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Methods of Fuzzy Multi-Criteria Decision Making for Controlling the Operating Modes of the Stabilization Column of the Primary Oil-Refining Unit

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Abstract: Many technological systems are characterized by fuzzy initial information, necessary for the development of their models, optimization and control of operating modes. Therefore, the purpose of this study is to formulate decision making problems for optimizing and controlling operating modes of such systems in a fuzzy environment and to develop methods for solving them. The developed heuristic methods of fuzzy multi-criteria decision making are based on the modification and combination of different principles of optimality. The proposed methods based on system models, knowledge and experience of the decision maker allow iterative improvement and effective decision making. On the basis of experimental–statistical methods, methods of expert evaluation, statistical and fuzzy models of the stabilization column have been developed. The conditions for judging the fuzzy model's effectiveness are determined and investigated. Using the proposed heuristic method based on the main criterion and maximin, the problem of two-criterion optimization of the stabilization column parameters in a fuzzy environment is solved. The results obtained confirm the advantages of the proposed method of fuzzy decision making in comparison with the results of known methods. The developed heuristic methods differ from known ones because they allow making adequate decisions in a fuzzy environment by maximizing the use of the collected fuzzy information.

Keywords: multicriteria optimization; fuzzy decision making; decision maker; principles of optimality; heuristic methods

MSC: 49N30; 76B75; 90B50

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1. Introduction

Technological processes of oil refining, petrochemistry, metallurgy and other industries are carried out mainly through chemical–technological systems (CTS). The complexity of such CTSs lies in a large number of interconnected technological units, which are characterized by many parameters, the influence of which on their operating modes, as well as on the volume and quality of products, is often non-formalizable and characterized by fuzziness [1,2]. This situation complicates the processes of developing mathematical models and optimizing the operating modes of CTS to control them [3]. Since CTSs are the main factor in the development of the economy of any country, increasing their efficiency through the use of modern mathematical methods of optimization and management is an important and urgent task of science and practice.

Multicriteria optimization problems in the presence of conflicting criteria are decision-making problems that are solved with the knowledge of a person—a decision maker (DM), taking into account his preferences. In practice, many complex production facilities are characterized by a lack and fuzziness of initial information. Such objects, in the presence of experienced operators—DMs, experts who manage the operating modes—are effectively managed due to their experience, knowledge and intuition. At the same time, many control criteria and restrictions are not clearly described in the natural language of DM experts. Therefore, to solve the problems of controlling the operating modes of fuzzy complex objects, it is more appropriate to use fuzzy multi-criteria decision making with the participation of DMs. It can be noted that at present, with the development of fuzzy set theory and decision-making methods, this approach has become more promising for solving decision-making problems for control in a fuzzy environment. Since many CTSs are described as fuzzy, the importance of research and development of decision-making methods in a fuzzy environment is increasing.

Currently, there are many works in which optimization and decision-making methods [4–19] are investigated and proposed, which are used or can be applied to optimize various complex CTSs, including in a fuzzy environment [20–27]. Let us consider the main results of studying the current state of the research area—tasks and decision-making methods, including in a fuzzy environment. One of the approaches to solving problems of multicriteria optimization is the application of the Pareto principle [4,5]. In this approach, not just a single solution is taken, but a set of solutions—points of the multidimensional search space that are the best from the point of view of one of the criteria. The resulting set of optimal points is the Pareto-optimal set (POS).

In [6], Petrovet. al. used a genetic algorithm related to the evolutionary method, which is characterized by high computational costs in order to optimize synchronous motors. The main reasons for this are that evolutionary methods are iterative methods that require large computational costs in the learning process [7]; to determine the best result, these methods iterate over all elements of the population through several iterations [8]. Saibal Majumder et al. in [9] proposed an efficient genetic algorithm based on uncertainties with a variable population for a random fuzzy maximum-flow problem.

Recently, numerous studies have been carried out, for example [10–12], aimed at reducing the computational costs of evolutionary methods due to a priori information about the initial population. Tan Liguo and Novikova in [10] proposed a multicriteria evolutionary algorithm with a step-by-step learning model, which allows a drastic reduction in the number of calculations compared to the classical genetic multivariate method. The peculiarity of this method is that the evolutionary search is purposeful due to the approximation of the POS based on the modification of the evolutionary genetic algorithm of multicriteria optimization using the stages of learning and forgetting. The paper [11] proposes a method of evolutionary multi-objective optimization for the control of generative adversarial networks, which makes it possible to reduce computational costs. Liand Kwong proposed an evolutionary method for multi-objective optimization through multiple learning, providing a reduction in the number of calculations [12]. Various multicriteria evolutionary algorithms (MEA) have been developed that allow obtaining an arbitrarily close approximation to the real continuous surface of the POS [13,14].

In order to provide direction to the evolutionary search, some researchers have attempted to extract knowledge from the regularity property of the POS in the MEA, which is based on learning. The regularity property of the POS was proved by Zhang et al. in [15]. This makes it possible to represent the POS as a continuous multi-criteria problem with m goals. Based on this statement, some authors of the study used various methods of supervised or unsupervised machine learning, such as Bayesian networks [16], Gaussian process [17], a restricted Boltzmann machine [17] and self-organizing maps [18], to evaluate POS.

The authors of [19] proposed an online strategy for self-adaptive pairing constraints based on agglomerative clustering. In this strategy, learning occurs only when the similarity between the populations of two successive generations is greater than the threshold value. In [20], Cheng et al. proposed an MEA using Gaussian process inverse modeling to reduce the number of Gaussian models. Li and Kwong incorporated manifold learning into multi-criteria optimization, which allows dimensionality reduction to extract the geometric properties of the structure of low-dimensional manifolds embedded in a high-dimensional space [12]. However, it should be noted that despite these efforts, the problems of excessive computational costs due to numerous iterations or retraining, especially at the completion stage, when the set of solutions approaches the Pareto set, are still not solved.

Decision making consists of evaluating possible solutions (alternatives) and choosing the best one according to given criteria. An important area of decision making is related to production. The larger the volume of production, the more difficult it is to make decisions when managing it, i.e., the problems of formalization and solving decision-making problems become relevant. The authors of [21,22] proposed decision-making methods for incomplete data based on the intuitionistic fuzzy approach, which allow making more adequate decisions in a fuzzy environment. The problems of fuzzy optimization and fuzzy decision making are studied in [23,24], where methods for their solution are proposed based on the α-level set of fuzzy set theories. The problems of applying multi-parameter fuzzy optimization in practice are studied in [25,26], and approaches to optimization in a fuzzy environment by oil refining are proposed in [27,28]. Aspects of technological systems such as power grids and the multi-criteria decision-making structure between the designer of a renewable power plant and an independent system operator are explored in [29,30]. The study [29] proposes an approach to improve fault tolerance, allowing power system planners to effectively manage multi-criteria sustainability indicators in a two-objective optimization planning model at the same time. In [30], a combination of a multi-criteria decision-making approach and a multi-agent modeling method was developed to obtain the maximum possible profit from a renewable generation plan. An approach to power market modeling based on fuzzy Q-learning coupled with a similarity order preference method (TOPSIS) is proposed as a new decision support system for renewable energy promotion.

Concretizing the literature review, it can be noted that in the above works [9,11–15, 17–19,22–25,30], the authors conducted studies using similar methods of multicriteria optimization and decision making. Furthermore, the authors of works [23,27,28] used in their research methods multicriteria optimization in a fuzzy environment to control the operating modes of technological systems. However, in these studies, fuzzy optimization problems, in contrast to our approach, are solved by converting them to a set of crisp problems based on the α-level set, which leads to the loss of some of the original fuzzy information and a decrease in the adequacy of the solutions obtained.

In the analyzed studies [23,27,28] for solving decision-making problems in a fuzzy environment, at the stage of formulation, a fuzzy task based on a α-level set is transformed into a set of equivalent crisp tasks. Further, the obtained crisp problems at α-levels are solved by known methods; then, by combining the obtained solutions, the solution of the original fuzzy problem is determined. However, such an approach can often lead to the loss of an important part of the original fuzzy information, which will lead to a decrease in the adequacy of the solutions obtained in a fuzzy environment. This motivates the development of a new approach that ensures the high adequacy of decisions made in a fuzzy environment.

