $\bar C_1 > 0$, $\bar C_2 > 0$ and $\nu_i := \min\{1 - \kappa_j(r_i+\tau)/r_j + \kappa_i,\ 2 \le j \le i,\ 1 \le i \le n\} > 0$ are some constants.

In what follows, we will employ the homogeneous domination approach to construct a global state feedback controller for system (7).

2.1 Stability Analysis

First, we construct a homogeneous state feedback controller for the nominal nonlinear system without considering the non-linearity of $\psi_i(\cdot)$, $i = 1,\ldots,n-1$ in (7), i.e.,

$$\dot z_i = Lz_{i+1}^{p_i}, \quad i = 1,\ldots,n-1, \quad \dot z_n = Lv, \quad y = z_1 \tag{10}$$

Using SIMILAR the approach in [11, 17-18], we can design a homogeneous state feedback stabilizer for (8), which can be described in the following Theorem1.

Theorem1. For a real given number $\tau \ge 0$, there is a homogeneous state feedback controller of degree τ such that the nonlinear systems (10) is globally asymptotically stable.

Proof. To prove the result, we use an inductive argument (recursive design method) to explicitly construct a homogeneous stabilizer for system (10).

Initial step1. Let $\xi_1 = z_1^{\sigma/r_1} - z_1^{*\sigma/r_1}$, where $z_1^* = 0$ and $\sigma \ge \max_{1\le i\le n}\{1, \tau + r_i\}$ is a positive number. Choose the Lyapunov function

$$V_1 = W_1 = \int_{z_1^*}^{z_1}\left(s^{\sigma/r_1} - z_1^{*\sigma/r_1}\right)^{(2\sigma-\tau-r_1)/\sigma} ds\,. \tag{11}$$

From (10), it follows that

$$\dot{V}_1 \le -nL\xi_1^2 + L\xi_1^{(2\sigma-\tau-r_1)/\sigma}\left(z_2^{p_1} - z_2^{*p_1}\right) \tag{12}$$

where z_2^* the virtual controller and it is chosen as

$$z_2^* = -n^{1/p_1} z_1^{(r_1+\tau)/p_1} := -\beta_1^{r_2/\sigma}\xi_1^{r_2/\sigma}, \ \beta_1 = n^{\sigma/(r_2 p_1)}. \tag{13}$$

Step $k\ (k = 2,\cdots,n)$. Suppose at the *step k-1*, there is a C^1, positive definite and proper Lyapunov function V_{k-1}, and a set of virtual controllers $z_1^*, \ldots, z_k^*$ defined by

$$\begin{aligned} z_1^* &= 0, & \xi_1 &= z_1^{\sigma/r_1} - z_1^{*\sigma/r_1} \\ z_i^* &= -\beta_{i-1}^{r_i/\sigma}\xi_{i-1}^{r_i/\sigma}, & \xi_i &= z_i^{\sigma/r_i} - z_i^{*\sigma/r_i}, \ i = 2,\cdots,k \end{aligned} \tag{14}$$

with $\beta_i > 0,\ 1 \le i \le k-1$ being constants, such that

$$\dot{V}_{k-1} \le -(n-k+2)L\sum_{l=1}^{k-1}\xi_l^2 + L\xi_{k-1}^{(2\sigma-\tau-r_{k-1})/\sigma}\left(z_k^{p_{k-1}} - z_k^{*p_{k-1}}\right). \tag{15}$$

We claim that (15) also holds at *Step k,* i.e., there is a C^1, proper, positive definite Lyapunov function defined by

$$V_k(\bar{z}_k) = V_{k-1}(\bar{z}_{k-1}) + W_k(\bar{z}_k), \quad W_k(\bar{z}_k) = \int_{z_k^*}^{z_k}\left(s^{\sigma/r_k} - z_k^{*\sigma/r_k}\right)^{(2\sigma-\tau-r_k)/\sigma} ds \tag{16}$$

and virtual controller $z_{k+1}^* = -\beta_k^{r_{k+1}/\sigma}\xi_k^{r_{k+1}/\sigma}$ such that

$$\dot{V}_k \le -(n-k+1)L\sum_{j=1}^{k}\xi_j^2 + L\xi_k^{(2\sigma-\tau-r_k)/\sigma}\left(z_{k+1}^{p_k} - z_{k+1}^{*p_k}\right). \tag{17}$$

