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Optimization of the production plan of drill bits with fuzzy constraints based on a heuristic method

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Abstract. The problem of optimizing the production plan under fuzzy constraints is formulated using the theory of fuzzy sets. To solve the obtained fuzzy optimization problem, a heuristic method is proposed, in which, on the basis of the use of expert information and the knowledge of experts, the decision-maker in the optimization process and the choice of the best solution, it makes possible to improve the solution of the fuzzy problem. The results obtained are implemented and applied when solving the problem of optimizing the production plan of a plant for the production of diamond drill bits under fuzzy constraints. It is shown that when solving the optimization problem based on the proposed fuzzy approach, the plant's profit from the sale of different types of bits increases to 8%.

1. Introduction

The tasks of optimizing production plans can be characterized by the fuzziness of the initial information. Accordingly, they are posed in the form of a fuzzy optimization problem and are solved using methods of fuzzy set theory a fuzzy approach that uses subjective information or the preferences of an expert, a decision maker (DM) when choosing an optimal solution. [1-5]. Fuzzy optimization problems can be formulated as fuzzy mathematical programming (FMP) problems [3, 6–9].

By the NMP problem we mean the problem of optimization of an objective function in an admissible set, taking into account the imposed constraints, and one or more objective functions or constraints are described indistinctly [3]. In these problems, when setting and solving in one way or another, methods of the theory of fuzzy sets are used [6].

Depending on the source of the fuzziness of the FMP problem, there can be different classes of FMP problems. We consider FMP problems with fuzzy constraints. Fuzzy constraints can be formalized as:

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It is desirable that the values of the constraint function be at least $\varphi_q(x) \tilde{<} b_q, q = \overline{1, L}$ (not more $\tilde{>}$, approximately equal $\tilde{\cong}$).

2. The problem of optimizing the production plan of drill bits with fuzzy constraints and a heuristic method for solving it

2.1. Statement of the problem of optimizing the production plan for different types of drill bits

The Atyrau Petroleum Equipment Manufacturing and Repair Plant produces two types of drill bits D_1 and D_2 for oil production enterprises. The production of bits is described by unclear constraints on artificial diamonds (raw materials) and processing time on machines.

For the production of each bit of type D_1, 4 kg of artificial diamond is required, and for the production of a bit of type D_2 - 5. Let us assume that the plant is supplied with up to 100 kg of artificial diamonds per month. In order to manufacture one unit of type D_1 bit, 12.5 hours of work are required on the machines, and for one unit of type D_2 bit, 10 hours on machines are required per month, you can work up to 240 hours of time.

It is necessary to optimize the production plan of the plant. How many bits of types 1 and 2 should be produced by the plant per month, if the profit from each bit of type D_1 is 5 thousand dollars, and from the production of each bit of type D_2 is 7 thousand dollars.

Let us formulate the mathematical setting of the problem. Let x_1 and x_2 be the number of products, respectively, of types D_1 and D_2, produced per month. It is necessary to determine the optimal production plan, i.e. find the best values of x_1 and x_2 that maximize the monthly profit $f(x)$.

Based on the above information and data, the monthly profit can be determined by the expression:

$$f(x) = 5x_1 + 7x_2 \tag{1}$$

Expression (1) is the objective function that needs to be maximized. Thus, we have obtained an optimization problem with one criterion, or if there are many criteria, then we believe that it is possible to obtain their convolution. It can be seen from the structure of the objective function that in order to increase profits, it is necessary to increase the number of products x_1 and x_2. However, the problem is that increases in these variables are limited by raw material and processing time constraints. Moreover, these restrictions may be unclear.

Since x_1 and x_2 represent the monthly production of type 1 and 2 bits, they must be integer and positive, i.e.:

$$x_1 \geq 0, x_2 \geq 0 \tag{2}$$

Restrictions on raw materials, i.e. for artificial diamonds and processing time can be mathematically written in the form of the following fuzzy inequalities:

$$4x_1 + 5x_2 \tilde{<} 100 \text{ (for raw materials)} \tag{3}$$

$$12.5x_1 + 10x_2 \tilde{<} 250 \text{ (for processing time)} \tag{4}$$

Thus, the task of optimizing production is to find and determine the number of produced bits of each type, i.e. values x_1 and x_2, which simultaneously satisfy the conditions of non-negativity (2), fuzzy constraints (3) and (4) and maximizes the objective function $f(x)$ (1).