In this regard, the main goal of this study is the formalization and formulation of fuzzy decision-making problems and the development of heuristic methods for solving them, which make it possible to obtain highly adequate solutions in a fuzzy environment. To do this, a fuzzy production problem based on the modification of various principles of optimality is posed and solved in a fuzzy environment without converting it to a set of crisp problems. At the same time, due to the experience, knowledge and intuition of DMs, the best decision is made, taking into account the current situation in production, the market for the sale of products, and the preferences of DMs. Thus, the main contribution of our research to the development of decision-making methods in a fuzzy environment is to provide the possibility of setting and solving fuzzy problems without reducing them to clear problems. This makes it possible to increase the adequacy and efficiency of decisions made when solving real production problems in a fuzzy environment.

The structure of the study is as follows. A review of the literature is presented in detail in Section 1. Section 2 describes the materials and the object of study and proposes methods for solving problems of multi-criteria decision making in a fuzzy environment. Section 3 describes the results obtained: models of the C-8 stabilization column of the EDP-AVT primary oil-refining unit; formulated and proved theorems on the adequacy of fuzzy models; the problem of fuzzy multi-criteria decision making for controlling the operating modes of the C-8 stabilization column of the primary oil refining-unit of Atyrau refinery was solved. Section 4 discusses the results of the study. Section 5 presents the study's limitations. Section 6 formulates conclusions and further directions for research.

2. Materials and Methods

The research materials in this work are experimental and statistical data, theoretical information and expert information, presented mainly in the form of fuzzy information, describing the states and operating modes of technological systems of oil refining production. These systems are a set of interconnected technological units with various streams and functioning as a whole, in which the chemical and physical processes of oil refining proceed and are often characterized by the fuzziness of some part of the initial information. To effectively control the operating modes of technological systems in a fuzzy environment, it is necessary to develop heuristic decision-making methods [3,22,31], that, based on models of the control object, allow DMs to choose the best operating mode.

In this paper, as a specific CTS, we study the EDP-AVT (Electric desalination plant-atmospheric-vacuum tubular) primary oil-refining unit using a simplified scheme, which is shown in Figure 1. The decision-making problem is solved in a fuzzy environment to control the operating modes of the C-8 stabilization column, where the target product is produced—stable gasoline.

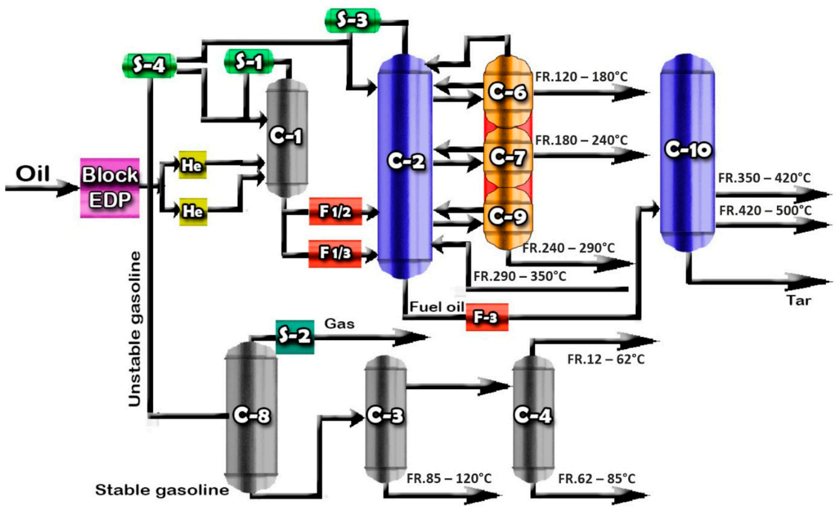

Figure 1. Simplified process flow diagram of the EDP-AVT unit.

The processed oil, having passed through the heat exchangers, enters the electric dehydrators of the EDP unit; then, after passing through the heat exchangers again, it enters C-1 oil topping column. Oil from C-1 outlet is heated in furnaces F-1/2, F-1/3 and transferred to C-2 atmospheric column. For the selection of narrow fractions of oil products 120–180, 180–240 and 240–290 °C, their overflows into the stripping columns C-6, C-7, C-9, respectively, are provided, from which, after cooling, these fractions are removed from the unit. The fuel oil from the C-2 column is heated in the F-3 furnace and enters the C-10 vacuum column. The gas is cooled in the C-8 column and enters the S-2 separator, from which the gas head is discharged. Stable gasoline from the bottom of the C-8 column enters the C-3 column, where gasoline fractions 85–120 °C are separated, then to C-4, where it is separated into gasoline fractions and 62–85 °C.

One of the main technological blocks of the EDP-AVT unit is the atmospheric unit, which includes columns C-1 and C-2. Fractions of light oil products are shipped from C-2 to the commodity park: 120–180; 180–240 and 240–290 °C, and unstable gasoline from columns C-1, C-2 is supplied as a raw material for the C-8 stabilization column and C-3 distillation column [26,28].

The described EDP-AVT primary oil-refining unit is characterized by the fuzziness of some of the initial information, which requires a fuzzy approach to make decisions on the management of its operating modes and the development and application of a fuzzy approach.

Fuzzy multi-criteria decision-making problems arise when it is required to choose the best solution at once according to several conflicting local criteria under conditions of fuzzy criteria and/or constraints. Since there is usually no solution that is best for all criteria at the same time, a reasonable compromise is necessary. Since only a person—a DM—can know which indicators are more important in the current situation, the solution of a multi-criteria fuzzy decision-making problem should be based on information about the preferences of the DM, his knowledge, experience and intuition.

For a unified description of the criteria, we will normalize them by reducing the range of their values to the segment [0, 1]. Such a description of the criteria is convenient for comparing their values with the values of the membership functions and for comparing the dimensionless values of various criteria. We present fuzzy decision-making statements and develop methods for solving them based on the heuristics and intelligence of a DM.

Let $\mu_c^1(\mathbf{x}), \ldots, \mu_c^m(\mathbf{x})$ be local criteria normalized in the interval [0, 1] that evaluate the efficiency of the modes of the control object, and $\varphi_q(\mathbf{x}) \tilde{\geq} b_q, q = \overline{1, L}$ be fuzzy constraints in the form of fuzzy instructions. Let us assume $\boldsymbol{\mu_q}(\mathrm{x}),\ q = \overline{1, L}$ as the membership functions of fuzzy constraints. As you can see, the local criterion and constraints depend on the vector of input, mode parameters $\mathbf{x} = (x_1, \ldots, x_n)$. All or some local criteria can also be fuzzy, in which case the corresponding $\mu_c^i(\mathbf{x})$ are their membership functions [3,23,24]. Let us assume that a number of local criteria priorities $I_C = \{1, \ldots, m\}$ and the vector of weight coefficients $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_L)$, reflecting the mutual importance of constraints, can be defined [3,10].

Then, modifying the ideas of the methods of the main criterion (MC) and maximin (MM) for working in a fuzzy environment, the decision-making problem for controlling the operating modes of an object can be written as the following formulation:

$$\max_{\mathbf{x} \in X} \mu_C^1(\mathbf{x}), \tag{1}$$

$$X = \left\{ \mathbf{x} : x \in \Omega \wedge \arg\left(\mu_C^i(\mathbf{x}) \geq \mu_R^i\right) \wedge \arg\left(\max_{\mathbf{x} \in X} \min_q \left(\beta_q \mu_q(\mathbf{x})\right), i = \overline{2, m}, q = \overline{1, L} \right) \right\} \tag{2}$$

where $\wedge$—logical "and", requiring the truth of all statements connected through it, μ_R^i—boundary values of local criteria $\mu_C^i(\mathbf{x}),\ i = \overline{2, m}$, set by the DM.

By changing the boundary values of local criteria $\mu_R^i,\ i = \overline{2, m}$ and the vector of weight coefficients $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_L)$ we can obtain a vector of solutions, i.e., input, mode parameters of the problem (1) and (2): $\mathbf{x}\left(\mu_R^i, \boldsymbol{\beta}\right)$. The choice of the best solution is made in an iterative mode with the participation of the DM based on his preference, experience, knowledge and intuition [3,16,30].