Since the prove of the claim (17) is very similar [4-5, 14], so omitted here.
Using the inductive argument above, we can conclude that at the -th step, there exists a state feedback controller of the form

$$v = -\beta_n^{r_{n+1}/\sigma}\xi_n^{r_{n+1}/\sigma} = -\left(\sum_{i=1}^{n}\bar{\beta}_i z_i^{\sigma/r_i}\right)^{r_{n+1}/\sigma} \tag{18}$$

with the C^1, proper and positive definite Lyapunov function,

$$V_n = \sum_{i=1}^{n}\int_{z_i^*}^{z_i}\left(s^{\sigma/r_i} - z_i^{*\sigma/r_i}\right)^{(2\sigma-\tau-r_i)/\sigma} ds \tag{19}$$

we arrive at

$$\dot{V}_n \le -L\sum_{j=1}^{n}\xi_j^2, \tag{20}$$

where $\xi_i = z_i^{\sigma/r_i} - z_i^{*\sigma/r_i}$ and $\bar{\beta}_i = \beta_n \cdots \beta_i,\ i = 1,\ldots,n$ are positive constants. Thus, the closed-loop system (10) and (18) is globally asymptotically stable.

2.2 Tracking control design for the time-delay nonlinear system (1)

Now, we are ready to use the homogeneous domination approach to design a global tracking controller for the system (1), i.e., state the following main result in this paper.

Theorem 2. For the time-delay nonlinear system (1) under Assumptions 1-2, the global practical output tracking problem is solvable by the state feedback controller $u = L^{\kappa_n+1}v$ in (7) and (18)

Proof. From (18), we have

$$v = -\beta_n^{r_{n+1}/\sigma}\xi_n^{r_{n+1}/\sigma} = -\left(\sum_{i=1}^{n}\overline{\beta}_i z_i^{\sigma/r_i}\right)^{r_{n+1}/\sigma}. \tag{21}$$

Now, we define the compact notations

$$z = \left(z_1, \ldots, z_n\right)^T, \quad E(z) = \left(z_2^{p_1}, \ldots, z_n^{p_{n-1}}, v\right)^T \text{ and } F(z) = \left(\varphi_1, \varphi_2/L^{\kappa_2}, \ldots, \varphi_n/L^{\kappa_n}\right)^T. \tag{22}$$

Using the same notation (7) and (22), the closed-loop system (7) - (18) can be written as the following compact form:

$$\dot{z} = LE(z) + F(z) \tag{23}$$

Moreover, by introducing the dilation weight $\Delta = (r_1, \ldots, r_n)$, from Definition 1, it can be shown that V_n is homogeneous of degree 2 − with respect to Δ.
Hence, adopting the same Lyapunov function (19) and by Lemm2 and Lemma 3, it can be concluded that

$$\dot{V}_n(z) = L\frac{\partial V_n}{\partial Z}E(z) + \frac{\partial V_n}{\partial Z}.\dot{F}(z) \le -m_1 L\|z\|_\Delta^{2\sigma} + \sum\nolimits_{i=1}^{n}\frac{\partial V_n}{\partial z_i}\psi_i \tag{24}$$

where $m_1 > 0$ is constant.

By (9), Assumption 1 and > 1, we can find constants $\delta_i > 0$ and $0 < \gamma_i \le 1$ such that

$$|\psi_i| \le \delta_i L^{1-\gamma_i}\left(\|z(t)\|_\Delta^{r_i+\tau} + \|z(t-d_j(t))\|_\Delta^{r_i+\tau}\right) + \overline{C}_2/L^{\kappa_i} \tag{25}$$

and noting that for $i = 1,\ldots,n$, by Lemma2, $\partial V_n/\partial z_i$ is homogeneous of degree $2\sigma - \tau - r_i$,

$$\left|\frac{\partial V_n}{\partial z_i}\right| \le m_2\|z\|_\Delta^{2\sigma-\tau-r_i}, \qquad m_2 > 0. \tag{26}$$

Therefore

$$\left|\frac{\partial V_n}{\partial z_i}\psi_i\right| \le m_2(1+\delta_i)L^{1-\gamma_i}\|z\|_\Delta^{2\sigma} + m_2\|z\|_\Delta^{2\sigma-\tau-r_i}\|z(t-d_j(t))\|_\Delta^{r_i+\tau} + \frac{\omega^{2\sigma/(\tau+r_i)}}{L^{1+\gamma_i}}, \tag{27}$$

where $\omega =: m_2\overline{C}_2$, $\frac{2\sigma-\tau-r_i}{2\sigma} \le 1$, $\frac{\tau+r_i}{2\sigma} \le 1$, and $\frac{2\sigma-(1-\gamma_i)}{\tau+r_i} - (1-\gamma_i) \ge 1+\kappa_i$.

