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## Maximum power tracking for wind energy conversion systems via a high-order optimal disturbance observer-based LQR without a wind speed sensor

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## ABSTRACT

A high-order optimal disturbance observer (HOODO) is introduced to precisely estimate the aerodynamic torque and the variable wind speed; and consequently calculate the optimal speed of the generator without measuring the wind speed. As the immeasurable wind speed is varying fast, the aerodynamic torque, which is considered as a disturbance, is also changed fast, then the conventional assumption that the disturbance is slowly-varying (i.e., its first-time derivative is zero), is not applicable. The proposed HOODO design considers the fast and stochastic characteristics of the wind speed by relaxing aforementioned conventional assumption. Moreover, via the linear optimal control theory, the parameters of the HOODO are tuned systematically by proper selection of the elements of the diagonal weighting matrices. In this study, a compromise between the observer's convergence rate and the ability to Gaussian noise suppression is also considered. This helps to solve the difficulties of many existing observers regarding the gain selection algorithms. Also in the article, for the first time, a control scheme is considered in combination with the proposed HOODO and the LQ controller. To maximize a power captured from the wind, a linear-quadratic regulator (LQR) is utilized to maintain the angular speed of the generator at the optimal speed reference. A stability analysis of the designed control scheme is also discussed. Comparative results of the HOODO under different orders as well as other observers are given to prove the effectiveness of the introduced estimation method. The obtained simulation results reveal that the HOODO has a superior estimation of disturbance. All the simulations are carried out in the MATLAB/Simulink software.

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## 1. Introduction

Wind energy is one of the main sources of renewable energy to solve the global warming and pollution associated with fossil fuels. In recent decades, the importance of wind energy has increased significantly among renewable sources [1–5]. Wind energy conversion systems (WECSs) are utilized to generate electric power from the aerodynamic power of the wind. In variable-speed WECSs, the permanent magnet synchronous generator (PMSG) is one of the widely used type of generators [6,7], since it has excellence in terms of high power factor, high power density, high efficiency, and high reliability [8]. However, controlling WECSs is challenging

as they have to face with nonlinearities, disturbance, model uncertainty, and modeling errors [9–11] of the system. Moreover, in variable-speed WECSs, the maximum power point tracking (MPPT) problem is essential. With MPPT, the optimum speed reference that the generator has to track is proportional to the wind speed. Anemometers are traditionally used to measure wind speed [12,13]; however, they cannot accurately measure wind speed for control purposes. A feasible solution is to estimate aerodynamic torque and then the wind speed by a disturbance observer (DO) [8,14]. Therefore, a DO-based control system is an effective solution to these issues.

In conventional linear and nonlinear DOs [15–22], a convergence of the estimation error was ensured by considering that a disturbance has slow dynamics with respect to the dynamics of the observer (i.e., the first order time-derivative of a disturbance equals to zero). This kind of assumption is a popular one and works in many practical cases. However, for some particular cases when disturbances have fast dynamics, this assumption is not appropri-

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ate and can lead to unsatisfied estimation performance. The WECS, in which the aerodynamic torque is related to the stochastic wind speed by cubic function relationship, belongs to such type of systems. Shotorbani et al. presented an adaptive nonlinear disturbance observer-based finite time control (FTC) [23]. According to this work, the proposed method has an ability to converge to an equilibrium point in the finite time and it shows better results comparing to the conventional asymptotic convergent controller (ACC). However, during the observer design, an unknown disturbance term is considered as slowly-varying. In the work of Mansouri et al. [24], an optimization of a nonlinear observer using a genetic algorithm is proposed and the speed is estimated by a phase locked loop (PLL). To enhance robustness of the proposed system, a super twisting algorithm is used instead of a traditional PI controller. However, implementation of this method seems complicated since it requires two observers and utilizes a genetic algorithm (GA) which may need additional calculation efforts, consequently make it difficult for hardware implementation. Recently, an optimal control solution to optimal tracking of power in PMSG-based WECS is presented [25]. A sum of squares (SOS) approach was applied to design a nonlinear observer to estimate an aerodynamic torque as well as  $i_d$  and  $i_q$  currents in WECS. In this proposed method, sum of squares conditions need to be chosen accurately. A nonlinear adaptive control for PMSG-based WECS was proposed in [26]. In this work, a high gain perturbation observer (HGPO) estimates lumped disturbance including nonlinearities, parameter variations, and other external disturbances. Basically, the proposed HGPO is designed based on the Luenberger observer. An improvement for a MPPT control scheme is found in the work of Dali et al. [27]. Although the obtained results of the proposed method show its superiority over traditional methods, detailed analysis about high-order derivatives of disturbances is not presented clearly. A high gain observer is designed for sensorless regulation of a PMSG-based WECS [28]. However, during the observer design, the aerodynamic torque is assumed to be as slowly varying. In another work, active disturbance rejection controller was designed for a WECS [29]. According to this method, the bandwidths of the controller as well as observer are taken as inverses of parameters of internal model controller (IMC) and are replaced by disturbance rejection and setpoint filters. The extended Kalman filter-based optimal controller to estimate and control PMSG-based WECS was presented in the work of Bakhtiari and Nazarzadeh [30]. The designed method shows better performance compared to the conventional PI and linear quadratic Gaussian (LQG). However, additional analysis on fast-changing disturbance conditions should be provided. The extended state observer-based adaptive disturbance rejection control (ADRC) was utilized [31] to estimate disturbances presented in PMSG-based WECS. However, as it is stated in the work of Das et al. [29], when the total disturbance is variable, the ESO cannot guarantee a perfect estimation of disturbance. Moreover, in WECSs with fast parametric changes, unknown external disturbance, and nonlinearities the accurate estimation of disturbance with the ESO is almost impossible. The article of Chen [32] considered a nonlinear disturbance observer-based control (NDOBC). From the presented results, this method is more stable and able to suppress disturbances accurately compared to other robust control methods. Also, the use of a high-order observer shows its robustness to parameter uncertainty, unmodeled errors, and any other type of fast-varying noise [33].

To achieve the desired control system performance, together with conventional proportional-integral-derivative (PID) controllers, many advanced control algorithms for a PMSG exist in the literature. For example, model predictive control (MPC) [34], proportional-integral (PI) control [35], sliding mode control (SMC) [12,36], linear quadratic (LQ) control [33], integral sliding

mode control (ISMC) [12,36,37], and neural network based control (NNDC) [38]. A backstepping control [39] was presented to enhance the performance of well-known vector-control (VC) scheme by replacing a traditional PI control widely used in WECS. The backstepping control algorithm shows satisfactory performance under various simulation as well as experimental scenarios, but the comparative results of the presented method with other existing methods is not shown clearly. In another work, a disturbance observer (DOB)-based SMC was utilized to robustly control of a PMSG [12]. Additionally, a fuzzy algorithm was used to switch the gains of the SMC appropriately. Comparing to the fixed-gain SMC, the proposed fuzzy SMC has advantages in terms of chattering reduction, however hardware implementation of this method might be difficult. A linear-quadratic regulator (LQR)-based optimal approach have been introduced by Le and Do [40]. In order to remove nonlinearities and disturbance terms by compensating them in the feedforward loop, the designed control scheme is combined with a high-order disturbance observer. The effectiveness of the presented controller requires a comprehensive analysis by comparing with other existing methods. A model-predictive control (MPC)-based on finite-control-set was designed to overcome the disadvantages existing in a conventional MPC [34]. Although this method shows improved results, its hardware realization needs a real-time setup based on a powerful controller such as field-programmable gate array (FPGA), which in turn adds additional cost to design of overall control system. Disturbance observer-based feedback linearisation control was proposed to improve transient performance under system uncertainty in WECS [35]. This method has a decoupled PI-like design with two extra parts, in which the first part is introduced as an antiwindup compensator, whereas the second part is used to eliminate the effect of sudden step variations in power during the transient period by using the reference jump information. Unfortunately, analysis of the system's performance with the proposed control method under a wind profile with fast dynamics is not shown in the results. Two SMC schemes [36] were presented to control both the mechanical speed of a generator as well as a dc-link voltage. In this work, in order to prove the efficacy of the designed controller over the traditional ones additional comparative studies should be conducted. A research group proposed a novel SMC-based control scheme for a PMSG [41], and the simulation outcomes reveal that with the presented control method a PMSG-based WECS become stable and robust in the region of MPPT. In a recent work [37], a composite controller consisting from a LQR-based ISMC and high-order DO has been presented. In this method, the LQR is designed in the nominal control part, while the ISMC is used to cope with system nonlinearities and parametric uncertainties. The above mentioned control methods greatly improve the control characteristics in various aspects. Although some of these methods are complex and difficult to implement (e.g. NNDC) [38], and for some, under the influence of the mismatched uncertainties, they lose their property as robustness (e.g. SMC) [42].