Based on the principles of MC (for criteria) and MM (for constraints), the following heuristic method is proposed for solving the fuzzy decision-making problem (1) and (2), formulated to control the operating modes of an object [32]. The main points of the proposed heuristic method for solving the problem are set out as follows (1) and (2):

1. The number of steps for each q-th coordinate $p_q, q = \overline{1, L}$ and a number of local criteria priorities $I_C = \{1, \ldots, m\}$ are set, where L and m are thenumber of constraints and criteria [33]. The main criterion should be given the highest priority of 1;
2. The DM sets the value of the weight vector $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_L)$, reflecting the relative importance of fuzzy constraints [33].
3. The DM assigns boundary values for local criteria $\mu_R^i, i = \overline{2, m}$, which are transferred to the composition of constraints and are taken into account as constraints.
4. To change the coordinates of the vector $\boldsymbol{\beta}$ according to the formula $h_q = \frac{1}{p_q},\ q = \overline{1, L}$ the values of each step specified in paragraph 1 are calculated [3].
5. A set of weight vectors $\boldsymbol{\beta}^1, \boldsymbol{\beta}^2, \ldots, \boldsymbol{\beta}^N, N = (p_1 + 1) \cdot (p_2 + 1) \cdot, \ldots, \cdot (p_L + 1)$ are built by changing the coordinates in the interval [0, 1] with a step h_q [10].
6. The term set is defined, for which 'fuzzy' describes the parameters of the object and the process [24,27].
7. Membership functions $\mu_q(x), q = \overline{1, L}$ of fuzzy constraints are constructed (fuzzification procedure) [3,27].
8. Based on the CTS mathematical models, a search is performed for the maximum of the main criterion (1) on the admissible set X, determined by the maximin principle (2), and the current solutions are determined: $\mathbf{x}(\mu_R^i, \boldsymbol{\beta}); \mu_C^1(\mathbf{x}(\mu_R^i, \boldsymbol{\beta})); \mu_C^2(\mathbf{x}(\mu_R^i, \boldsymbol{\beta})), \ldots, \mu_C^m(\mathbf{x}(\mu_R^i, \boldsymbol{\beta})), i = \overline{2, m}; \mu_1(\mathbf{x}(\mu_R^i, \boldsymbol{\beta})), \ldots, \mu_L(\mathbf{x}(\mu_R^i, \boldsymbol{\beta}))$ [7,10].
9. The results obtained are presented to the DM. If the current solutions do not satisfy the DM, then new values $\mu_R^i, i = \overline{2, m}$; are assigned to them and/or the values of the constraint weight vector $\boldsymbol{\beta}$ are adjusted and a return to step 3 follows. Otherwise, go to the next item [3].
10. The search for a solution stops; the results of the final choice of DM are displayed: the optimal values of the vector of parameters $\mathbf{x}^*(\mu_R^i, \boldsymbol{\beta})$; maximum value of the main criterion $\mu_C^1(\mathbf{x}^*(\mu_R^i, \boldsymbol{\beta}))$; required values of local criteria $\mu_C^2(\mathbf{x}^*(\mu_R^i, \boldsymbol{\beta})), \ldots, \mu_C^m(\mathbf{x}^*(\mu_R^i, \boldsymbol{\beta}))$ and maximum degrees of fulfillment of fuzzy constraints $\mu_1(\mathbf{x}^*(\mu_R^i, \boldsymbol{\beta})), \ldots, \mu_L(\mathbf{x}^*(\mu_R^i, \boldsymbol{\beta}))$, $i = \overline{2, m}$.

The preferences, weights, or priorities assigned to the decision maker's criteria significantly influence the decision-making process. The DM's preferences in the decision-making process in the described MC + MM heuristic method are taken into account by determining priorities and assigning weights to the criteria in paragraphs 1,2. At the same time, the DM determines priorities and weighting factors based on his experience and knowledge, taking into account the current situation in production and the market for the products produced.

Now suppose there are conditions for applying the principles of Pareto optimality and the ideal point. Then, modifying the principles of Pareto optimality and the ideal point, for fuzziness, we can write the following formulation of fuzzy decision making (3) and (4): [4,5]:

$$\max_{\mathbf{x} \in X} \boldsymbol{\mu}_C(\mathbf{x}),\ \boldsymbol{\mu}_C(\mathbf{x}) = \sum_{i=1}^{m} \gamma_i \mu_C^i(\mathbf{x}), \tag{3}$$

$$X = \left\{ \mathbf{x} : x \in \Omega \wedge \arg\left(\mu_q(\mathbf{x}) \geq \min \| \mu_q(\mathbf{x}) - \mu_q^p \|_D \right),\ q = \overline{1, L} \right\}, \tag{4}$$

where $\boldsymbol{\mu}_C(\mathbf{x})$ is theintegrated criterion obtained on the basis of the Pareto optimality principle, taking into account the weight of local criteria; $\gamma_i,\ i = 1, m$ are theweight coefficients of local criteria; Ω is the initial set of alternatives; μ_q^p are thecoordinates of an ideal point—the required degree of fulfillment of fuzzy constraints. In this formulation of the fuzzy decision-making problem, $\| \mu_q(\mathbf{x}) - \mu_q^p \|_D$ is a selectable metric D, which determines the distance between the current and ideal degrees of fulfillment of fuzzy constraints, wherein $\mu_q^p = (\max \mu_1(\mathbf{x}), \ldots, \max \mu_L(\mathbf{x}))$. If $\mu_q(\mathbf{x}), q = \overline{1, L}$ is normal, then $\mu_q^p = (1, \ldots, 1)$.

To solve the obtained decision-making problem in a fuzzy environment, the following heuristic method is proposed, developed on the basis of a modification of the Pareto optimality (PO) and ideal point (IP) principles [3–5]. The block diagram of the PO + IP heuristic method is shown in Figure 2.

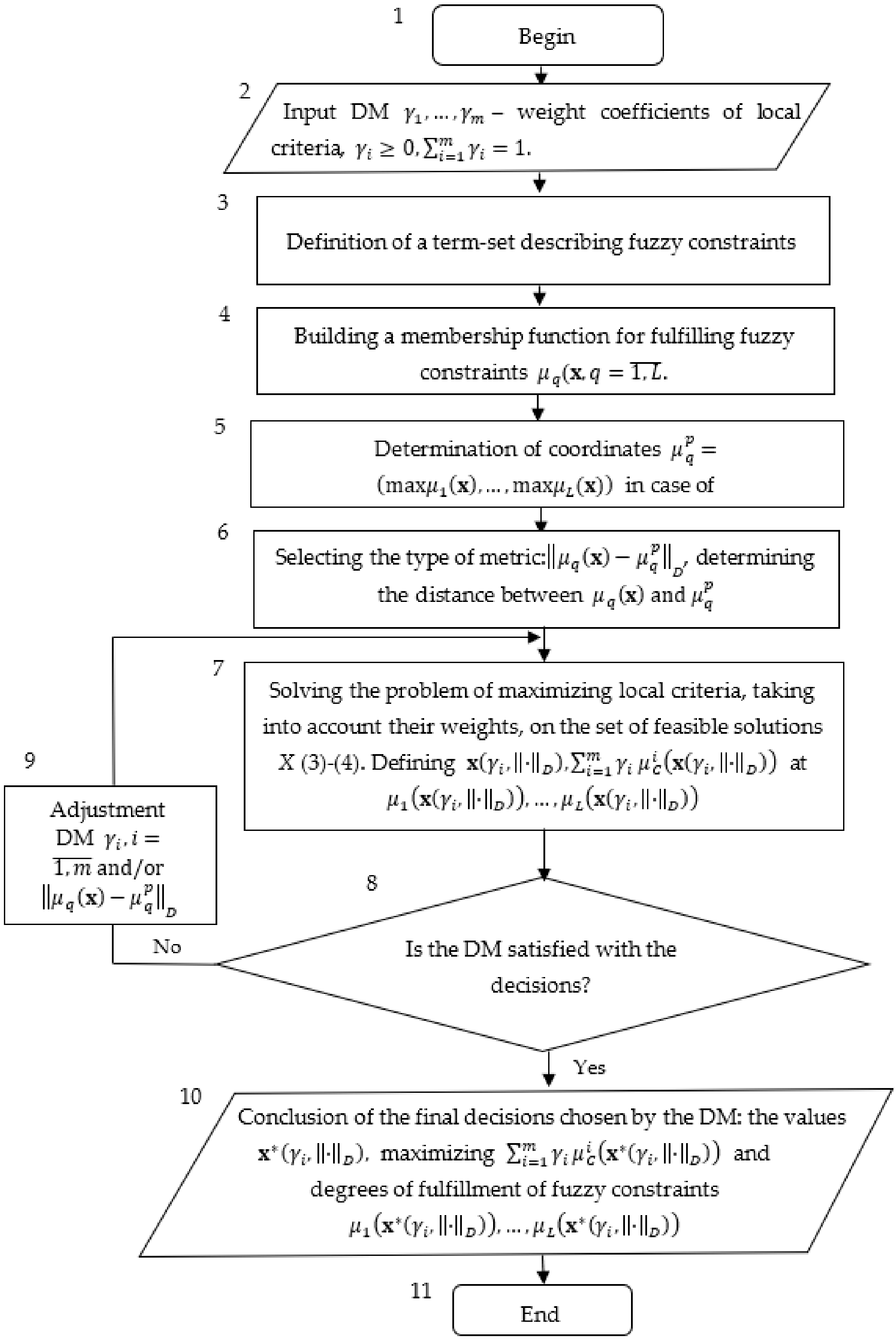

Figure 2. PO + IP heuristic block diagram.

