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Analytical and numerical solutions for temperature distribution in topsoil layers

T. Musabaev, T. Kayupov, D. Seilkhanova & G. Khafizova
Eurasian Institute of Technology at L.N. Gumilyov Eurasian National University, Astana, Kazakhstan

ABSTRACT: The article analyzes the analytical and numerical solutions of heat conduction under the theory of inhomogeneous bodies. The distribution of sub-zero temperature in an inhomogeneous half-space and accounting for the continuous inhomogeneity of the heat conduction rate and internal heat dissipation sources are given for the first time. The evaluation of the obtained results and the known solutions as per the European and national standards are reviewed. The comparison of numerical and analytical solutions for the test problems proves the accuracy of the obtained results. Given the availability of appropriate coefficients, these solutions are also correct for solving problems of chemical reactions with the release of heat, moisture transmission, diffusion, corrosion cracking, and other problems described by the equation of heat conduction.

1 INTRODUCTION

In winter, the atmospheric temperature in the northern regions of the country, in Nur-Sultan city, goes down to -40°C, but the depth of ground freezing is accepted to be 1,800 mm. At the same time, frosty days are changed to snowfall when the temperature rises to -10°C, and there is an alternation of day and night. Also, the ground is covered with snow. Is this standard depth of ground freezing sufficient?

The survey site is in the left-bank part of Astana, between the avenues of Kabanbay Batyr and Mangilik El, and is characterized by absolute levels (at the mouths of drilled wellbores) within the range of 348.33 - 348.74 m. This site has an embedded structure, next to which snow is not removed. The standard thickness of snow in the area of 100 cm will be taken as the thickness of the stale snow in the clearing. There is a 50 cm thick topsoil all around under the snow, consisting of interwoven plant roots and air pockets. We model this layer as peat heat insulating slabs per Table 1 SN RK [1] (Figure 1).

Table 1. Coefficient of heat conduction of materials.

Name of materials listed in [3]	Coefficient of heat conduction λ, W/moC	Isıl İletkenlik Hesap Değeri λ (W/mK)
Interior plaster (İÇ SIVA)	0.76	0.7
Brick wall (DOLGU DUVAR)	0.41	0.45
External plaster (DIŞ SIVA)	0.76	1.6
Waterproofing (SU YALITIMI)	-	0.19
Heat insulation (ISI YALITIM)	0.035	0.035
Heat insulating plaster (ISI YALITIM SIVASI)	-	0.35

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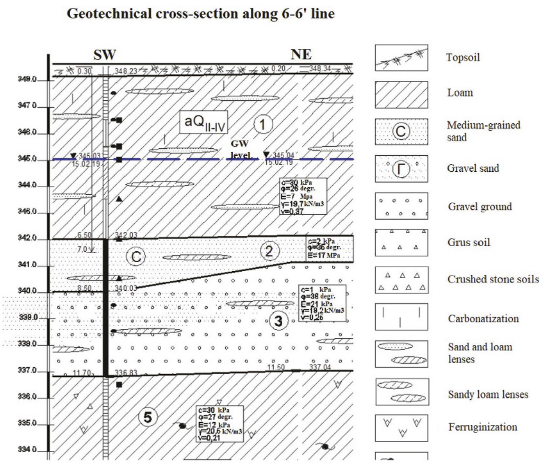

Figure 1. Geotechnical cross-section of the ground of the embedded structure on Mangilik El Avenue in Astana.

According to Eurocode 1, in case there are no internal sources of heat dissipation, the temperature distribution in layered structures shall be determined by the following formula (1) [2]:

$$T(x) = T_{in} - \frac{R(x)}{R_{tot}}(T_{in} - T_{out}) \tag{1}$$

where T_{in} is the indoor air temperature; T_{out} is the outside air temperature; R_{tot} is the total thermal resistance of an element, including the resistances of both surfaces (2); $R(x)$ is the thermal resistance from the inner surface and layers on the inner surface to the given point with coordinate x (Figure 1).