Considering the previous research works, this paper introduces a high-order optimal DO (HOODO) for estimating aerodynamic torque which, in turn, helps to estimate the wind speed. This paper also utilizes an linear quadratic regulator (LQR), for the speed tracking problem of the generator. The efficiency of the presented optimal composite control scheme is justified by computer simulation in MATLAB/Simulink. The main contribution of this work are: 1) The proposed HOODO relaxes the assumption that disturbance is slowly changed. By doing this, it creates a theoretical base for estimation of fast-varying disturbances such as aerodynamic torque of WECSs precisely; 2) Moreover, to systematically select the gains of the proposed HOODO, an optimal algorithm is utilized. Via tuning the diagonal of weighting matrices, the gains are easily selected for satisfied performance. Note that the gain tuning rule is

a big issue in most of the existing observers; 3) Four different orders of the HOODO, i. e. first, second, third, and fourth order, are compared, analysed and investigated in detail; 4) A straightforward gain tuning rule is implemented for the LQR, which is practical to implement in real hardware; 5) Detailed analysis on stability of the closed-loop HOODO-based LQR is presented explicitly using the Lyapunov approach.

## 2. Control of a WECS

This section introduces the PMSG-based WECS model [33].

### 2.1. Wind turbine modelling

The aerodynamic power ( $P_a$ ) which a wind turbine (WT) can be extracted from the wind is expressed by the following non-linear relationship

$$P_a = \frac{1}{2} \rho \pi R^2 C_p(\lambda, \beta) v^3 \quad (1)$$

where  $\rho$  represents the air density,  $R$  is the radius of a WT's blade,  $v$  is a speed of the wind,  $C_p(\lambda, \beta)$  is a WT's power coefficient, which describes the ability of the WT to obtain a mechanical energy from the energy of the wind,  $\lambda$  is a tip-speed ratio (TSR),  $\beta$  represents a pitch angle of the WT's blade.

The power coefficient  $C_p$  relates to the TSR  $\lambda$  and blade pitch angle  $\beta$  by nonlinear relationships. The coefficient in most cases are calculated experimentally, hence it is usually reported by the WT's manufacturer. The dependence of the power coefficient  $C_p(\lambda, \beta)$  on the TSR  $\lambda$  at a different angle  $\beta$  for a certain blade profile is given in Fig. 1. The figure shows that for each curve, there is one optimal  $\lambda$ , named  $\lambda_{opt}$  at which the power coefficient  $C_p(\lambda, \beta)$  reaches its maximum value.

The TSR  $\lambda$  is defined as [43]

$$\lambda = \frac{\omega_t R}{v} \quad (2)$$

where  $\omega_t$  is the angular speed of the WT's shaft.

Then from (2), the optimal reference angular shaft speed of the WT is determined as

$$\omega_{t,ref} = \frac{\lambda_{opt} v}{R} \quad (3)$$

Here,  $\lambda_{opt}$  is the optimal TSR, at which the power coefficient  $C_p$  achieves its maximum value. The ratio in the gearbox of the WT is defined using the following formula,

$$n_{gb} = \frac{\omega}{\omega_t} = \frac{T_a}{T_g} \quad (4)$$

where  $\omega$  is the generator's angular shaft speed,  $T_g$  is a torque of the generator's rotor,  $T_a$  is a aerodynamic torque.

On the other hand, the WT's aerodynamic torque depends on the angular speed  $\omega_t$  through the expression

$$T_a = \frac{P_a}{\omega_t} = \frac{1}{2} \rho \pi R^3 C_q(\lambda, \beta) v^2 \quad (5)$$

where  $C_q(\lambda, \beta) = \frac{C_p(\lambda, \beta)}{\lambda}$ .

### 2.2. A model of the PMSG

A structural analysis of the design diagrams of low-power synchronous generators, carried out in [44], showed that multi-pole machines with excitation from permanent magnets outperform other types of generators in terms of efficiency and stability. This is due to the high performance of modern permanent magnets. In this paper, we will consider a PMSG-based WECSs.

We will use the dynamic model of the PMSG as given in [35]:

$$\begin{cases} J\dot{\omega} = -B\omega - T_e + \frac{1}{n_{gb}} T_a \\ \dot{i}_q = -\frac{R_s}{L_s} i_q - P\omega i_d - \frac{\psi_m P}{L_s} \omega + \frac{1}{L_s} u_q + \bar{d}_q \\ \dot{i}_d = -\frac{R_s}{L_s} i_d + P\omega i_q + \frac{1}{L_s} u_d + \bar{d}_d \end{cases} \quad (6)$$

where  $i_d$  and  $i_q$  are the currents of a generator's stator,  $u_d$  and  $u_q$  are the voltages in the generator's stator in the direct and quadrature axes, respectively.  $R_s$  is a nominal value of the stator's resistance;  $L_s$  denotes the nominal value of the stator's inductance;  $B$  is a viscous friction coefficient of the shaft of PMSG;  $\psi_m$  defines a magnetic flux linkage;  $J$  is the inertia of the rotating parts including the wind turbine;  $P$  is the pole pairs number;  $T_a$  and  $T_e$  represent the aerodynamic torque and the electromagnetic torque, respectively. Also, where  $\bar{d}_q$  and  $\bar{d}_d$  are the terms including parameter uncertainties, noise and modeling errors, which are calculated as follows:

$$\begin{aligned} \bar{d}_q &= \left( \frac{R_s}{L_s} - \frac{R_s + \Delta R_s}{L_s + \Delta L_s} \right) i_q + \left( \frac{\psi_m}{L_s} - \frac{\psi_m + \Delta \psi_m}{L_s + \Delta L_s} \right) P\omega + \left( \frac{1}{L_s} - \frac{1}{L_s + \Delta L_s} \right) u_q + \bar{d}_{qn} \\ \bar{d}_d &= \left( \frac{R_s}{L_s} - \frac{R_s + \Delta R_s}{L_s + \Delta L_s} \right) i_d + \left( \frac{1}{L_s} - \frac{1}{L_s + \Delta L_s} \right) u_d + \bar{d}_{dn} \end{aligned} \quad (7)$$

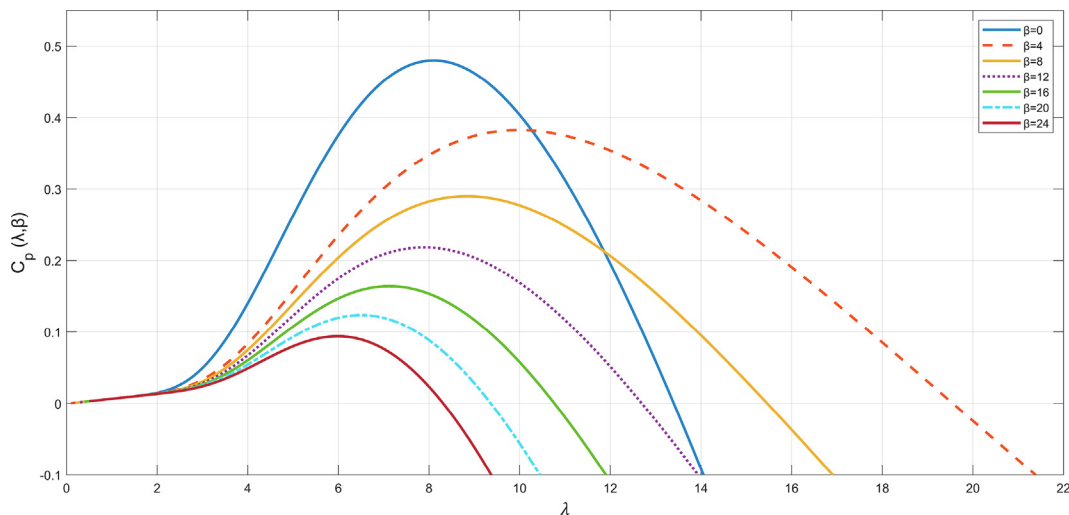


Fig. 1. Relation between  $C_p$  and tip speed ratio  $\lambda$  at different value of blade pitch angle  $\beta$ .

where  $\Delta R_s$  and  $\Delta L_s$  are the variations of stator resistance and inductance, respectively,  $\tilde{d}_{qn}$  and  $\tilde{d}_{dn}$  are the noise and modeling errors.

In the system (6), the electromagnetic torque  $T_e$  is determined by the formula:

$$T_e = K\tilde{i}_q \quad (8)$$

where  $K = \frac{3}{2}(\psi_m P)$  is the torque coefficient.

In next sections, the control system design is being carried out under the following assumptions:

*Assumption 1:*  $\omega, i_d, i_q$  are available.

*Assumption 2:*  $v$  and  $T_a$  are not available.