In block 2, with the help of the DM and experts based on Pareto optimality [5], the weight coefficients of the criteria are set, which evaluate their importance.

In blocks 3 and 4, if expert evaluation is required, a term set describing fuzzy constraints is selected, and membership functions are built for them [3]. Membership functions can be built using the Fuzzy Logic Toolbox application of the MATLAB system [34].

In blocks 5 and 6, with the participation of DM, the coordinates of ideal points (solutions) are determined, i.e., the necessary degrees of fulfillment of fuzzy constraints, and the type of metric is selected. The chosen metric should determine the minimized distance between ideal points and the obtained degrees of fulfillment of fuzzy constraints [33].

In block 7, with the participation of the DM, the integrated criterion is maximized, taking into account the weight of local criteria (3) on the set of feasible solutions X (4) [33]. As a result, the values of the vector of input, mode parameters $\mathbf{x}(\gamma_i, \|\cdot\|_D)$, the corresponding values of the integrated criterion $\boldsymbol{\mu}_C(\mathbf{x})$ and degrees of fulfillment of fuzzy constraints $\mu_1(\mathbf{x}(\gamma_i, \|\cdot\|_D)), \ldots, \mu_L(\mathbf{x}(\gamma_i, \|\cdot\|_D))$ are determined.

In block 8, the DM analyzes the resulting solutions to select the final, best solution. If the current results do not satisfy the DM, then in order to improve the solution, the values $\gamma_i, i = \overline{1, m}$ and/or type of the metric $\|\mu_q(\mathbf{x}) - \mu_q^p\|_D$ (block 9) are adjusted and the iteration of the search for the best solution is repeated starting from block 7. In the case of obtaining a solution that satisfies the DM, he chooses the best solution and to derive the final solution, the transition to block 10 is carried out.

Block 10 displays the selected final decision $\mathbf{x}^*(\gamma_i, \|\cdot\|_D)$, providing local criteria $\sum_{i=1}^{m} \gamma_i \mu_C^i(\mathbf{x})$ (3) as the maximum in the area of feasible solutions (4) and the maximum value of the degrees of fulfillment of fuzzy constraints $\mu_q(\mathbf{x}), q = \overline{1, L}$.

The work also uses methods of multicriteria optimization [10,13–17,35,36], and for the collection and processing of fuzzy information, methods of expert assessment [37,38] and fuzzy set theory [21–26,39–41] are used. To develop mathematical models of the optimization and control object, experimental and statistical methods [42–44] for developing fuzzy models [27,45] are used.

Let us note the advantage of the proposed approach to solving problems of decision-making in a fuzzy environment. This approach, based on the modification and combination of various principles of optimality, depending on the nature of the available information and the possibilities available to the DM, allows the choice of the most appropriate way to solve a specific decision-making problem in the current production situation.

3. Results

3.1. Mathematical Models of the C-8 Stabilization Column of the EDP-AVT Primary Oil Refining Unit

Based on the method of successive regressors [46] and experimental data and expert information, the following model structures have been identified that estimate the yield of gasoline and gas (y_1, y_2) and are characterized by the fuzzy quality of stable gasoline ($\overline{y}_3, \overline{y}_4$) from the output of the C-8 column:

$$y_j = a_0 + \sum_{i=1}^{5} a_i x_i + \sum_{i=1}^{5} \sum_{k=i}^{5} a_{ik} x_i x_k, \qquad j = 1, 2, \tag{5}$$

$$\widetilde{y}_j = \widetilde{a}_0 + \sum_{i=1}^{5} \widetilde{a}_i x_i + \sum_{i=1}^{5} \sum_{k=i}^{5} \widetilde{a}_{ik} x_i x_j, \qquad j = 3, 4, \tag{6}$$

where x_i, $i = 1, 2$ arethe temperature of the top (x_1) and bottom (x_2) of the C-8 column; by changing these, an increase in the yield of stable gasoline is achieved, i.e., control variables; x_i, $i = \overline{3, 5}$ are disturbing influences, namely the flow x_3, temperature x_4 and raw material density x_5.

After identifying the parameters of multiple regression models (5) based on the least-squares method [43], a model for estimating the yield of stable gasoline was obtained:

$$y_1 = 111.78 - 0.047x_1 + 0.13x_2 - 0.05x_3 - 0.01x_4 - 43.073x_5 - 0.003x_1^2 + 0.011x_2^2 - 0.001x_1x_2 + 0.022x_2x_4\,, \tag{7}$$

$$y_2 = 32.804 + 0.406x_1 - 0.302x_2 + 0.113x_3 - 0.084x_4 + 0.085x_5 - 0.001x_1^2 + 0.00007x_2^2 - 0.002x_2x_4\,, \quad (8)$$

To identify fuzzy parameters (regression coefficients) of models (6), fuzzy models are first presented as a set of crisp models based on the α-level set:

$$y_j^{\alpha_q} = a_0^{\alpha_q} + \sum_{i=1}^{5} a_i^{\alpha_q} x_i + \sum_{i=1}^{5}\sum_{k=1}^{5} a_{ik}^{\alpha_q} x_i x_k, \quad j = 3,4 \quad (9)$$

where $\alpha_q,\ q = \overline{1,3}$ is the set of the α-level; we have chosen three levels, since the chosen function of the Gaussian type of the α-level has left and right levels, i.e., α = (0.5; 08; 1; 0.8: 0.5).

After identifying the regression coefficients at the α-levels of the model (9) based on the least-squares method, they are combined according to the formula [27,47]:

$$\tilde{a}_{ij} = \bigcup_{\alpha\in[0.5\div 1]} a_{ij}^{\alpha_l} \text{ or } \mu_{\tilde{a}_{ij}}(a_{ij}) = \sup_{\alpha\in[0.5\div 1]} \min\left\{\alpha_l, \mu_{a_{ij}}^{\alpha_l}(a_{ij})\right\}, \text{ where } a_{ij}^{\alpha_l} = \left\{\tilde{a}_{ij} | \mu_{\tilde{a}_{ij}}(a_{ij})| \geq \alpha\right\}.$$

As a result, the following models for estimating the beginning of the boiling of gasoline y_2 and the end of its boiling y_2 are obtained, suitable for computer simulation, optimization and control of the operating modes of the C-8 column:

$$y_3 = 243.06 - 0.04x_1 - 0.057x_2 - 0.002x_3 + 0.001x_4 - 50.27x_5 - 0.001x_1^2 - 0.001x_2^2 - 0.0002x_1x_2 + 0.011x_2x_4, \quad (10)$$

$$y_4 = 15.87 + 0.05x_1 - 0.087x_2 + 0.0004x_3 + 0.032x_4 + 23.33x_5 + 0.0003x_1^2 - 0.002x_2^2 - 0.011x_1x_2 + 0.01x_1x_4\,, \quad (11)$$

In the obtained models (7), (8), (10) and (11), regressors with zero or near-zero coefficients are considered neglible.

3.2. Determination of the Adequacy of Fuzzy Models of Technological Systems

The efficiency of the process of optimization and control of the operating modes of technological systems operating in a fuzzy environment mainly depends on the adequacy of their fuzzy models. In this regard, the issue of assessing the adequacy of fuzzy models of technological systems is very important. To study the adequacy of fuzzy models, we consider that the system being modeled is described by a set of input $\{X_1, \ldots X_n\} \subset U$ and output $\{Y_1, \ldots Y_m\} \subset V$ fuzzy sets, where U and V are universes defining the input and output spaces of the system. Fuzzy sets $X_i, i = \overline{1,n}$ and $Y_j, j = \overline{1,m}$ are defined through their membership functions:

$$X_i = \{(x, \mu_{X_i}(x))\}; Y_j = \left\{\left(y, \mu_{Y_j}(y)\right)\right\}.$$

Then technological systems can be described by a fuzzy equation (model):

$$Y_j = X_i \circ R_{ij},\ i = \overline{1,n},\ j = \overline{1,m},$$

where ∘ is the sign of a composition operation, or membership function:

$$\mu_{Y_j}(y) = \underset{x}{maxmin}\left\{\mu_{X_i}(x),\ \mu_{R_{ij}}(x,y)\right\}, \quad (12)$$

where R_{ij} is a fuzzy binary relation between the input and output parameters of the system on $U \times V$. It can be represented as a set of control rules with the structure:

$$IF\ X_i\ \ THEN\ Y_j,\ i = \overline{1,n},\ j = \overline{1,m} \quad (13)$$

defined as follows: $R_{ij} = \underset{i=\overline{1,n},\ j=\overline{1,m}}{U} X_i \times Y_j$ or by the membership function: $\mu_{R_{ij}}(x.y) = maxmin\left\{\mu_{X_i}(x),\ \mu_{Y_j}(y)\right\}$,

In (12), the fuzzy relation R_{ij} is known. The use of such a fuzzy relation shows that the estimate $\hat{Y}_j$, which is determined by expression (12), does not coincide with the original fuzzy set of output parameters of the object Y_j [28]. This means the fuzzy relation R_{ij} does not adequately reflect the given statement (13).