A similar formula can be found in formula (29) of SN RK [1], where the thermal resistance to heat transmission of layered structures can be determined by a similar one with (2) [2]:

$$R_0^{mp} = \frac{1}{\alpha_H} \cdot + \cdot \frac{h_1}{\lambda_1} \cdot + \cdot \frac{h_2}{\lambda_2} \cdot + \cdot \ldots \cdot + \cdot \frac{h_n}{\lambda_n} \cdot + \cdot \frac{1}{\alpha_B} \tag{2}$$

where α_B=8.7 is the coefficient of heat transfer from the inner surface of the land cover, taken according to Table 1 [1] as the coefficient of heat transfer from the inner surface of the walls; α_H=6 is the coefficient of heat transfer of the outer surface of the ground (snow cover), taken according to Table 1 [1] as the covering over the unheated basement; h_i is the thickness of the i-th layer, m; λ_i is the heat conduction of the material of the i-th layer, taken based on the results of tests in an accredited laboratory [1].

A comparison of the heat conduction values of the materials in the composition of the wall [3] as given in the Turkish and Kazakh national appendices to the Eurocode is listed in Table 1. As Table 1 shows, there is no significant difference in the numerical values of the heat conduction of materials, with the exception that in practice, the presence of some materials marked with the minus sign in Table 1 is not taken into account in thermo-technical calculations.

In the climatic conditions of Astana city, due to the presence of castle clay under the clay loam layer, the surface soils, except for the topsoil for the winter, are getting waterlogged and in some places are connected by an aquifer. The topsoil passes through the moisture like a sieve, supplies water to vegetation, and desiccates before the weather is cold. During waterlogging, all soil interstices are filled with water and freeze in winter, forming a monolith of soil and ice. Therefore, the thermal properties of the frozen soil layers can be equated with those of ice [4]. It is seen from Figure 2 that the coefficient of heat conduction of ice is strongly dependent on temperature. For this reason, the value of the heat conduction coefficient of frozen ground is a function of temperature.

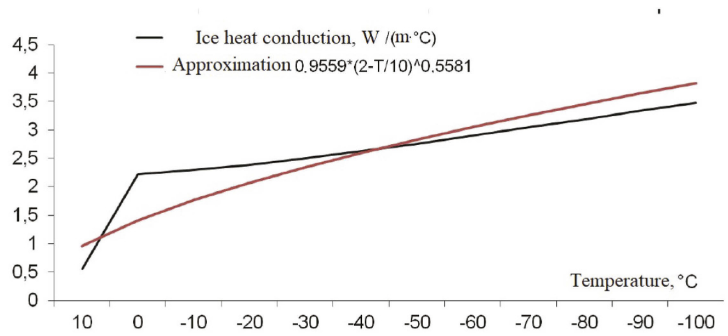

Figure 2. Ice heat conduction coefficient and the approximation of its graph by a power function.

As a rule, the depth of ground freezing does not reach the aquifer. Most likely that the aquifer is formed by the following processes such as ice formation, melting, ice surfacing in the water lens, raising and lowering of the surface soil layer due to the lunar attraction, as well as sand settlement to the bottom of the water lens.

Assume that the ground surface is covered with fresh snow with a heat conduction coefficient of λ_1 = 0.15 W/m °C [5] with a thickness of δ_1 = 0.2 m and a stale snow cover of λ_2 = 0.5 W/m °C [5] with a thickness of δ_2 = 0.8 m. The surface coating of the layered half-space is split into a topsoil layer (modeled as peat heat-insulating slabs of λ_3 = 0.064 W/m °C [1]) with a thickness of δ_3 = 0.5 m, frozen loam (as ice at -10°C λ_4 = 2 W/m °C [4]) with a thickness of δ_4 = 1.5 m, and other layers from Table 2. From [1] and [5], we will take the thermo-physical properties of other soil layers, as per the geotechnical cross-section (Figure 1).

Table 2. Heat transmission resistance in the multilayer land cover.