### 3. LQR design

The system (6) can be written in terms of an error-based dynamics as below:

$$\begin{cases} \dot{\tilde{\omega}} = -\frac{B}{J}\tilde{\omega} - \frac{1}{JK}\tilde{i}_q \\ \dot{\tilde{i}}_q = -\frac{R_s}{L_s}\tilde{i}_q - \frac{\psi_m P}{L_s}\tilde{\omega} + \frac{1}{L_s}(u_q - u_{ffq}) \\ \dot{\tilde{i}}_d = -\frac{R_s}{L_s}\tilde{i}_d + \frac{1}{L_s}(u_d - u_{ffd}) \end{cases} \quad (9)$$

where  $\tilde{\omega}$  is a speed tracking error of the rotor,  $\tilde{i}_q$  is the  $q$ -axis stator's current tracking error, and  $u_{ffq}$  and  $u_{ffd}$  are control inputs serving as compensating terms in the feedforward loop. The tracking errors are determined by,

$$\begin{aligned} \tilde{\omega} &= \omega - \omega_{ref}; \omega_{ref} = \omega_{t,ref} \cdot n_{gb} = \frac{\omega_{opt}}{R} v \cdot n_{gb} \\ \tilde{i}_q &= i_q - i_{q,ref}; i_{q,ref} = \frac{1}{Kn_{gb}} T_a - \frac{B}{K} \omega_{ref} - \frac{1}{K} \omega_{ref} \\ u_{ffq} &= R_s i_{q,ref} + L i_{q,ref} + \psi_m P \omega_{ref} + P L_s \omega \tilde{i}_d \\ u_{ffd} &= -P L_s \omega \tilde{i}_q \end{aligned} \quad (10)$$

where  $\omega_{ref}, i_{q,ref}$  are a reference speed of a generator and a reference of a stator current in the  $q$ -axis, respectively.

Applying Eqs. 9,10, the mathematical model in the Eqs. (9) and (10) takes the form:

$$\begin{cases} \dot{\tilde{\omega}} = -\frac{B}{J}\tilde{\omega} - \frac{1}{J}\tilde{T}_e \\ \dot{\tilde{T}}_e = -\frac{R_s}{L_s}\tilde{T}_e - \frac{\psi_m PK}{L_s}\tilde{\omega} + \frac{K}{L_s}(u_q - u_{qc}) \\ \dot{\tilde{i}}_d = -\frac{R_s}{L_s}\tilde{i}_d + \frac{1}{L_s}(u_d - u_{dc}) \end{cases} \quad (11)$$

$$\begin{aligned} \tilde{T}_e &= T_e - T_{e,ref}; T_{e,ref} = \frac{1}{n_{gb}} T_a - B \omega_{ref} - J \omega_{ref} \\ u_{qc} &= \frac{R_s}{K} T_{e,ref} + \frac{L_s}{K} T_{e,ref} + \psi_m P \omega_{ref} + P L_s \omega \tilde{i}_d \\ u_{dc} &= -\frac{P L_s}{K} \omega T_e \end{aligned} \quad (12)$$

We note that  $\omega_{ref}$  and  $i_{q,ref}$  are unknown, since  $v$  and  $T_a$  are unknown by Assumption 2. To estimate  $T_a$  and  $v$ , details on design of the HOODO will be given in the following sections.

The system in (11) can be rewritten in the state-space form as follows,

$$\dot{x} = Ax + B_c(u - u_c) \quad (13)$$

with

$$A = \begin{bmatrix} -\frac{B}{J} & \frac{1}{J} & 0 \\ -\frac{\psi_m PK}{L_s} & -\frac{R_s}{L_s} & 0 \\ 0 & 0 & -\frac{R_s}{L_s} \end{bmatrix}, B_c = \begin{bmatrix} 0 & 0 \\ \frac{K}{L_s} & 0 \\ 0 & \frac{1}{L_s} \end{bmatrix}. \quad (14)$$

where  $x = [\tilde{\omega} \ \tilde{T}_e \ \tilde{i}_d]^T, u = [u_q \ u_d]^T, u_c = [u_{qc} \ u_{dc}]^T$ .

The optimal control law is defined as [45]:

$$u = u_c + K_u x, \quad (15)$$

where  $K_u = -R^{-1}B_c^T P_u$  is a calculated gain of the optimal controller, where a positive definite symmetric matrix  $P_u > 0$  is a solution of the following algebraic Riccati equation (ARE):

$$P_u A + A^T P_u - P_u B_c R^{-1} B_c^T P_u + Q = 0 \quad (16)$$

with  $Q \geq 0$  is a  $3 \times 3$  matrix and  $R > 0$  and is a  $2 \times 2$  matrix. From (15) and (16), it is seen that  $K_u$  has dependency on the weight matrices  $Q$  and  $R$ . Increase of  $Q$  positively affects to reference tracking efficiency with more control efforts applied. Increase of  $R$  leads to a small control value and poor reference tracking performance. It is recommended to take into account the indicated influence of  $Q$  and  $R$  on  $K_u$ , and to come to a compromise between control efficiency and energy consumption [3]. The weighting matrices are typically chosen as a diagonal ones with non-negative elements on the main diagonal. In this case, however, the presence of some zero values on its main diagonal is allowable.

**Theorem 1.** The optimal control law in (15) provides an exponential convergence of the state vector  $x$  in system (13) to zero.

**Proof.** Let us define a Lyapunov candidate function  $V(x)$  as follows,

$$V(x) = x^T P_u x. \quad (17)$$

Provided that  $V(x)$  is radially unbounded positive definite function. Using (13), (15) and (16), the time derivative of  $V(x)$  is obtained:

$$\begin{aligned} \dot{V}(x) &= \frac{d}{dt} x^T P_u x = 2x^T P_u (A + B_c K_u) x \\ &= 2x^T P_u (A - B_c R^{-1} B_c^T P_u) \\ &= x^T P_u (P_u A + A^T P_u - 2P_u B_c R^{-1} B_c^T P_u) x \leq -x^T Q x. \end{aligned} \quad (18)$$

As shown in (18) the first-order time derivative of  $V$  is negative for all non-zero  $x$ , which, in turn, means that the state vector  $x$  exponentially converges to zero. That is, for a given control law (15), the considered system (13) is stable, which was required to be proved.

In the work [46], the deviation of parameters is considered, using  $\Delta A$  as the value of system parameter variations, the state-dependent coefficient matrix  $A$  is rewritten as  $A' = A + \Delta A$ . Therefore, (13) can be rewritten into the following error dynamics:

$$\dot{x} = A'x + B_c(u - u_c). \quad (19)$$

The derivative of Lyapunov function in (17) is:

$$\begin{aligned} \dot{V}(x) &= 2x^T P_u \dot{x} \\ &= x^T (P_u A + P_u \Delta A + \Delta A^T P_u + A^T P_u - 2P_u B_c R^{-1} B_c^T P_u) x \\ &< 0. \end{aligned} \quad (20)$$

Using (16), the derivative becomes

$$\dot{V}(x) = x^T (P_u \Delta A + \Delta A^T P_u - Q - P_u B_c R^{-1} B_c^T P_u) x. \quad (21)$$

If the following inequality holds for the considered  $\Delta A$ :

$$P_u \Delta A + \Delta A^T P_u < P_u B_c R^{-1} B_c^T P_u + Q. \quad (22)$$

then  $x$  exponentially converges to zero.

Extensive simulation results considering parameter uncertainties will be included in Section 5.

### 4. High-order optimal disturbance observer-based control

This section presents the designing of the proposed high-order optimal observer for estimating aerodynamic torque as rapidly

changing disturbances. To estimate  $T_a$ , the speed dynamics equation in (6) is considered:

$$\dot{\omega} = \left( -\frac{B}{J}\omega - \frac{1}{J}T_e + \frac{1}{Jn_{gb}}T_a \right) \quad (23)$$

**Assumption 3:** A disturbance term, defined as,  $d = \frac{1}{Jn_{gb}}T_a$  is assumed to be continuous and its  $r^{th}$ -order time-derivative is bounded, i.e.,  $|d^{(r)}| \leq \xi$ , where  $d^{(r)}$  is the  $r^{th}$ -order time derivative of the disturbance, and  $\xi > 0$  represents an unknown number.

#### 4.1. High-order optimal disturbance observer design

Combining (23) and Assumption 2, the following extended system can be achieved:

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{M}d^{(r)} \\ y = \bar{C}\bar{x} \end{cases} \quad (24)$$

where  $\bar{x} = [\omega \quad d \quad d^{(1)} \quad \dots \quad d^{(r)}]^T$ ,  $u = -\frac{1}{J}T_e$ ,

$$\bar{A}_{(r+1) \times (r+1)} = \begin{bmatrix} -\frac{B}{J} & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \bar{B}_{(r+1) \times 1} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (25)$$

$$\bar{C}_{1 \times (r+1)} = [1 \quad 0 \quad \dots \quad 0], \bar{M}_{(r+1) \times 1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix},$$

It is straightforward to see that for any positive integer value of  $r$  a pair  $(\bar{A}, \bar{C})$  is observable.

The proposed high-order optimal disturbance observer is designed as:

$$\frac{d\hat{\bar{x}}}{dt} = \bar{A}\hat{\bar{x}} + \bar{B}u + L_g(y - \bar{C}\hat{\bar{x}}) \quad (26)$$

where  $L_g = P_0\bar{C}^T R_0^{-1}$  is the observer gain with  $P_0$  as the solution of the ARE below:

$$\bar{A}P_0 + P_0\bar{A}^T - P_0\bar{C}^T R_0^{-1} \bar{C}P_0 + Q_0 = 0 \quad (27)$$

where  $Q_0 \geq 0, R_0 > 0$  and both are symmetric matrices.

The error dynamics form of the proposed observer is obtained as:

$$\frac{d\tilde{\bar{x}}}{dt} = (\bar{A} - L_g\bar{C})\tilde{\bar{x}} + \bar{M}d^{(r)} \quad (28)$$

where  $\tilde{\bar{x}}$  is the estimation error,  $\tilde{\bar{x}} = \bar{x} - \hat{\bar{x}}$ .