The criterion for evaluating the adequacy of the fuzzy model (12) to the initial fuzzy output parameters of the object can be the following expression:

$$J = min \sum_{j=1}^{m} \left(\mu_{Y_j}(y) - \mu_{\hat{Y}_j}(y)\right)^2. \quad (14)$$

Criterion (14) minimizes the deviation of the membership functions of the output parameters of the control object $\mu_{Y_j}(y)$, which are used in the formation of the fuzzy mapping R_{ij}, from the membership functions of the fuzzy sets $\mu_{\hat{Y}_j}(y)$, calculated by (12).

The main reason for the inadequacy of the fuzzy model for the observed parameters of the technological system is the inaccurate implementation of the compositional inference rule in (12).

Let us consider the conditions for the exact fulfillment of the compositional inference rule, which minimizes the values of the fuzzy model adequacy criterion J (14).

Let us represent the condition for the adequacy of the fuzzy model. Let the fuzzy relation $\boldsymbol{R} = R_{ij} = \underset{i=\overline{1,n},\ j=\overline{1,m}}{U} X_i \times Y_j$ be given. Then, if for each $X_i,\ i = \overline{1,n}$:

$$Y_j = X_i \circ R_{ij},\ i = \overline{1,n},\ j = \overline{1,m}, \quad (15)$$

then the compositional inference rule is fulfilled exactly, i.e., the fuzzy model is adequate; otherwise, it is approximate, which indicates the inadequacy of the fuzzy model.

Now we give theorems on the adequacy of the fuzzy model, i.e., on the exact fulfillment of the compositional inference rule, which minimizes the criterion for the adequacy of the fuzzy model (14).

Theorem 1. $R_{ij} = \underset{i=\overline{1,n},\ j=\overline{1,m}}{U} X_i \times Y_j$ *is a regular relation matrix*, i.e., $n = m$ *and* $det\left(R_{ij}\right) \neq 0$. *Then If* X_i *and* Y_j *are normal* ($max\mu_{X_i}(x) = 1,\ max\mu_{Y_j}(y) = 1$), *then condition (15) is exactly satisfied and the fuzzy model is adequate.*

Proof of Theorem 1. It is known that fuzzy sets X_i, Y_j and relation R_{ij} are described through their membership functions:

$$X_i = \{(x,\ \mu_{X_i}(x\ ))\}; Y_j = \left\{\left(y,\ \mu_{Y_j}(y)\right)\right\}; R_{ij} = \left\{\left(\mu_{R_{ij}}(x,y), (x,y)\right)\right\}.$$

According to Tong's theorem [48], if the fuzzy sets X_i and Y_j are normal, and the relation matrix R_{ij} is regular, then R_{ij} has a column x^* and a line y^*, for which

$$\mu_{X_i}(x) = \mu_{R_{ij}}(x^*, y);\ \ \mu_{y_j}(y) = \mu_{R_{ij}}(x, y^*), \quad (16)$$

or

$$\mu_{X_i}(x) = \max_{y} \mu_{R_{ij}}(x,y);\ \ \mu_{y_j}(y) = \max_{x} \mu_{R_{ij}}(x,y), \quad (17)$$

Whence it follows that for the elements (x, y) of the relation R_{ij}, the following relations are met $\mu_{R_{ij}}(x^*, y) \geq \mu_{R_{ij}}(x, y)$ *and* $\mu_{R_{ij}}(x, y^*) \geq \mu_{R_{ij}}(x, y)$; consequently:

$$min\left\{\mu_{R_{ij}}(x^*, y), \mu_{R_{ij}}(x, y)\right\} = \mu_{R_{ij}}(x, y); \ \ min\left\{\mu_{R_{ij}}(x, y^*), \mu_{R_{ij}}(x, y)\right\} = \mu_{R_{ij}}(x, y). \quad (18)$$

Since

$$\mu_{Y_j}(y) = \underset{x}{max} min\left\{\mu_{X_i}(x),\ \mu_{R_{ij}}(x, y)\right\}, \quad (19)$$

then, substituting (16) into (19), taking into account (17) and (18), we obtain:

$$\mu_{Y_j}(y) = \underset{x}{max} min\left\{\mu_{X_i}(x),\ \mu_{R_{ij}}(x, y)\right\} = \underset{x}{max} min\left\{\mu_{R_{ij}}(x^*, y),\ \mu_{R_{ij}}(x, y)\right\} = \underset{x}{max} \mu_{R_{ij}}(x, y) = \mu_{Y_j}(y).$$

□

Let us give a crisp explanation of Equations (15) and (19) given above. Equation (15) is a compositional inference rule known from fuzzy set theories [24,27,38]. Equations (16) and (17) mean that if fuzzy sets X_i and Y_j are normal, i.e., the maximum value of their membership function reaches 1, then their membership functions are determined by the maximum values of the membership function of the relation matrix R_{ij}. In this case, the ratio matrix R_{ij} must be regular. In B (16), x^*, y^* column and row, these conditions are satisfied, and are proved by Tong's theorem [46]. Equation (18) follows from condition (17) and shows that the minimum values of the membership function of fuzzy relations $\mu_{R_{ij}}(x^*, y)$, $\mu_{R_{ij}}(x, y)$ and $\mu_{R_{ij}}(x, y^*), \mu_{R_{ij}}(x, y)$, i.e., their intersections will be equal to $\mu_{R_{ij}}(x, y)$. Equation (19) represents the record of the compositional inference rule (15) through the membership functions of the fuzzy sets Y_j and X_i and the relation matrix R_{ij}.

Theorem 2. *Let* $\boldsymbol{R} = \underset{i=\overline{1,n},\ j=\overline{1,m}}{U} X_i \times Y_j$ *be an irregular ratio matrix*, i.e., $R = \underset{i=\overline{1,n}}{U} R_i$, *where* $R_i = X_i \times Y_i$ *is a regular relation matrix. If fuzzy sets* X_i, Y_i, $i = \overline{1, n}$ *are normal* ($max\ \mu_{X_i}(x) = 1$, $max \mu_{Y_i}(y) = 1$) *and* $\underset{i=\overline{1,n}}{\cap} X_i = \varnothing$, *and* $\underset{j=\overline{1,n}}{\cap} Y_i = \varnothing$, *then condition (15) is exactly satisfied and the fuzzy model is adequate.*

Proof of Theorem 2. We represent fuzzy sets X_i, Y_i in terms of their membership function $X_i = \left\{\left(\mu_{X_i}(x), x\right)\right\}; Y_i = \left\{\left(\mu_{Y_i}(y), y\right)\right\}$. Then, the fuzzy model is defined as

$$\mu_{Y_i}(y) = \underset{x}{max} min\{\mu_{X_i}(x),\ \mu_R(x, y)\}. \quad (20)$$

Matrix $\boldsymbol{R} = \underset{i=\overline{1,n}}{U} R_i$ is unregular, where

$$R_i = X_i \times Y_i. \quad (21)$$

By the conditions of the theorem

$$\underset{i=\overline{1,n}}{\cap} X_i = ,\ \underset{j=\overline{1,n}}{\cap} Y_i = . \quad (22)$$

Then from expression (21), taking into account condition (22), we can write:

$$\mu_{R_i} = \begin{cases} \mu_{R_i}, & if\ (x, y) \in \text{supp}\ X_i \times Y_i; \\ 0, & if\ (x, y) \notin \text{supp}\ X_i \times Y_i, \end{cases}$$