The optimization problem (1) - (4) can be rewritten in the standard form for integer programming (CPU) problems:

$$f(x) = 5x_1 + 7x_2 \tag{5}$$

$$4x_1 + 5x_2 \tilde{<} 100 \tag{6}$$

$$12.5x_1 + 10x_2 \tilde{\lesssim} 250 \quad (7)$$

$$x_1 \geq 0, x_2 \geq 0, \quad x_1, x_2 - \text{integer} \quad (8)$$

To illustrate the obtained CPU problem with fuzzy constraints graphically, we construct the domains of feasible solutions defined by the non-negativity condition (8) and fuzzy constraints (6) and (7) (figure 1).

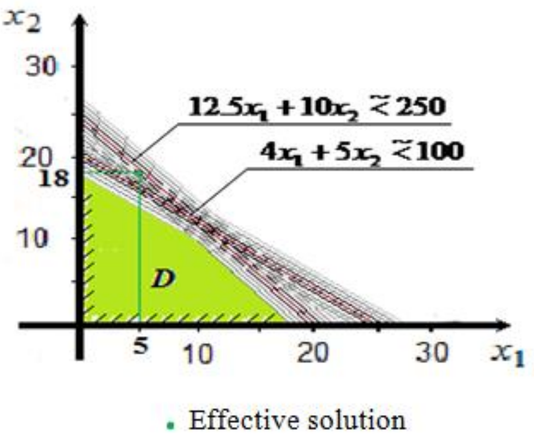

Figure 1. Illustration of the solution to the FMP problem with fuzzy constraints.

Constraints $x_1 \geq 0, x_2 \geq 0$ indicate that the solution is in the first quadrant. We draw lines for other restrictions. To construct the limit line (6) using two points with coordinates: x_1 = 0; x_2 = 25 and x_1 = 20; x_2 = 0, draw points with the following coordinates (0, 20), (25, 0) on the graph and draw a fuzzy line $\tilde{\varphi}_1(x)$. In order to determine which part of the plane is determined by the fuzzy inequality $4x_1 + 5x_2 \tilde{\lesssim} 100$, substitute an arbitrary point with coordinates (30, 30), we obtain a contradiction (120+150>100). Therefore, this inequality defines a half-plane that does not contain a point with coordinates (30, 30). Now you can similarly build a line for the fuzzy constraint (7) along the coordinates of two points: (0; 25) and (20; 0) and set the direction of the admissible plane.

2.2. Solving the problem of optimizing a production plan with fuzzy constraints based on a deterministic approach and a heuristic method (fuzzy approach)

Let us consider various approaches to solving the formulated problem of optimization (maximization) of the profit of a plant for the production of drill bits:

2.2.1. Deterministic approach. Setting hard constraints, we represent the original fuzzy problem as a clear problem, i.e. we get the following CPU task:

$$f(x) = 5x_1 + 7x_2 \rightarrow \max$$

$$4x_1 + 5x_2 \leq 100$$

$$12.5x_1 + 10x_2 \leq 250$$

$$x_1 \geq 0, x_2 \geq 0 \text{ and integers.}$$

So, it turned out an integer programming problem, you can apply one of the methods for solving integer programming problems.

To solve the formulated CPU problem, we used the MANAGER software package, which implements the branch and bound method [10].

As a result of solving the obtained CPU task for optimizing the production plan for different types of bits using the MANAGER program, $x_1 = 0$, $x_2 = 20$ were obtained. For these values of the independent variables, the maximum value of the objective function is achieved, i.e. income is equal to 140 thousand c.u. Thus, the results show, under the given strict restrictions, if the plant produces 20 bits of D_2 type per month, and bits of D_1 type are not produced ($x_1 = 0$), then the plant will receive the maximum, i.e. 140 thousand $ profit per month. With any other version of the production plan for bits, the plant receives less profit.