**Theorem 2.** The proposed high-order optimal disturbance observer (HOODO) designed with (26)–(27) can guarantee a bounded estimation error.

**Proof.** Let us introduce a Lyapunov function as  $V(\tilde{\bar{x}}) = \tilde{\bar{x}}^T G \tilde{\bar{x}}$ , where  $G = P_0^{-1}$ . The first-order time-derivative of the candidate Lyapunov function written in terms of the observer's error dynamics (28) is determined by

$$\begin{aligned} \dot{V} &= \frac{d}{dt} \tilde{\bar{x}}^T G \tilde{\bar{x}} = 2\tilde{\bar{x}}^T (G\bar{A} - P_0\bar{C}^T R^{-1}C) \tilde{\bar{x}} + 2\tilde{\bar{x}}^T G \bar{M} d^{(r)} = \\ &\tilde{\bar{x}}^T G (\bar{A}P_0 + P_0\bar{A}^T - 2P_0\bar{C}^T R^{-1} \bar{C}P_0) \times G \tilde{\bar{x}} + 2\tilde{\bar{x}}^T G \bar{M} d^{(r)} \leq \\ &-\tilde{\bar{x}}^T G Q_0 \tilde{\bar{x}} + 2\tilde{\bar{x}}^T G \bar{M} d^{(r)} \leq -\|\tilde{\bar{x}}\| (\lambda_{m1} \|\tilde{\bar{x}}\| - 2\|\bar{G}\bar{M}\|\xi) \end{aligned} \quad (29)$$

where  $\lambda_{m1}$  denote the smallest eigenvalue of  $GQ_0G$ . Therefore, an appropriate tuning of  $Q_0$  and  $R_0$  can provide a bounded norm of the estimator after sufficiently large duration of time, i. e.

$$\|\tilde{\bar{x}}\| \leq \lambda_1 \quad (30)$$

where  $\lambda_1 = 2\|\bar{G}\bar{M}\|\xi/\lambda_{m1}$ .

We note that the estimation performance of the observer depends on the value of  $\lambda_1$ . Its value can be selected by changing  $Q_0$  and  $R_0$ . Hence, by referring to [47], the observer can guarantee definitive boundedness and uniform stability of a small ball with the center at  $\tilde{\bar{x}} = 0$ .

**Remark 1:** In a previous work [48], a high-order observer (HOO) was presented to estimate the high-order disturbance with the assumption that  $d^{(r)} = 0$ . This somehow solves the conventional assumption associated with conventional DO that  $d^{(1)} = \dot{d} = 0$ ; however, does not reflect a broad range of time-varying disturbance, such as that in aforementioned Assumption 1. For example, when the disturbance includes a fast-varying sinusoidal signal, the assumption in HOO is not applicable.

**Remark 2:** The gains in HOO is designed based on a loose guideline, in which the observer gains are selected such that the characteristic equation meets the Hurwitz criterion. That means as long as the gains are positive, it is satisfied. This does not help the designer to select appropriate gains for their applications other than trial-and-error. On the other hand, the proposed HOODO is designed based on the optimal control framework, has a systematic way of selecting the weighting matrices, i.e., selecting the gains of the observer via a straight-forward and implementable rule.

#### 4.2. High-order optimal disturbance observer-based tracking control

With  $\hat{d} = \frac{1}{Jn_{gb}}\hat{T}_a$  estimated by using the HOODO and the system as in (11); via (1), (3), (5) and (10), the estimation of the generator's reference speed is calculated as

$$\hat{\omega}_{ref} = n_{gb} \sqrt{\frac{\hat{T}_a}{k_{opt}}} \quad (31)$$

where  $k_{opt} = \frac{1}{2} \rho \pi R^5 \frac{C_{pmax}}{\lambda_{opt}^3}$ . Based on estimated aerodynamic torque  $\hat{T}_a$ , the compensating terms as well as reference speed tracking error are derived as

$$\begin{aligned} \tilde{\omega} &= \omega - \hat{\omega}_{ref}; \tilde{T}_e = T_e - \hat{T}_{e,ref}; \\ \hat{T}_{e,ref} &= \frac{1}{n_{gb}} \hat{T}_a - B\hat{\omega}_{ref} - J\dot{\hat{\omega}}_{ref} \end{aligned} \quad (32)$$

$$\hat{u}_{qc} = \frac{R_s}{K} \hat{T}_{e,ref} + \frac{L_s}{K} \dot{\hat{T}}_{e,ref} + \psi_m P \hat{\omega}_{ref} + P L_s \omega \dot{i}_d$$

Control law (15) now becomes

$$u = \hat{u}_c + K_u \hat{x} \quad (33)$$

where  $\hat{x} = [\hat{\omega} \quad \hat{T}_e \quad \hat{i}_d]^T$ ,  $\hat{u}_c = [\hat{u}_{qc} \quad u_{dc}]^T$ .

Then, from (31)–(33), the following equations are achieved,

$$\begin{aligned} \dot{\hat{x}} &= x + Fx_e \\ \dot{\hat{u}}_c &= u_c + Hx_e \end{aligned} \quad (34)$$

where

$$\begin{aligned} x_e &= [e \quad \dot{e}]^T, e = T_a - \hat{T}_a, F = \begin{bmatrix} n_1 & n_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T, H = \begin{bmatrix} n_3 & n_4 \\ 0 & 0 \end{bmatrix}^T. \\ n_1 &= \frac{n_{gb}}{\sqrt{k_{opt}}(\sqrt{T_a} + \sqrt{\hat{T}_a})}, \quad n_2 = \frac{1}{n_{gb}} - Bn_1 - Jg, \quad \text{with} \\ g &= \frac{n_{gb}}{2\sqrt{k_{opt}T_a\hat{T}_a}} \left( \sqrt{T_a} - \frac{\hat{T}_a}{T_a + \hat{T}_a} \right), \end{aligned}$$

$$n_3 = -\frac{R_s n_2}{K} + \frac{LB}{K} \dot{g} - \psi_m P n_1, \quad n_4 = -\frac{L_s}{K} \left( \frac{1}{n_{gb}} - Jg \right).$$

### 4.3. Closed-loop stability analysis

Let us rewrite (33) based on (34) as

$$u = u_c + K_u x + E x_e \tag{35}$$

where  $E = K_u F + H$ .

On the other hand, we have

$$x_e = U \tilde{x} \tag{36}$$

$$\text{where } U_{2 \times (r+1)} = \begin{bmatrix} 0 & 1 & 0 \dots & 0 \\ 0 & 0 & 1 \dots & 0 \end{bmatrix}.$$

**Theorem 3.** A state vector  $x$  and its estimation error  $\tilde{x}$  are ultimately bounded around the origin.

**Proof.** Let us take a Lyapunov function of the form:

$$V(x, \tilde{x}) = x^T P_u x + \gamma \tilde{x}^T G \tilde{x} \tag{37}$$

where  $\gamma > 0$ .

The time-derivative of the Lyapunov function is

$$\begin{aligned} \dot{V}(x, x_e) &= 2x^T P_u (Ax + B_c u - B_c u_c) + 2\gamma \tilde{x}^T G (\bar{A} - L_g \bar{C}) \tilde{x} = \\ &= 2x^T P_u (Ax - B_c K_u x + B_c E U \tilde{x}) + 2\gamma \tilde{x}^T G (\bar{A} - L_g \bar{C}) \tilde{x} + 2\gamma \tilde{x}^T G M d^{(r)} \leq \\ &= -x^T Q x + 2x^T P_u B_c E U \tilde{x} - \gamma \tilde{x}^T G Q_0 G \tilde{x} + 2\gamma \tilde{x}^T G M \varepsilon \end{aligned} \tag{38}$$

We further denote  $x_a = [x \quad \tilde{x}]^T$ , so

$$x = T_1 x_a, \quad \tilde{x} = T_2 x_a \tag{39}$$

where  $T_1 = [I_3 \quad 0_{3 \times (r+1)}], T_2 = [0_{(r+1) \times 3} \quad I_{(r+1)}]$ .