Layers	h, m	λ, W/m°C	Rlayer
Heat transfer coefficient of the outer snow surface	α_{out}=	6	0,17
Fresh snow	0.2	0.15	1.33
Stale snow cover	0.8	0.5	1.60
Topsoil: peat insulation	0.5	0.064	7.81
Frozen loam	1.5	2	0.75
Water saturated loam - aquifer	2	1.33	1.50
Clay	0.5	0.8	0.63
Dry loam	1.5	0.5	3.00
Coefficient of inner surface heat transfer	$\alpha_{in.}$=	8.7	0.11
Overall thickness of the concerned multilayer land cover	7	ΣR0 =	16.91

In establishing the thickness of the land cover, it is essential to identify the mark of the beginning of the temperature of 10°C for the considered region. Normally, the temperature of 10°C is observed at a depth of 10 m, 20 m, and 30 m as well. In this case, we accept the thickness of the land cover to be 7 m.

Figure 3 depicts plots of the distribution of coefficient of heat conduction of land cover layers by the depth and its approximation by a power function calculated using the least-squares method and a polynomial function, selected visually.

It is seen from Figure 3 that the standard values of the coefficient of heat conduction of land cover layers have a step pattern of distribution. With the polynomial approximation, it is possible to partially describe the ascending and descending branches of the normative graph

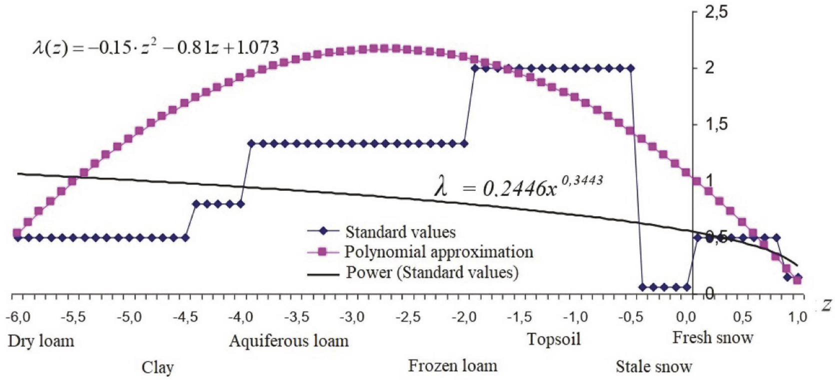

Figure 3. Graph of the variation of the coefficient of heat conduction of land cover layers by the depth and its approximation by the power and polynomial functions.

and the initial and final value. With the power approximation, it is possible to approach the standard values only on the surface layers. The efficiency of the approximation can be determined after plotting the temperature distribution with different methods, comparing and evaluating the obtained results.

The differential equation of heat conduction of one-dimensional problem (the properties of layers are given only along the z-axis which is perpendicular to the ground surface, and the properties and thickness do not change in each layer) in the orthogonal coordinates is written as follows:

$$\frac{\partial}{\partial r}\left(\lambda(r,z)\cdot\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(\lambda(r,z)\cdot\frac{\partial T}{\partial z}\right)=\gamma\cdot c\cdot\frac{\partial T}{\partial \tau}-q(r,z) \tag{3}$$

where γ is the bulk weight, kg/m^3, c is the specific heat capacity, J/(kg °C), $\lambda(r,z)$is the heat conduction coefficient; $q(r,z)$ is the density of internal heat dissipation sources [6, 7, 8].

Let there are no internal heat dissipation sources, a steady-state problem of heat conduction ($\partial T/\partial\tau=0$) is considered, and the coefficient of heat conduction of soils is changing in the following manner:

$$\lambda(z)=\lambda_0\cdot z^m \tag{4}$$

where λ_0=0.2446, m=0,3443 – approximation by power function, Figure 3.

By integrating (3) by z, we will get:

$$\lambda_0\cdot z^m\frac{\partial T}{\partial z}=C_1;\ \ T=C_1\cdot\frac{z^{1-m}}{1-m}+C_2 \tag{5}$$

where C_1 and C_2 are arbitrary constants determined from boundary conditions.

In this case, as boundary conditions, the laws of convective heat transfer of the medium with the wall surface are used [7]:

$$\lambda_0\cdot a^m\frac{\partial T}{\partial Z}=\alpha_B(T-T_B);\lambda_0\cdot b^m\frac{\partial T}{\partial Z}=-\alpha(T-T_H) \tag{6}$$

Here, a and b are, respectively, the coordinates of the outer and inner surfaces of the considered land cover; α_B, α_H are the heat transfer coefficients; T_B,T_H is the temperature of the environment near the inner and outer surfaces of the considered land cover.