From the last entry

$$\mu_{\mathbf{R}} = \begin{cases} \mu_{R_1}, & if\ (x,y) \in \text{supp } X_1 \times Y_1; \\ \mu_{R_2}, & if\ (x,y) \in \text{supp } X_2 \times Y_2; \\ . & . \\ . & . \\ . & . \\ \mu_{R_n}, & if\ (x,y) \in \text{supp } X_n \times Y_n \end{cases}$$

therefore, for all $\mu_{\mathbf{R}} = \mu_{R_j},\ j = \overline{1,n}, \min(\mu_{X_i}, \mu_{\mathbf{R}}) = 0$, except $\mu_{\mathbf{R}} = \mu_{R_i},\ j = i$,

Taking this into account, in (20) we obtain:

$$\mu_{Y_i}(y) = \max_x \min\{\mu_{X_i}(x),\ \mu_{\mathbf{R}}(x,y)\} = \max_x \min\{\mu_{X_i}(x),\ \mu_{R_i}(x,y)\}.$$

Since according to the condition $X_i,\ i = \overline{1,n}$ are normal fuzzy sets, $R_i = X_i \times Y_i$—a regular relation matrix, according to Theorem 1, condition (15) is strictly satisfied; i.e., the fuzzy model is adequate. □

3.3. Solving the Problem of Fuzzy Multi-Criteria Decision-Making for Controlling the Operating Modes of the C-8 Stabilization Column of the Primary Oil Refining Unit of Atyrau Refinery

To support the proposed concept of solving problems of multi-criteria decision making in a fuzzy environment based on the developed heuristic method PO + IP of synthesized models of the C-8 stabilization column of Atyrau refinery, we will give the following example from real life.

In real practice, the task of fuzzy multi-criteria decision making of the process of controlling the operating modes of the stabilization column is to determine such values of the components of the vector $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5)$, providing that:

$$\max_{\mathbf{x} \in X} f_i(\mathbf{x}), i = 1, 2, \tag{23}$$

$$X = \{\mathbf{x} : \mathbf{x} \in \Omega \wedge (\varphi_1(x) \tilde{\geq} b_1, \varphi_2(x) \tilde{\leq} b_2)\}, \tag{24}$$

where $f_i(\mathbf{x}),\ j = 1, 2$ are local criteria for evaluating the yield of stable gasoline and gas from the C-8 column; $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)$ are the vector of input, mode parameters, and changes which optimize the operating modes of the object; and $\Omega = \lceil x_i^{min}, x_i^{max} \rceil,\ i = \overline{1,5}$ are the minimum and maximum values of the vectors specified by the object regulations. The C-8 operating procedure defines the following intervals for changing parameters x_i, $i = \overline{1,5} : x_1 = \lceil 140,\ 160 \rceil$, °C; $x_2 = \lceil 250,\ 260 \rceil$; °C; $x_3 = \lceil 210,\ 350 \rceil$, $\text{m}^3/\text{h}; x_4 = \lceil 70,\ 90 \rceil$, °C; $x_5 = \lceil 0.70,\ 1.00 \rceil,\ \text{kg/cm}^2$. The quality of gasoline is described by fuzzy constraints in the form of fuzzy instructions "beginning of boiling of gasoline $\tilde{y}_2$ should not be lower than 30 °C", "end of boiling of gasoline $\tilde{y}_3$ should not be higher than 180 °C". These fuzzy constraints in (23) and (24) are formalized as $\varphi_1(\boldsymbol{x}) \tilde{\geq} b_1$ and $\varphi_2(x) \tilde{\leq} b_2$.

The current values of the input, parameters relating to operation and affecting the performance of operating modes, as well as the decision-making process are available and determined using appropriate measuring instruments. Their accuracy is guaranteed by the accuracy classes of these instruments, and the intervals of their values are determined by the technological regulations of the primary oil-refining unit [32]. Furthermore, information related to fuzzy constraints is evaluated as fuzzy by DM experts, and membership functions are built for them, allowing them to process such fuzzy information [3]. The quality of fuzzy information is determined by the experience, knowledge and intuition of DM experts and, if necessary, can be improved through additional surveys and examinations [37].

We note the importance when solving a more general decision-making problem of taking into account production management and additional restrictions, such as restrictions on different resources, performance indicators, or expert knowledge, so that decision-making methods provide more specific solutions.

We present the Pareto front in a graphical format for this decision-making problem with fuzzy restrictions in the presence of two criteria, $f_i(\mathbf{x})$, $j = 1, 2$, estimating the yield of stable gasoline and gas from the stabilization column.

The set of Pareto optimal non-dominated solutions is called the Pareto front (boundary). Non-dominated solutions are a pair of solutions x and $x\prime$, for which $\exists i \in \overline{1,K}$: $f_i(x) > f_i(x\prime) \wedge \exists j \in \overline{1,K}$, $j \neq i$: $f_j(x\prime) > f_i(x)$. Pareto front P(X) includes points x satisfying the conditions P(X) = {x ∈ X:{ $x\prime$ ∈ X: $x\prime \leq x$, $x\prime \neq x$} = 0}. The most obvious is the Pareto front, i.e., the compromise curve is presented for the case of two mutually contradictory criteria, as is the case of our task; for example, in the form of a graph (Figure 3).

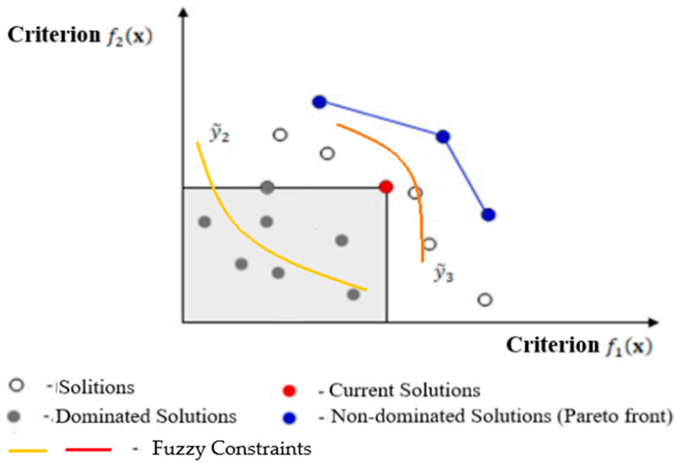

Figure 3. Pareto front for a two-criteria decision problem with two fuzzy constraints.

Having normalized local criteria and based on the formulation of the problem of fuzzy multi-criteria decision making (1) and (2), based on the principles of MC and MM, problems (23) and (24) can be written in the following form:

$$\max_{\mathbf{x}\in X} \mu_C^1(\mathbf{x}), \tag{25}$$

$$X = \left\{ \mathbf{x} : \mathbf{x} \in \Omega \wedge \arg\left(\mu_C^2(\mathbf{x}) \geq \mu_R^2\right) \wedge \arg\left(\max_{\mathbf{x}\in X}\min_{q}\left(\beta_q \mu_q(\mathbf{x})\right), q = \overline{1,2}\right)\right\}, \tag{26}$$

where $\mu_C^1(x)$ is the main criterion, in our case, the yield of stable gasoline, and its value is determined by the model y_1 (7); $\mu_C^2(x)$ is the criterion estimating the gas yield, determined by the model y_2 (8), counted as a constraints; μ_R^2 is the boundary value set by the DM for the gas outlet, in our case $\mu_R^2 = 28$ m^3/h; β_q, $q = 1, 2$ are weight coefficients of fuzzy constraints; and $\mu_q(x)$, $q = 1, 2$ are membership functions that evaluate the degree of fulfillment of fuzzy constraints.

The problem of fuzzy multicriteria decision making (25) and (26) for controlling the operating modes of the C-8 gasoline stabilization column based on the models obtained in Section 3 can be rewritten as a problem of fuzzy mathematical programming:

$$y_1 = 111.78 - 0.047x_1 + 0.13x_2 - 0.05x_3 - 0.01x_4 - 43.073x_5 - 0.003x_1^2 + 0.011x_2^2 - 0.001x_1x_2 + 0.022x_2x_4 \ \rightarrow \max$$

under constraints:

$$y_2 = 32.804 + 0.406x_1 - 0.302x_2 + 0.113x_3 - 0.084x_4 + 0.085x_5 - 0.001x_1^2 + 0.00007x_2^2 - 0.002x_2x_4 \ \geq 28;$$

$$y_3 = 243.06 - 0.04x_1 - 0.057x_2 - 0.002x_3 + 0.001x_4 - 50.27x_5 - 0.001x_1^2 - 0.001x_2^2 - 0.0002x_1x_2 + 0.011x_2x_4 \tilde{\geq} 30;$$

$$y_4 = 15.87 + 0.05x_1 - 0.087x_2 + 0.0004x_3 + 0.032x_4 + 23.33x_5 + 0.0003x_1^2 - 0.002x_2^2 - 0.011x_1x_2 + 0.01x_1x_4 \widetilde{<} 180;$$

$$140 \le x_1 \le 160; 250 \le x_2 \le 260; 210 \le x_3 \le 330; 70 \le x_4 \le 90; 0.70 \le x_5 \le 1.00.$$

Let us present the results of solving the decision-making problem in a fuzzy environment (25) and (26) based on the MC+MM heuristic method proposed in Section 2.