Then,

$$\begin{aligned} & -x^T Q x + 2x^T P_u B_c E U \tilde{x} - \gamma \tilde{x}^T G Q_0 G \tilde{x} + 2\gamma \tilde{x}^T G M \varepsilon = \\ & -x_a^T T_1^T Q T_1 x_a + 2x_a^T T_1^T P_u B_c E U T_2 x_a - \gamma x_a^T T_2^T G Q_0 G T_2 x_a + 2\gamma x_a^T T_2^T G M \varepsilon \leq \\ & -\lambda_{m2} \|x_a\|^2 + 2\gamma \|T_1^T P_u B_c E U T_2\| \cdot \|x_a\|^2 + 2\gamma \varepsilon \|T_2^T G M\| \|x_a\| = \\ & -\|x_a\| \left[ (\lambda_{m2} - 2\gamma \|T_1^T P_u B_c E U T_2\|) \|x_a\| - 2\gamma \varepsilon \|T_2^T G M\| \right] \end{aligned} \tag{40}$$

where  $\lambda_{m2}$  denote the smallest eigenvalue of  $(T_1^T Q T_1 + \gamma T_2^T G Q_0 G T_2)$ . From (40), it is seen that we can always obtain  $\gamma$  to assure that

**Table 1**  
Parameters of the WECS [49].

Symbol	Quantity	Value	[Unit]
$P_{rated}$	Rated power	5	kW
$R_s$	Resistance of a stator	0.3676	$\Omega$
$L_s$	Inductance of a stator	3.55	mH
$\psi_m$	Magnet flux linkage	0.2867	V · s/rad
$J$	Mechanical inertia	7.856	kg · m <sup>2</sup>
$P$	Pole pairs	14	-
$B$	Viscous friction coefficient	0.002	kg · m <sup>2</sup> /s
$R$	Rotor radius	1.84	m
$\rho$	Air density	1.25	kg/m <sup>3</sup>
$n_{gb}$	Gear box ratio	1	-

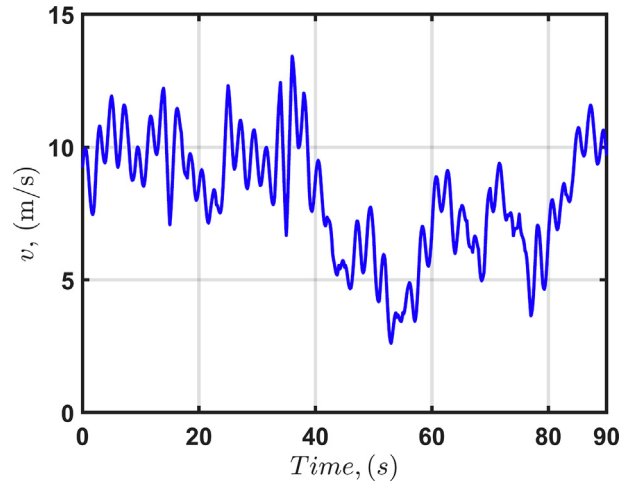


Fig. 3. Wind profile used in the study.

**Table 2**  
Parameters of controllers and observers.

	Parameters
LQR	$Q = \text{diag}(1e5, 1, 1), R = \text{diag}(1, 1)$
HOODO, $r = 1$	$Q = 5e2 \times \text{diag}(1e1, 1), R = 1$
HOODO, $r = 2$	$Q = 5e2 \times \text{diag}(1e1, 1e2, 1), R = 1$
HOODO, $r = 3$	$Q = 5e2 \times \text{diag}(1e1, 1e2, 1e2, 1), R = 1$
HOODO, $r = 4$	$Q = 5e2 \times \text{diag}(1e1, 1e2, 1e2, 1e2, 1), R = 1$
HOO, $r = 1$	$L_0 = 1e2, L_1 = 1e3$
HOO, $r = 2$	$L_0 = 1e2, L_1 = 1e3, L_2 = 3e4$
HOO, $r = 3$	$L_0 = 1e2, L_1 = 1e3, L_2 = 3e4, L_3 = 1e1$
HOO, $r = 4$	$L_0 = 1e2, L_1 = 1e3, L_2 = 3e4, L_3 = 1e1, L_4 = 1e1$

**Table 3**  
Tracking error of the reference speed.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
$\Delta R_s, \%$	0	+20	+40	+40
$\Delta L_s, \%$	0	-1	-15	-15
$\Delta \psi_m, \%$	0	0	0	-2
$\tilde{\omega}_{LQR+HOO}, \text{rad/s}$	$r = 1$ 19.0570	18.8910	23.6818	22.6737
	$r = 2$ 2.6650	2.6022	3.6882	3.5078
	$r = 3$ 2.6650	2.6022	3.6883	3.5085
	$r = 4$ 2.6650	2.6022	3.6883	3.5085
$\tilde{\omega}_{LQR+HOODO}, \text{rad/s}$	$r = 1$ 9.1160	9.0555	11.7513	11.2662
	$r = 2$ 0.4724	0.4726	0.4715	0.4717
	$r = 3$ 0.4822	0.4823	0.4813	0.4815
	$r = 4$ 0.4713	0.4714	0.4703	0.4706

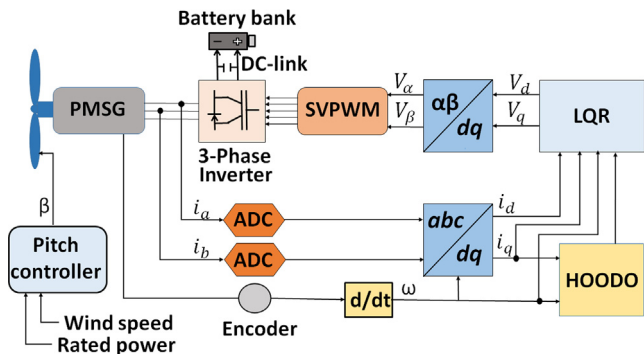


Fig. 2. General control scheme of the PMSG-based WECS with the proposed HOODO-based LQR.

$(\lambda_{m2} - 2\gamma\|T_1^T P_u B_c EUT_2\|) > 0$ . So, after sufficiently long time with appropriately selected  $Q_0, R_0, Q$ , and  $R$ ,  $\|x_a\|$  is bounded as

$$\|x_a\| \leq \lambda_2 \tag{41}$$

where  $\lambda_2 = 2\gamma\epsilon\|T_2^T \overline{GM}\| / (\lambda_{m2} - 2\gamma\|T_1^T P_u B_c EUT_2\|)$ . Hence, the state vector  $x$  and its estimation error  $\tilde{x}$  are bounded, and by appropriate tuning of the weighting matrices of both controller and observer, it is possible to lower that bound. Finally, the stability of closed-loop composite control based on the HOODO is proven in the sense of [47].

### 5. Simulation results

The control scheme of the PMSG-based WECS is shown in Fig. 2. The diagram includes a PMSG, a three-phase inverter, a DC-link with battery bank. The control system requires an encoder and two current sensors. The signals of current is fed to analog-to-digital (ADC) block. There are also Clark and Park blocks required for the frame transformations. The control system includes the proposed HOODO and LQR. The output of controller is then sent to a block for frame transformation and then to the space vector pulse-width modulation (SVPWM) to generate high-frequency switching commands for the three phase inverter. A pitch controller is designed to prevent the WECS system from damage when the wind speed is above the rated value.

The WECS parameters are shown in Table 1. On the other hand, a profile of the time-varying wind speed, with an average speed equals to 8.43 m/s, is depicted in Fig. 3. The profile has been gen-

erated by adding four periodic signals with various amplitudes and frequencies and a Gaussian noise. The generated wind profile signal varies rapidly from a minimum equal to 2.6 m/s to a maximum equal to 14.4 m/s. The maximum power factor  $C_{p_{max}} = 0.4412$  was calculated using  $\lambda_{opt} = 8.09$ .

Table 2 shows the selected weighting matrices of the LQR, proposed HOODO, and HOO [50].

#### 5.1. Discussion of results

Two control schemes are tested under four different scenarios, where performances of the HOO and HOODO are compared for variations in  $L_s, R_s$ , and  $\psi$ . In Scenario 1, we studied performance of each control scheme applied for the WECS with nominal values of each parameter ( $L_s, R_s, \psi$ ). In Scenario 2, we applied the HOO and HOODO for the WECS where the stator inductance  $L_s$  decreased by  $-1\%$  and the stator resistance  $R_s$  increased by  $20\%$ , whereas the magnetic flux linkage  $\psi$  remained to be constant. Scenario 3 considers parameter variations similar to [50], where the stator inductance decreased by  $15\%$  and resistance increased by  $40\%$ , while the magnetic flux linkage did not change. In Scenario 4, the variations in the stator resistance and inductance are taken similar to the amount of variations in Scenario 3, and the magnetic flux linkage is decreased by  $-2\%$  with assumption that the motor operates under high temperature for a long time [51,52]. The results of each scenario are summarized in Table 3 and Table 4. Here, the HOO [50] is implemented with the same controller and the same scenarios for the purpose of comparison.

Figs. 4–15 present the simulation results of the proposed HOODO-based LQR method under Scenario 2. Figs. 6, 7 illustrate

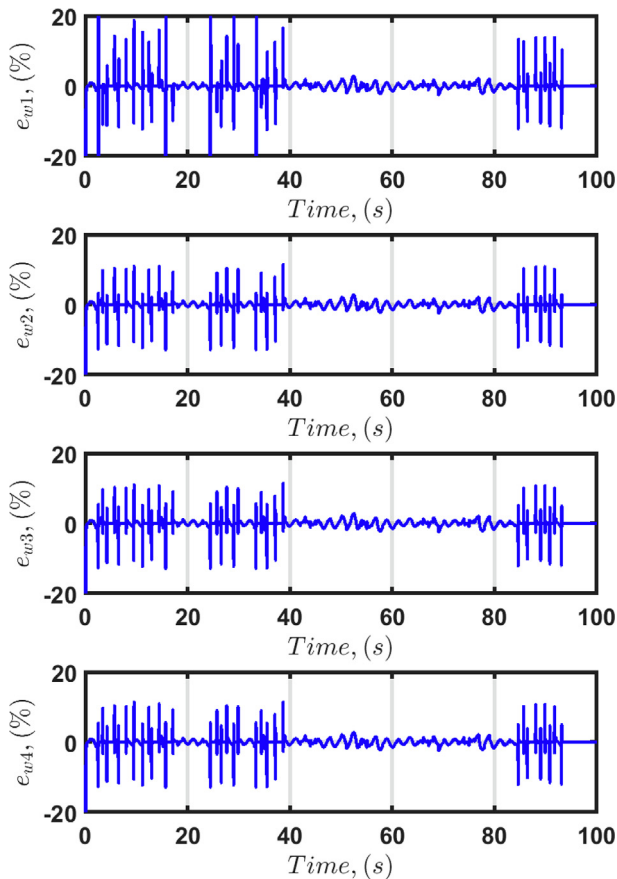


Fig. 4. Scenario 2. Errors of speed estimation with the HOODO-based LQR for  $r = 1, 2, 3, 4$ .