We substitute the solution of (4) and (5) into (6), and then the solution of equation (3) is determined based on the solution of the system of equations:

$$C_1 \cdot \left(\frac{a^{1-m}}{1-m} - \frac{\lambda_0}{\alpha_B}\right) + C_2 = T_B; C_1 \cdot \left(\frac{b^{1-m}}{1-m} + \frac{\lambda_0}{\alpha_H}\right) + C_2 = T_H \tag{7}$$

$$C_1 = (T_H - T_B) / \left(\frac{b^{1-m} - a^{1-m}}{1-m} + \frac{\lambda_0}{\alpha_H} + \frac{\lambda_0}{\alpha_B}\right); C_2 = T_B - C_1 \cdot \leq \left(\frac{a^{1-m}}{1-m} - \frac{\lambda_0}{\alpha_B}\right) \tag{8}$$

Let the heat transfer coefficient of the wall change in the following manner:

$$\lambda(z) = -0,15 \cdot z^2 - 0.81z + 1,073 \tag{9}$$

By integrating (3) by z, we will get:

$$\frac{\partial T}{\partial z} = \frac{C_1}{-0,15 \cdot z^2 - 0.81z + 1,073}; \; T = C_1\left(0,88ln\left|\frac{z+6,5}{z-1,1}\right|\right) + C_2 \tag{10}$$

By solving the quadratic equation (9), we get two roots: $z_1 = 1.1$ and $z_2 = -6.5$, which gives zero in the denominator. These points are beyond the range of consideration.

To solve the formulated boundary value problem (3) with boundary conditions (6) using the numerical method, the variation-difference method is applied. Following this method, the ground depth interval $a \leq z \leq b$ of unit height and length is arbitrarily subdivided into M - 1 parts, i.e. an irregular rectangular mesh is introduced (Figure 4).

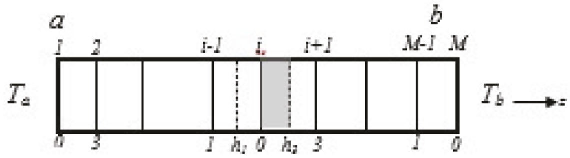

Figure 4. Difference mesh.

The numbering at the top refers to the difference mesh node number, and at the bottom - 0, 1, 3 - are the local numbers of the i-th node cells. The numbers in the cell number indicate which node the given cell is located near h1 and h3 are the mesh steps near the cells. The basic mesh is shown with solid lines, and the additional - with dotted lines, and the typical right cell 03 is shaded.

The equivalent functional of the Ritz method [9] is as follows:

$$I(T) = \int_V \left(\lambda(z) \cdot \frac{\partial T}{\partial z}\right)^2 dv - \alpha_n \oint_S \left(T^2 - 2T_n T\right) \cdot ds \tag{11}$$

where T is the temperature, °C, αn is the surface heat transfer coefficient; T_n is the ambient temperature near the surface with the outer normal n.

An approximate finite-difference expression of potential energy (11) is obtained under the assumption that all functions are constant in each cell. For instance, for a typical cell 03, derivatives are replaced by difference expressions, and the functions take the following values:

$$T = T_0, \frac{\partial T}{\partial Z} = \frac{T_3 - T_0}{h_3}, T_n = T_a, \alpha_n = -\alpha_a, \; dv = \frac{h_3}{2} \cdot 1 \cdot 1, \; ds = h_3 \cdot 1 \tag{12}$$

The values of the surface heat transfer coefficient αn for the cells where the direction of the outer normal n does not coincide with the direction of the z-axis are taken with the opposite sign. E.g., for cell 01:

$$\alpha_n = \alpha_b \quad (13)$$

The sum of potential energies of all cells will be the total potential energy of the whole body. The boundary integrals are calculated using the trapezoid formula. The minimum of this function is attained subject to the following conditions:

$$\frac{\partial I(T)}{\partial T_0} = 0 \quad (14)$$

By performing differentiation, one can obtain a difference approximation of the boundary value problem (3), (6), (11).