2.2.2. Fuzzy approach. Now, taking into account the vagueness of the constraint, we will consider a fuzzy approach to solving the problem of maximizing the monthly profit using the proposed heuristic method for solving the FMP problem. Since constraints (3), (4) are fuzzy, violation of them up to an acceptable value is allowed.

Let us assume that the membership functions are determined on the basis of expert judgment $\mu_q(x), q = 1,2$, evaluating the degree of fulfillment of fuzzy constraints, as well as the values of the priority series $I = \{1, 2\}$ for these restrictions. Then, applying the idea of the main criterion method, problem (5) - (7) can be written as the following FMP problem:

$$\max_{x \in X} \mu_0(x),$$

$$X = \{x : x \in \Omega \wedge \arg(\mu_q(x) \geq \mu_q^R), q = 1,2\}$$

Here and below, $\mu_0(x)$ – objective function normalized by expression:

$$\mu_0(x) = \frac{f(x) - \inf\limits_{x \in X} f(x)}{\sup\limits_{x \in X} f(x) - \inf\limits_{x \in X} f(x)} = \frac{f(x) - 0}{155 - 0} = \frac{f(x)}{155}$$

where 155 is the upper limit (sup) of the profit value.

Then the problem of optimizing the production plan of products in order to maximize the monthly profit under fuzzy constraints is written as:

$$\max_{x \in X} \mu_0(x) = \frac{\max\limits_{x \in X}(f(x)}{155},$$

$$X = \{x : x \in \Omega = [x_i \geq 0, i = 1,2] \wedge \arg(\mu_1(x) = 1.0) \wedge \arg(\mu_2(x) \geq 0.90)\},$$

where 1.0 and 0.90 are given boundary values for the degrees of fulfillment of fuzzy constraints (4), (3), having priorities 1 and 2, respectively.

2.2.3. Heuristic method. To solve the obtained FMP problem, we have chosen the heuristic method proposed by us, developed on the basis of a modification of the idea of the main criterion method for fuzzy constraints. As a result of the implementation of the points of this heuristic method, the following are obtained:

- Based on expert judgment, a number of priorities for constraints have been identified: $I_R = \{1, 2\}$ where 1 is the priority of the restriction on processing time (4), 2 is the priority of the restriction on raw materials (3).
- A term-set was selected, and membership functions were determined to assess the degree of fulfillment of fuzzy constraints: $\mu_q(x), q = 1,2$:

$$\mu_1(x)=\begin{cases}1, \text{ if } 12{,}5x_1+10x_2\le 245\\ 1-\dfrac{255\text{-}(12.5x_1+10x_2)}{5}, \text{ if } 245<12.5x_1+10x_2<255\\ 0, \text{ if } 12{,}5x_1+10x_2\ge 245\end{cases}$$

$$\mu_2(x)=\begin{cases}1, \text{ if } 4x_1+5x_2\le 95\\ 1-\dfrac{111\text{-}(4x_1+5x_2)}{15}, \text{ if } 95<4x_1+5x_2<111\\ 0, \text{ if } 4x_1+5x_2\ge 111\end{cases}$$

In the above functions, d = 5, d = 15 means the value of the allowable threshold.

- With the involvement of a decision maker, the initial values of the boundary constraints of the membership function are given, which estimate the degree of fulfillment of the constraints: $\mu_q^{R(l)}$, $q=1,2,\ l=1$: $\mu_1^{R(1)}=1.0;\ \mu_2^{R(1)}=0{,}90$.
- The problem of determining the maximum criterion is solved: $\max_{x\in X}\mu_0(x)$, evaluating the monthly profit of the plant from the sale of manufactured bits $\mu_0(x)$ on the set X and a set of solutions is defined: $x(\mu_q^{R(l)}),\ \mu_0(x(\mu_q^{R(l)})), \mu_1(x(\mu_q^{R(l)})), \mu_2(x(\mu_q^{R(l)})), x=(x_1,x_2), q=1,2, l=1,2,...,$.
- Current solutions are presented to decision makers for analysis and selection of the final best solutions.
- The decision maker, in order to search for the best solutions, corrected the values of the boundary values of fuzzy constraints 4 times and made the reverse transition to point 4.
- In the fifth iteration, as a result of determining the optimal values x_1, x_2 - the number of bits 1 and 2 of type, the decision maker, taking into account the fuzziness of the constraint, determined the best value of the objective function, which is greater than the profit obtained with a clear solution of the optimization problem.