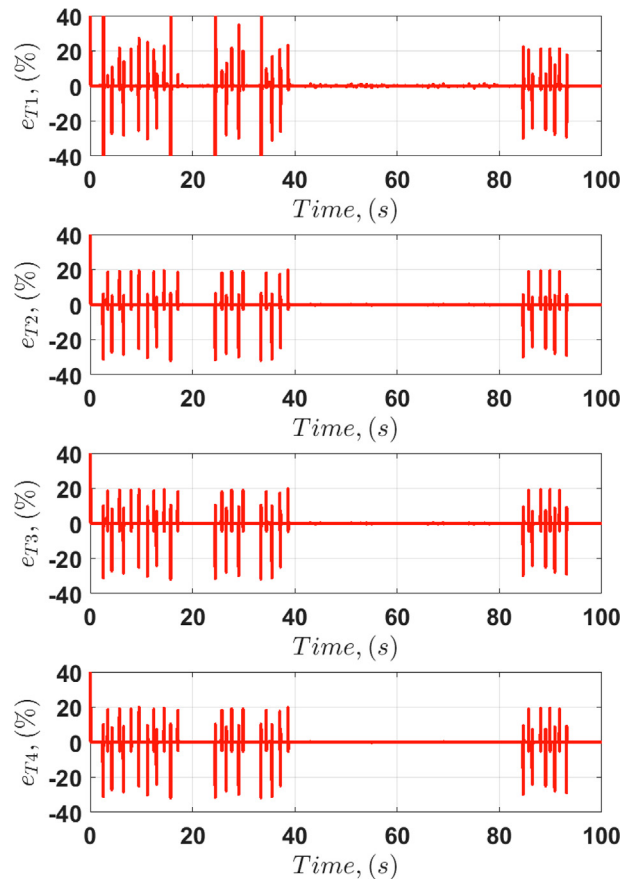
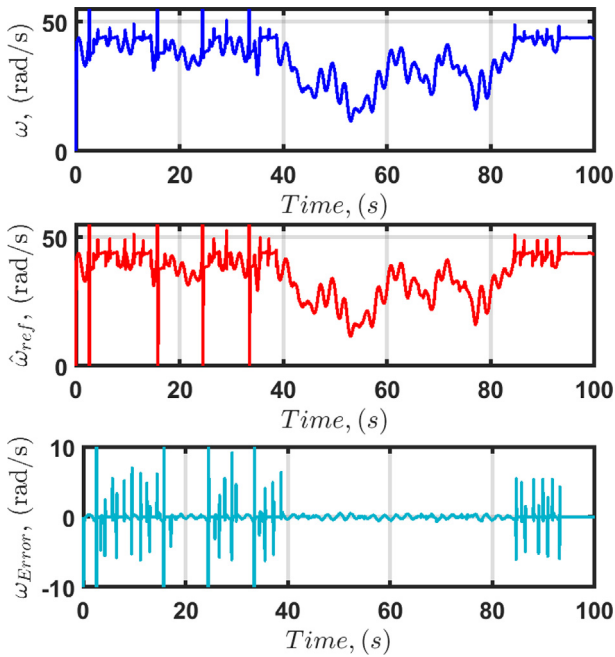


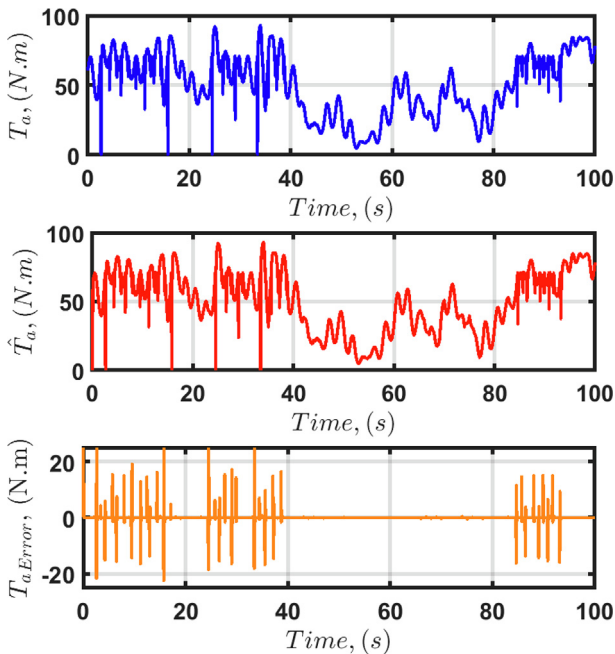
Fig. 5. Scenario 2. Errors of aerodynamic torque estimation with the HOODO-based LQR for  $r = 1, 2, 3, 4$ .

**Table 4**  
Tracking error of  $T_a$ .

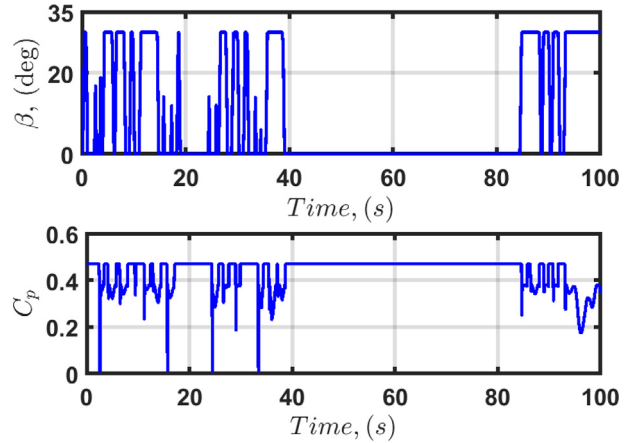
	Scenario 1	Scenario 2	Scenario 3	Scenario 4	
$\Delta R_s, \%$	0	+20	+40	+40	
$\Delta L_s, \%$	0	-1	-15	-15	
$\Delta \psi_m, \%$	0	0	0	-2	
$\hat{T}_{a_{LQR+HOO}}, N \cdot m$	$r = 1$	1.5688	1.5666	1.6366	1.6329
	$r = 2$	1.3932	1.3916	1.4284	1.4181
	$r = 3$	1.3932	1.3916	1.4284	1.4181
	$r = 4$	1.3932	1.3916	1.4284	1.4181
$\hat{T}_{a_{LQR+HOODO}}, N \cdot m$	$r = 1$	1.5149	1.5136	1.5908	1.5872
	$r = 2$	0.6273	0.6273	0.6276	0.6276
	$r = 3$	0.6896	0.6896	0.6900	0.6899
	$r = 4$	0.6251	0.6251	0.6253	0.6253



**Fig. 6.** Scenario 2. Speed tracking performance with the proposed HOODO-based LQR for  $r = 1$ .

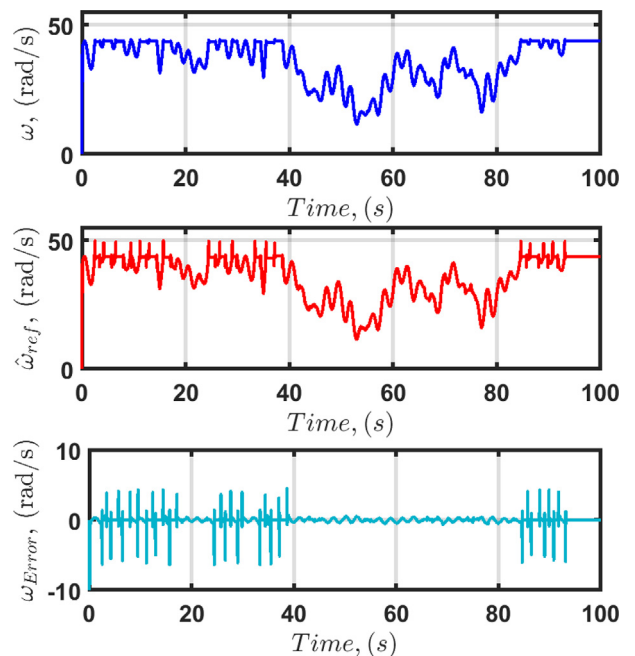


**Fig. 7.** Scenario 2. Estimation of  $T_a$  with the HOODO-based LQR for  $r = 1$ .



**Fig. 8.** Scenario 2. Pitch-angle and power coefficient with the proposed HOODO-based LQR for  $r = 1$ .

the controller and observer performance with the proposed HOODO-based LQR with  $r = 1$ . It is seen from these figures that the estimated  $T_a$  and tracking error are sufficiently accurate for the proposed LQR with HOODO observers. Fig. 8 presents the pitch angle control performance for  $r = 1$ . Figs. 9–17 show the estimation and control performances with  $r = 2, r = 3$ , and  $r = 4$ , respectively. Despite of the rapid change of the wind speed and pitch angle in this gust wind, the HOODO-based LQR is able to show its good performance in all cases. Figs. 18–21 illustrate the performance of the HOO for comparison. For the convenience of the reader to track the difference of these methods, Tables 3 and 4 summarize the results for each case. It can be seen that, as the order increase higher than two, the performance is slightly improved for both HOO and the proposed HOODO. Also, it is clear from the table that, with HOODO, both the wind speed tracking and disturbance estimation performances are better compared to those of the HOO. For example, with  $r = 2$  in Scenario 1, the root