After reducing the constants, the equation for the right cell 03 has the following finite-difference expression:

$$(\lambda_0 + \lambda_3)\frac{T_3 - T_0}{2h_3} = \alpha_a(T_0 - T_a) \quad (15)$$

The equation is similar for the left cell 01:

$$(\lambda_0 + \lambda_1)\frac{T_1 - T_0}{2h_1} = -\alpha_b(T_0 - T_b) \quad (16)$$

The equations for the other cells are different in the absence of boundary elements and the presence of both right and left cells so that they are obtained by summing the summands of formulas (15) and (16)

$$(\lambda_0 + \lambda_1)\frac{T_1 - T_0}{h_1} + (\lambda_0 + \lambda_3)\frac{T_3 - T_0}{h_3} = 0. \quad (17)$$

The matrix of algebraic equations will be tridiagonal. That is why it is convenient to present the system of linear algebraic equations as follows [7]:

$$a_i T_{i-1} - c_i T_i + b_i T_{i+1} = -f_i \quad (18)$$

where a_i are the coefficients preceding T_1, c_i are the coefficients preceding T_0, b_i are the coefficients preceding T_3, fi is the product of coefficients preceding T_a and T_b and their values. The sweep method [10] is used to calculate the temperature.

The results of computing the temperature distribution in an inhomogeneous half-space under the snow cover, which are calculated by formulas: (1), (8), (10), and the numerical solution, are presented in Figure 5. The considered thickness of 7 m of the land cover is divided into 70 steps with a length of 0.1 m.

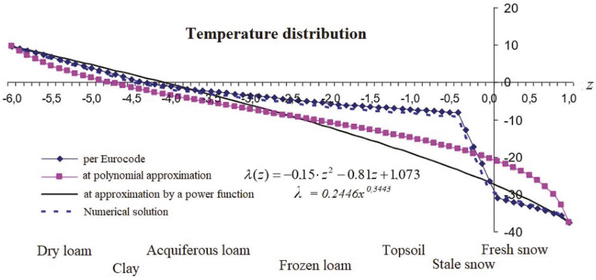

Figure 5. Temperature distribution in an inhomogeneous half-space under the snow cover.

It is seen from Figure 5 that formula (2) of Eurocode 1 [2] and formula (29) of SN RK [1] adequately describe the temperature distribution in the multilayer structure of the land cover formation. A linear law of temperature distribution is observed in each layer. The numerical calculation qualitatively repeats the temperature distribution result; quantitatively, there are differences of less than 2 % in some nodes.

The temperature determined using the power function qualitatively differs from other solutions, and the quantitatively adequate result is obtained at the point of temperature sign change. Such differences may be caused by the approximation of the heat conduction coefficient by a power function, which qualitatively differs from the set heat conduction coefficient variation law. The temperature distribution using formula (10) in the approximation of the heat conduction coefficient by a quadratic polynomial better conveys the behavior of the change in the Eurocode solution. But the dew point position by (8) is better compared to formula (10). Two analytical solutions based on the theory of inhomogeneous bodies, i.e., inhomogeneous models of the structure adequately describe the physical process of heat conduction in a multilayer half-space under the snow cover on the effect of sub-zero temperature.

2 CONCLUSION

1. As can be seen from the figure, all temperature distribution curves turn out to be qualitatively similar; in quantitative terms, there are differences near the surface layer.
2. The solutions of (8) and (10) types can be used as a test task to evaluate the results of physical and numerical experiments with complex boundaries or in two- and three-dimensional settings.
3. Based on the results of calculations in different models in a one-dimensional setting for Nur-Sultan city, the standard depth of ground freezing of 1,800 mm is not sufficient, as the dew point in Figure 5 goes down to 4 m.
4. The results obtained may change in case of using other models of soil, snow, ice, the thickness of the considered cover, heat transfer coefficients, as well as in case of two-dimensional or nonlinear non-steady solution taking into account heating pipe mains, sewerage systems, power supply networks as sources of heat and temperature fluctuations in the winter season.

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