1. The main criterion and the number of steps along each q-th coordinate are $p_q, q = \overline{1,2} : p_1 = 5$; and $p_2 = 4$; an order of priority of local criteria $I_C = \{1, 2\}$ is determined, where 1 is the priority of the main criterion $\mu_C^1(\mathbf{x})$, estimating the yield of stable gasoline, 2—the priority of the criterion estimating the yield of gas from C-8 column;
2. DM introduced the value of the weight vector $\boldsymbol{\beta} = (0.55, 0.35)$, reflecting the mutual importance of fuzzy constraints on the beginning of the boil and the end of the boil of the produced stable gasoline.
3. DM has assigned a boundary value for the local criterion μ_R^2, which is taken into account as constraints: μ_R^2 = 8 m^3/h;.
4. To change the coordinates of the vector $\boldsymbol{\beta}$ the steps $h_1 = \frac{1}{p_1} = \frac{1}{5} = 0.20, h_2 = \frac{1}{4} = 0.25$ are calculated.
5. A set of weight vectors $\boldsymbol{\beta}^1, \boldsymbol{\beta}^2, \ldots, \boldsymbol{\beta}^N, N = (5+1)\cdot(4+1) = 30$ is constructed by changing the coordinates in the interval [0, 1] with steps $h_1 = 0.20$ and $h_2 = 0.25$.
6. The term set is defined, which is fuzzy and described by the instructions for the constraints: "not lower than 30 °C", "not higher than 180 °C".
7. Membership functions of fuzzy constraints $\mu_q(\mathbf{x}), q = \overline{1,2}$ are constructed, which are presented in the form of exponential functions $\mu_q^t(\widetilde{y}_j) = \exp\left(Q_{\widetilde{B}_j}^t \left| \left(y_j - y_j^{md}\right)^{N_{\widetilde{B}_j}^t} \right|\right)$, where t is the quantum number; $Q_{\widetilde{B}_j}^t$—is a parameter characterizing the degree of fuzziness, whose value is identified when constructing the membership function; $N_{\widetilde{B}_j}^t$ is a coefficient that allows more accurate approximation of the graph of the membership function; y_j^{md} is a fuzzy variable that more closely matches the selected term and is defined by the expression $\mu_{\widetilde{B}_j}^t(\widetilde{y}_j) = \max_j \mu_{\widetilde{B}_j}^t(y_j)$.

$$\mu_1^1(\widetilde{y}_3) = \exp\left(0.5\left|(y_3 - 25)^{0.6}\right|\right); \mu_2^1(\widetilde{y}_3) = \exp\left(0.5\left|(y_3 - 170)^{0.6}\right|\right);$$

$$\mu_1^1(\widetilde{y}_3) = \exp\left(0.7\left|(y_3 - 30)^{0.8}\right|\right); \mu_2^2(\widetilde{y}_3) = \exp\left(0.6\left|(y_3 - 180)^{0.7}\right|\right);$$

$$\mu_1^1(\widetilde{y}_3) = \exp\left(0.8\left|(y_3 - 35)^{0.9}\right|\right); \mu_2^3(\widetilde{y}_3) = \exp\left(0.7\left|(y_3 - 190)^{0.8}\right|\right);$$

8. On the basis of mathematical models of the C-8 stabilization column and using the Manager software package, a search was made for the maximum of the main criterion, taking into account the imposed constraints. In each iteration, the current solutions are determined: the vector of control and disturbing actions $\mathbf{x}(\mu_R^i, \boldsymbol{\beta})$, maximizing the main criterion $\mu_C^1(\mathbf{x}(\mu_R^i, \boldsymbol{\beta}))$, providing the required value of the criterion $\mu_C^2(\mathrm{x}(\mu_R^i, \beta))$ and degrees of fulfillment of fuzzy constraints $\mu_1(\mathbf{x}(\mu_R^i, \boldsymbol{\beta})), \mu_2(\mathbf{x}(\mu_R^i, \boldsymbol{\beta}))$.
9. The DM analyzes the received solutions. Since, in the first four iterations, the DM is not satisfied with the current solutions, in order to improve the solution, he corrects μ_R^2 and/or β_1, β_2 and the transition returns to step 8. On the 5th cycle, results are obtained that satisfy the DM and the transition to the next step 10 was carried out.
10. The best solutions selected by DM are derived: the optimal value $\mathbf{x}^*(\mu_R^2, \boldsymbol{\beta})$, which maximizes the main criterion $\mu_0^1(\mathbf{x}^*(\mu_R^2, \boldsymbol{\beta}))$, providing the required criterion value $\mu_0^2(x^*(\mu_R^2, \boldsymbol{\beta}))$ and maximum degrees of fulfillment of fuzzy constraints $\mu_1(x^*(\mu_R^2, \boldsymbol{\beta}))$, $\mu_2(x^*(\mu_R^2, \boldsymbol{\beta}))$. These results are listed in Table 1.

Table 1. Comparison of the results of optimizing the operating modes of the stabilization column according to the proposed MC + MM heuristic method, according to the known method and real data on the object optimization.

Criteria and Constraint Values	Known Results [49]	Proposed MC + IP Heuristic Method	Real Data
The yield of stable gasoline—the main criterion $y_1 = f_1(\mathbf{x}^*)$, m^3/h	125.3	128.7	126
Gas yield $y_2 = f_2(\mathbf{x}^*)$, m^3/h	-	28.5	28.3
Fuzzy constraint on the onset of boiling of stable gasoline, $\mu_1(\mathbf{x}) = y_3$, °C	-	$\mu_1(\mathbf{x}) = 1$, $y_3 = 35$	(33) ^{L}
Fuzzy constraint at the end of boiling of stable gasoline, $\mu_2(\mathbf{x}) = y_4$, °C	-	$\mu_2(\mathbf{x}) = 1$, $y_4 = 178$	(180) ^{L}
Optimal values of input and operating parameters $\mathbf{x}^* = (x_1^*, x_2^*, x_3^*, x_4^*, x_5^*)$: x_1^*—temperature of the top of the C-8 stabilization column, °C;	160	157	158
x_2^*—temperature of the bottom of the C-8 stabilization column, °C;	258	250	253
x_3^*—raw material consumption, m^3/h;	158	158	158
x_4^*—raw material temperature, °C;	90	87	88
x_5^*—raw material density, kg/cm^2.	0.95	0.95	0.95

Notes: $(\cdot)^L$ means that these quality indicators are determined by the laboratory and require human participation; (-) means that these indicators cannot be determined by known methods. The solution search time in the compared methods is the same: about one minute.

Consider the results of the sensitivity analysis to the problem from the point of view of the variation of the membership function of fuzzy constraints in paragraph 7. The variation in the value of the membership function built according to the exponential formula proposed in this paragraph and the shape of its graph depends on the tuning coefficients $Q^t_{\tilde{B}_j}$ и $N^t_{\tilde{B}_j}$. The variation in the values of these coefficients allows quickly (roughly) and slowly (exactly) selecting the values that are more consistent with the graph of the constructed membership function. Thus, the values of the membership function are sensitive to changes in the values of these coefficients, allowing the building of membership functions that are more adequate for fuzzy descriptions.

Let us give an illustration of the contradictory nature of the set of goals (criteria). The goals of the solved problem are to maximize the value $f_i(\mathbf{x}),\ j = 1,2$, i.e., criteria that estimate the volume of stable gasoline and gas from the outlet of the stabilization column using the developed models (7), (8). At the same time, it is necessary to maximize $f_i(\mathbf{x}),\ j = 1,2$ taking into account the imposed fuzzy restrictions on the quality of gasoline, estimated according to the models (10) and (11) developed in Section 3.1.

It follows from the general material balance of products that an increase in the volume of stable gasoline leads to a decrease in the volume of gas, i.e., these criteria are mutually contradictory. In addition, maximizing the volume of gasoline on a Pareto optimal set leads to a deterioration in its quality indicators. An illustration of the contradictory nature of the criteria for problem (23) and (24) is shown in Figure 4.