**Fig. 9.** Scenario 2. Speed tracking performance with the proposed HOODO-based LQR for  $r = 2$ .



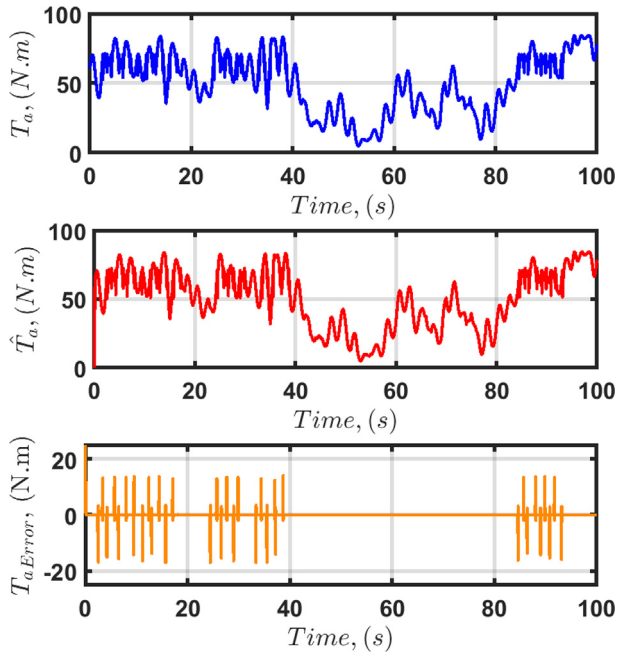


Fig. 10. Scenario 2. Estimation of  $T_a$  with the HOODO-based LQR for  $r = 2$ .

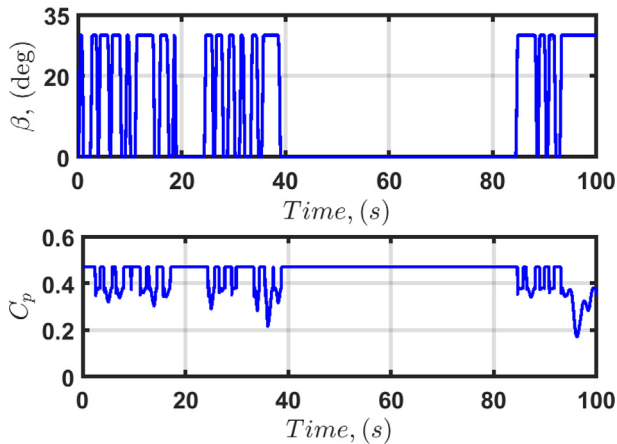


Fig. 11. Scenario 2. Pitch-angle and power coefficient with the proposed HOODO-based LQR for  $r = 2$ .

mean of squared error (RMSE) of the wind speed tracking and aerodynamic torque estimation with the proposed HOODO is 0.4724% and 0.6273%, respectively; whereas those with HOO are larger, 2.6650% of the wind speed tracking and 1.3932% of aerodynamic torque estimation. In Scenario 2, LQR with HOO shows the RMSE value of 2.6022% for the wind speed tracking and 1.3916% for aerodynamic torque estimation, whereas the RMSE values for the torque estimation and wind speed tracking for the proposed HOODO control scheme did not change from Scenario 1 to Scenario 2. In scenarios 3 and 4, we consider more severe condition by considering larger parameter variations. In both scenarios, the first order HOO and HOODO degrade in performance of speed estimation, and show more stable results starting from the order two. Nevertheless, starting from the second order, the HOODO gives much less estimation error comparing to the HOO, which means

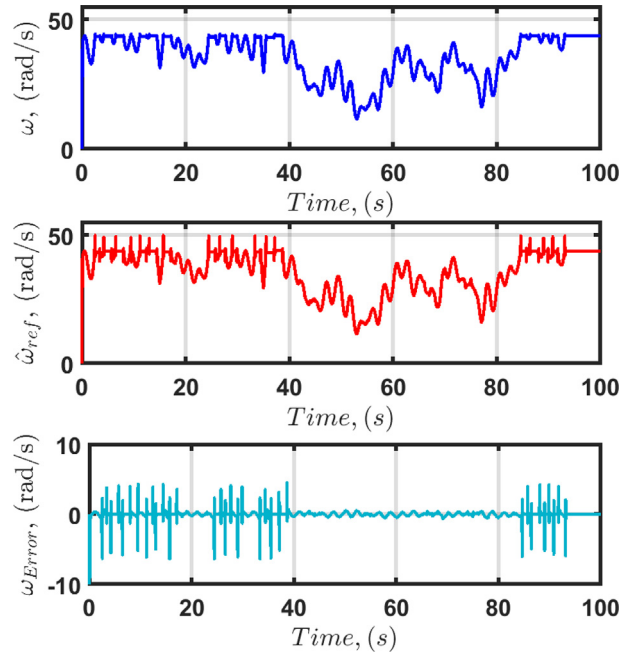


Fig. 12. Scenario 2. Speed tracking performance with the proposed HOODO-based LQR for  $r = 3$ .

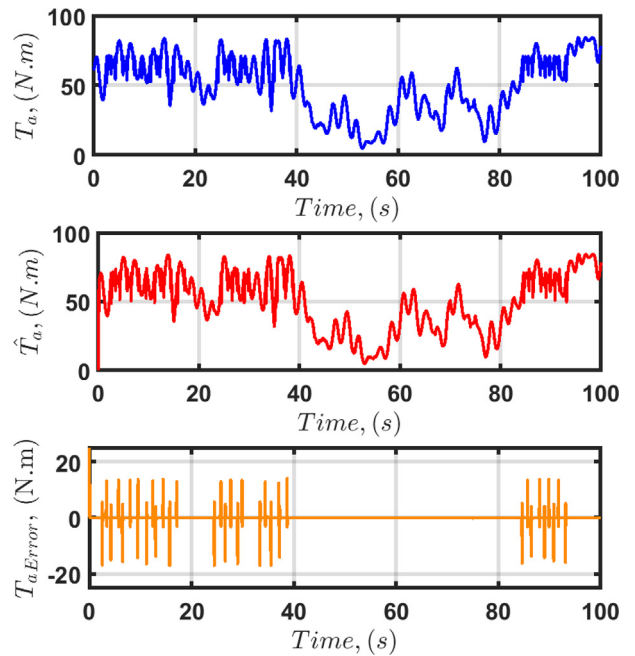


Fig. 13. Scenario 2. Estimation of  $T_a$  with the HOODO-based LQR for  $r = 3$ .

that the HOODO is more accurate than the HOO under the severe parameter variations.

Further comparison of the proposed HOODO with the HOO observer from the simulation results shows that for the HOO, the error in estimating the aerodynamic torque of four values of  $r$  for the HOO-based LQR control scheme are  $(e_r)_{max} = 1.5688\%$ , 1.3932% and 1.3932%, 1.3932% for  $r = 1, 2, 3$  and 4, respectively. For the wind speed estimation error  $(\dot{\omega})_{max} = 19.0570\%$ , 2.6650% and 2.6650%, 2.6650% for  $r = 1, 2, 3$  and 4, respectively. While the HOODO-based LQR control results show that the aerodynamic torque estimation error and angular speed estimation error are

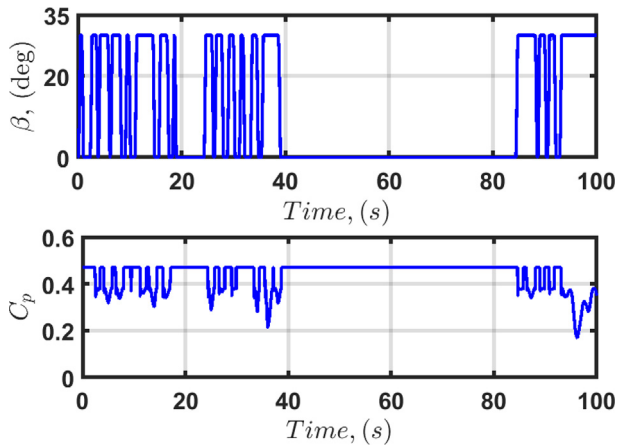


Fig. 14. Scenario 2. Pitch-angle and power coefficient with the proposed HOODO-based LQR for  $r = 3$ .