As a result of determining the values $x^*=(x_1^*,x_2^*)$ taking into account the vagueness of the restrictions, it is determined $x_1^*=5$; $x_2^*=18$ (figure 1) and the volume of monthly profit increased by 11 thousand dollars (by 7.9%):

$$f(x)=5x_1+7x_2=5\cdot 5+7\cdot 18=25+126=151 \text{ thousand dollars.}$$

As, $12.5x_1+10x_2=12.5\cdot 5+10\cdot 18=242.5\le 245$ the membership function of the fulfillment of a fuzzy constraint on the processing time $\mu_1(x)=1$, and the value of the membership function describing the degree of fulfillment of the fuzzy constraint on the raw material is calculated by the expression given in clause 2:

$$\mu_2(x)=1-\frac{111-(4\cdot x_1+5\cdot x_2)}{15}=1-\frac{111-(20+90)}{15}=1-\frac{1}{15}=0{,}94$$

Thus, as a result of optimization of the production plan for the production of different types of products, we obtain the following optimal solution: $x^*(\mu_q^{R(l)})=(5{,}18)$ – number of produced bits of the first x_1=5 pcs. and the second type x_2=18 pcs.; $\mu_0(x^*(\mu_q^{R(l)}))=0.97\Rightarrow f(x^*)=151$ – the maximum value of the objective function; $\mu_1(x^*(\mu_q^{R(l)}))=1$ и $\mu_2(x^*(\mu_q^{R(l)}))=0{,}94$ – maximum values of the membership function describing the degree of fulfillment of fuzzy constraints

3. Discussion of results

From the results obtained, it can be seen that when solving the optimization problem based on a fuzzy approach, taking into account the fuzzy constraint, the best solutions were obtained. Objective function value, i.e. the volume of the monthly profit of the plant due to the sale of produced bits of different types, in comparison with the result of a clear approach to solving this problem, increased.

Since $\mu_1(x)=1$, unclear limitation on processing time $12.5x_1+10x_2 \tilde{\leq} 250$, which has the priority 1, is fully satisfied. A membership function that estimates the degree of fulfillment of a fuzzy constraint on raw materials $4x_1+5x_2 \tilde{\leq} 100$, which has priority 2 is $\mu_2(x)=0.94$.

Since the specified boundary value of this function is $\mu_2^{R(1)}=0{,}90$, condition is satisfied $\mu_2(x)\geq 0.90$, which shows a sufficiently high degree of fulfillment of the 2nd fuzzy constraint.

4. Conclusion

The FMP task is formulated for finding an effective production plan for drill bits of different types in the presence of fuzzy constraints. By modifying the main criterion method, taking into account fuzziness, a heuristic method for solving the formulated problem has been developed. The results of solving the problem of optimizing the production plan for the production of drill bits on the basis of a deterministic and fuzzy approach are compared. It is shown that in the presence of fuzzy initial information, the use of a fuzzy approach provides a more efficient solution.

The originality and novelty of the method of problem setting and methods for their solution proposed in this work, in contrast to the known methods, is reflected in the following: based on the maximum use of the initial fuzzy information, a more adequate and better solution of the fuzzy problem is achieved. The theoretical contribution of the work is determined in the development of a heuristic method for solving the FMP problem. The practical significance of the work lies in the fact that the proposed approach allows one to obtain more effective solutions to real production problems in conditions of indistinctness of the initial information.

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