Substituting (27) into (24) yields

$$\dot{V}_n(z) \le -L\left(m_1\|z\|_\Delta^{2\sigma} - (1+m_2(1+\delta)\sum_{i=1}^{n}L^{-\gamma_i}\|z\|_\Delta^{2\sigma} - m_2\sum_{i=1}^{n}L^{-\gamma_i}\|z\|_\Delta^{2\sigma-r_i-\tau}\|z(t-d)\|_\Delta^{r_i+\tau}\right) + \sum_{i=1}^{n}\frac{\omega^{2\sigma/(\tau+r_i)}}{L^{1+\gamma_i}} \tag{28}$$

By Lemma4, there exists a constant $m_3 > 0$ such that

$$m_2\|z\|_\Delta^{2\sigma-r_i-\tau}\|z(t-d)\|_\Delta^{r_i+\tau} \le \|z\|_\Delta^{2\sigma} + m_3\|z(t-d)\|_\Delta^{2\sigma}, \tag{29}$$

which yields

$$\dot{V}_n(z) \le -L\left(m_1\|z\|_\Delta^{2\sigma} - (2+m_2(1+\delta))\sum_{i=1}^{n}L^{-\gamma_i}\|z\|_\Delta^{2\sigma} - m_3\sum_{i=1}^{n}L^{-\gamma_i}\|z(t-d)\|_\Delta^{2\sigma}\right) + \sum_{i=1}^{n}\frac{\omega^{2\sigma/(\tau+r_i)}}{L^{1+\gamma_i}}, \tag{30}$$

Construct a Lyapunov-Krasovskii functional as follows:

$$V(z(t)) = V_n(z(t)) + \int_{t-d}^{t}\|z(s)\|_\Delta^{2\sigma}\,\eta ds, \tag{31}$$

where η is a positive constant. Let $\eta = m_3\sum_{i=1}^{n} L^{1-\gamma_i}$ follows from (29) and (30) that

$$\dot{V} \le -L\left(m_1 - (2+m_2(1+\delta)+m_3)\sum_{i=1}^{n} L^{-\gamma_i}\right)\|z(t)\|_{\Delta}^{2\sigma} + \frac{\rho_1}{L^{1+\gamma}}. \tag{32}$$

Hence, by choosing a large enough L as $L > \max\left\{1, (((2+m_2(1+\delta)+m_3)/m_1))^{-\gamma}\right\}$, where $\gamma = \min_{1\le i\le n}\{\gamma_i\}$ and $\rho_1 = \sum_{i=1}^{n}\alpha^{2\sigma/(\tau+r_i)}$.

Then, there exists a constant $\rho_2 > 0$, such that (30) becomes

$$\dot{V}\,(z(t)) \le -\rho_2\,\|z(t)\|_{\Delta}^{2\sigma} + 2\rho_1. \tag{33}$$

Moreover, $V_n(z)$ and $\int_{t-d}^{t}\|z(s)\|_{\Delta}^{2\sigma}\,\eta ds$ are homogeneous of degree $2\sigma-\tau$ and 2σ with respect to Δ , respectively. Therefore, by Lemma2, there are positive constants λ_1, λ_2 , λ_3 and λ_4, such that

$$\lambda_1\|z(t)\|_{\Delta}^{2\sigma-\tau} \le V_n(z(t)) \le \lambda_2\|z(t)\|_{\Delta}^{2\sigma-\tau}, \qquad \lambda_3\|z(t)\|_{\Delta}^{2\sigma} \le \int_{t-d}^{t}\|z(s)\|_{\Delta}^{2\sigma}\,\eta ds \le \lambda_4\|z(t)\|_{\Delta}^{2\sigma} \tag{34}$$

Therefore combining (33) and (34) yields

$$\dot{V}\,(z(t)) \le -\rho_2^{-1}V(z(t)) + \tilde{\rho}_1, \tag{35}$$

where $\rho_2 = (\lambda_4 + (2\delta-\tau)/2\sigma)$ and $\tilde{\rho}_1 = \tau\lambda_2^{(\tau-2\sigma)/\tau}\Big/\left(2\sigma\rho_2 L^{(2\sigma-\tau)/\tau}\right) + \rho_1/L^{1+\gamma}$.

From (35) it is not difficult to show that there is a finite time $T > 0$, such that

$$V(z) \le 3\tilde{\rho}_1, \quad \forall t \ge T \tag{36}$$

from which it is clear that z_1 can be rendered smaller than any positive tolerance with a sufficiently large L .

4 Conclusion

In this paper, we have studied the practical output tracking problem for a class of uncertain nonlinear time-delay systems under a homogeneous condition. First, we design a homogeneous state feedback controllers have been constructed with adjustable scaling gains. Then, with the help of a homogeneous Lyapunov-Krasovskii functional, we've redesigned the homogeneous domination approach to tune the scaling gain for the overall the closed loop systems. It is shown that an appropriate choice of gain will enable us to globally track for a class of uncertain non-linear systems in finite time.

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