When determining the main criterion, the value of the weight vectors and the boundary value of the local criterion transferred to the composition of the restriction, eight specialists in the object of study and the subject area participated. The participants included three senior operators of the primary oil refinery (DM) and five subject area experts (head, technologist and plant engineer, as well as two specialists of the plant laboratory for determining the quality of gasoline). All participants are employees of the Atyrau Oil Refinery, where primary oil-refining units operate; their age ranged from 35 to 50 years; in terms of gender, two participants from the laboratory are female and the rest are male.

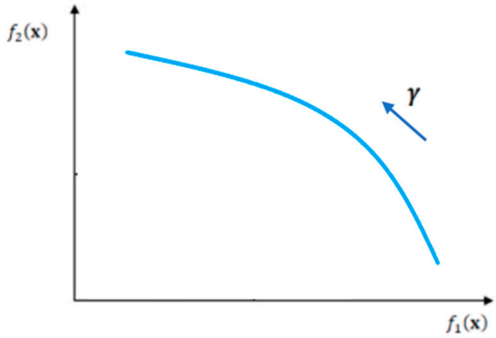

Figure 4. Contradictory character of criteria $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$, γ—parameter that takes into account the ratio of the importance of criteria (weight coefficients).

In Table 1, for comparison, in addition to the well-known results of optimizing the operating modes of the stabilization column under deterministic conditions [49] and the results of fuzzy optimization, real optimization data from the object of study are also listed. Real data were obtained experimentally on the C-8 column of the EDP-AVT Atyrau refinery.

Theoretical contributions of this research are formulated and theorems on the adequacy of fuzzy models are proved, as well as by modifying, combining various principles of optimality, new formulations of problems of fuzzy multi-criteria decision making are formulated and heuristic methods for their solution are developed. The business contribution of the study lies in the ability of the DM to make more efficient decisions in a fuzzy environment based on their preferences, while also taking into account the current production situation and the product market. In addition, the proposed approach can be exported to control the operation modes of other fuzzy objects of various industries.

4. Discussion

The formulated problem statements for fuzzy multi-criteria decision making (1) and (2) and (3) and (4) are based on the modification and combination of various principles of optimization for fuzziness and for choosing the best compromise solution depending on the production situation. The proposed heuristic methods for solving the formulated problems of fuzzy multi-criteria decision making are iterative and based on the involvement of a DM in the decision process and in the choice of the final decision. At the same time, the DM selects the best solutions based on his preferences, experience, knowledge and intuition, as well as taking into account the current situation in production and the market.

The proposed heuristic method for the problem of fuzzy multi-criteria choice (1) and (2) is obtained by modifying the principles of optimization of the main criterion (for criteria) and maximin (for fuzzy constraints). This method is effectively used where there is a possibility of highlighting the main criterion, determining the boundary values for the rest of the local criteria and the need for guaranteed degrees of fulfillment of fuzzy constraints. Furthermore, the PO+IP heuristic method for solving problem (3) and (4) is based on the modification and combination of the Pareto optimality principles and the ideal point method. Accordingly, this method allows to effectively solve the problem of fuzzy multi-criteria decision making with the possibility of DM involvement to determine the weight coefficients of local criteria and the ideal points—the degrees of fulfillment of fuzzy constraints.

The structure of mathematical models of the C-8 stabilization column of the EDP-AVT primary oil-refining unit, used to make decisions on managing its operating modes, has been identified based on modified methods for the sequential inclusion of regressors. In addition, the identifications of crisp and fuzzy model parameters are identified using the REGRESS software package [50] which implements the least-squares method.

The formulated problem of fuzzy multi-criteria decision making for controlling the operating modes of the stabilization column based on its developed models is presented in the form of an FMP problem. Then, the obtained FMP problem was solved based on the proposed MC+MM heuristic method and the Manager software package with the participation of an experienced operator DM of the EDP-AVT Atyrau refinery primary oil-processing unit.

Analysis of the comparison results shows that the proposed fuzzy decision-making approach based on the heuristic MC+MM method allows obtaining better and more efficient solutions compared to the known deterministic method. In addition, the results of the fuzzy approach more closely match the real data obtained experimentally when optimizing the control object. As a result of the analysis and discussion of the data in Table 1, as an advantage of the proposed approach to solving problems of fuzzy decision making to control the operating modes of the object of study, we can note:

- the yield of the target product, stable gasoline (the main criterion), is increased by 3.4 m^3/h (2.7%), or allows the production of 81.6 m^3 more stable gasoline per day;
- The MC+MM heuristic method allows determining the yield of a by-product—gas from the top of the stabilization column—which makes it possible to determine the material balance of the column;
- the proposed fuzzy approach to solving decision-making problems allows estimation of the degree of fulfillment of fuzzy constraints ($\mu_1(\mathbf{x}),\ \mu_2(\mathbf{x})$), which are not determined by other methods;
- the proposed approach makes it possible to obtain better solutions with lower thermal energy (temperature), which ensures the effectiveness of the approach;

In addition, when solving the problem of fuzzy decision-making to control the operating modes of the object of study based on the proposed heuristic method, the adequacy of the decision is increased. This is achieved by taking into account additional fuzzy information (experience, knowledge, intelligence of the DM and experts) that more fully describes the real situation without idealization.

5. Study Limitations

The main limitations of the proposed heuristic approach to solving problems of multi-criteria decision making in a fuzzy environment include some difficulties in assessing the degree of belonging of fuzzy constraints to fuzzy sets, adequately describing them and possible difficulties for less experienced DMs in the process of choosing the best solution.

In the future, these limitations can be eliminated by developing a special system for assessing the degree of belonging of fuzzy indicators to fuzzy sets and preparing and training DMs for the decision-making process. For the development of this study, it is planned to automate and algorithmize the process of evaluating and choosing the best solution as much as possible.

6. Conclusions

The problems of setting and solving problems of fuzzy multi-criteria decision making for controlling the operating modes of technological systems on the basis of a heuristic approach are studied. Details of the main findings of the study and conclusions include:

(1) By modifying the principles of the main criterion and maximin, as well as Pareto optimality and the ideal point, mathematical formulations of fuzzy multi-criteria decision-making problems are obtained and heuristic methods for their solution are developed. The proposed heuristic method, in contrast to the well-known ones based on the transformation of the original fuzzy problem into a set of crisp problems, allows the fuzzy problem to be set and solved in a fuzzy environment without converting to crisp problems. Due to the preservation and maximum use of the collected fuzzy information, i.e., the knowledge, experience and intuition of expert DMs, the proposed methods allow obtaining more efficient and adequate solutions to a production problem in a fuzzy environment;

(2) Based on the adaptation of the methods of successive inclusion of regressors and least squares, effective mathematical models of the C-8 stabilization column of the EDP-AVT primary oil refining unit were developed, taking into account the available

fuzzy information. The main reason for choosing the method of sequential inclusion of regressors in our case is the availability of a priori information about the structure of the developed models, which facilitates the process of developing models;

(3) The questions of determining the adequacy of fuzzy models are investigated and the corresponding theorems and their proofs are given.

(4) Based on the developed models and the proposed heuristic method MC + MM, the problem of two-criteria decision making with fuzzy constrains for controlling the operating modes of the C-8 stabilization column is solved

The main contribution of this study to the development of decision-making methods in a fuzzy environment and the novelty of the proposed heuristic methods for solving such problems lies in the effective use of knowledge, experience and intuition of DMs and experts in solving fuzzy problems of multicriteria optimization. The proposed approach, due to the maximum use of available fuzzy information in the form of DM experience and intuition, allows obtaining more efficient and adequate solutions when making a decision on the management of real production facilities.

As a direction for further research, the authors plan to:

- develop methods for fuzzy multi-criteria decision making based on the modification for fuzziness of other principles of optimality; for example, the lexicographic principle of optimality, the principles of equality, quasi-equality, concessions, etc. Such methods can effectively solve decision-making problems when the proposed methods are difficult due to a lack or absence of the necessary information for their application. Thus, the DM will be able to choose the most efficient method for the fuzzy decision problem under various possible situations and depending on the availability of various types of initial information;
- create and apply neuro-fuzzy systems [51,52], which allow combining the learning ability of a neural network with the representation of fuzzy logic knowledge, which will help to further enhance the effect of solving decision-making problems under conditions of uncertainty;
- apply the following methods to solve similar problems of decision making: Analytic Hierarchy Process (AHP), Elimination and Choice Translating REality (ELECTRE), Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE), Fuzzy TOPSIS [53–55].

The future field of application of the obtained research results is solving decision-making problems for controlling the operating modes of other complex, fuzzy described objects of oil refining, petrochemistry, metallurgy and other industries.

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