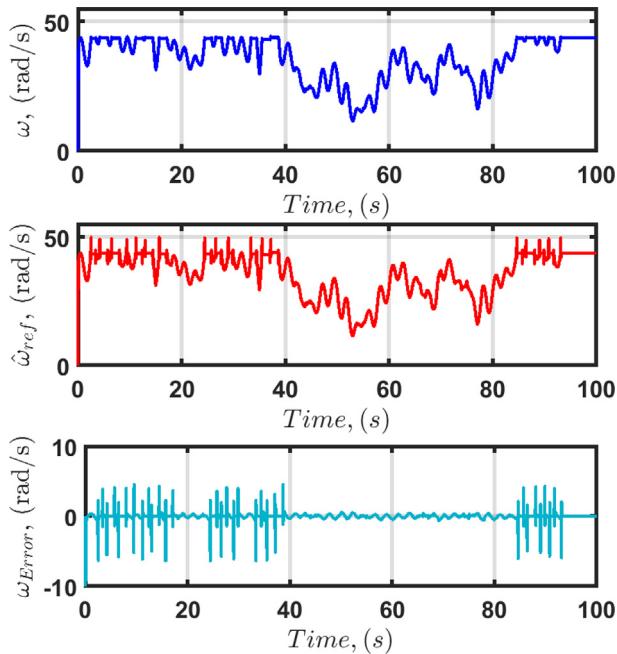


Fig. 15. Scenario 2. Speed tracking performance with the proposed HOODO-based LQR for  $r = 4$ .

unchanged for all  $r = 1, 2, 3$  and  $4$ :  $(e_r)_{max} = 9.1160\%, 0.4724\%, 0.4822\%, 0.4713\%$  and  $(\hat{\omega})_{max} = 1.5149\%, 0.6273\%, 0.6896\%, 0.6251\%$ , respectively, and are illustrated in Fig. 4. The aerodynamic torque estimation errors for each  $r$  are plotted in Fig. 5. By looking at the aerodynamic torque estimation performance in Table 4, one can conclude that the proposed HOODO scheme can be used to achieve much less estimation error than the HOO. Further, to validate robustness of the HOODO performance to different parameter variations, we run 10 random simulations with the same wind profile for all cases, where  $R_s$  is randomly increases in the range  $[1\%, 40\%]$ ,  $L_s$ , and  $\psi_m$  randomly decrease in the range  $[-15\%, 0\%]$  and  $[-2\%, 0\%]$ , respectively. The amount of variations for each parameter were determined using a uniform probability distribution within the given ranges. The simulation results of

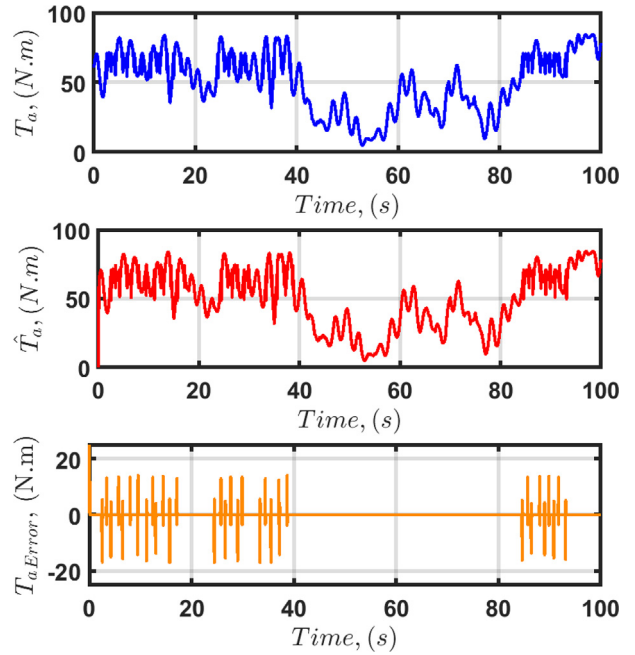


Fig. 16. Scenario 2. Estimation of  $T_a$  with the HOODO-based LQR for  $r = 4$ .

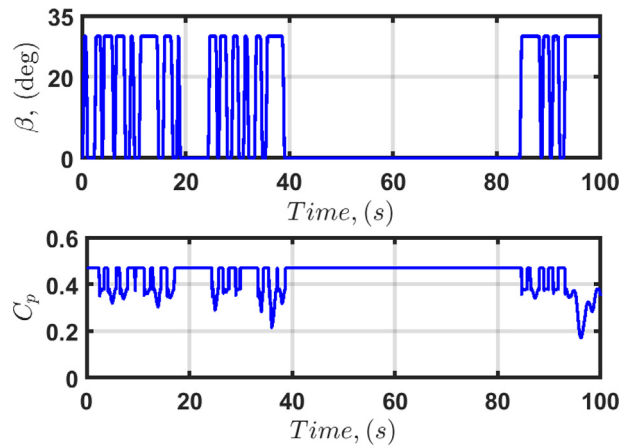
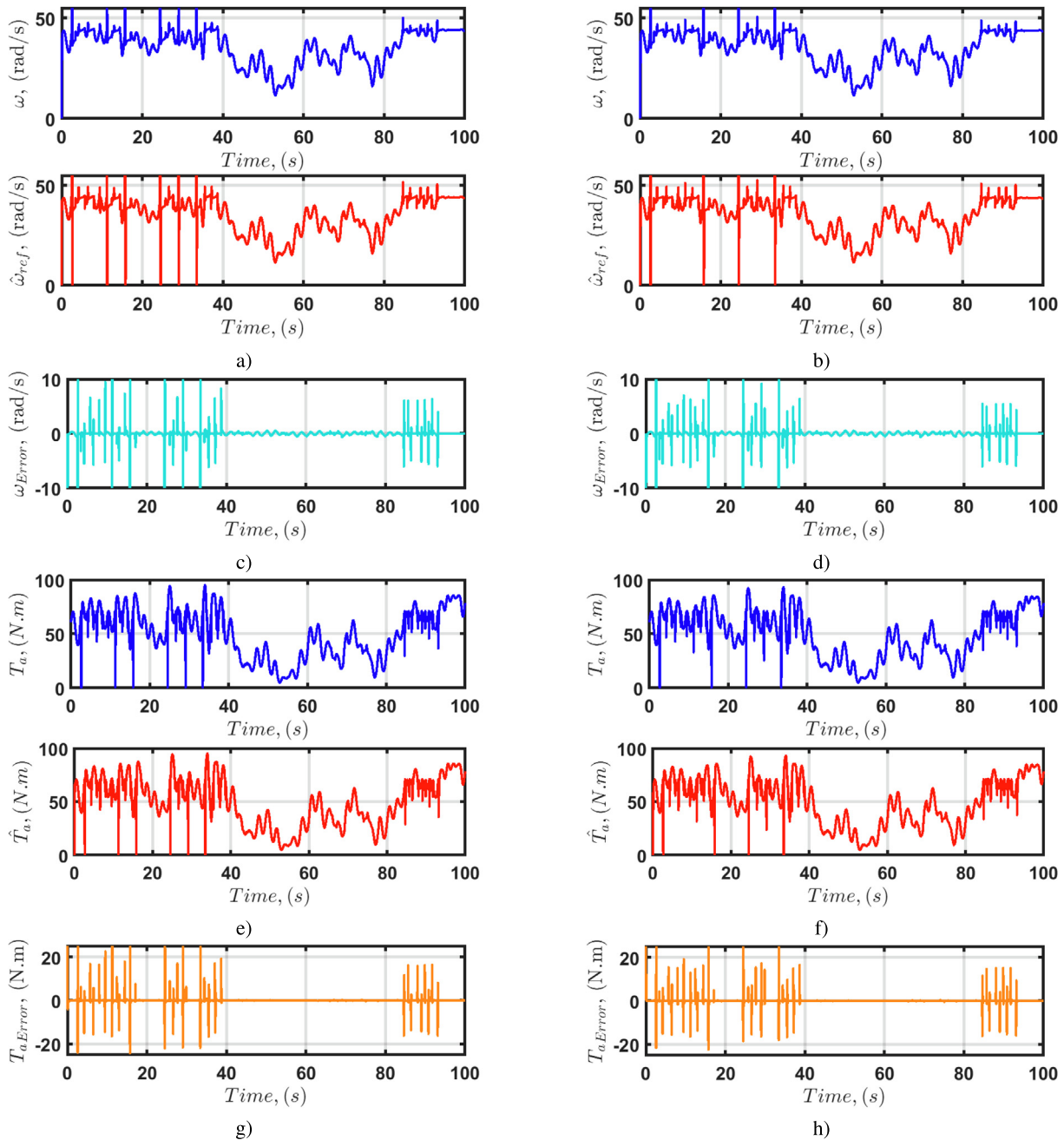


Fig. 17. Scenario 2. Pitch-angle and power coefficient with the proposed HOODO-based LQR for  $r = 4$ .

speed estimation error and aerodynamic torque estimation error are respectively shown in Tables 5 and 6. The estimation performance for both speed and aerodynamic shows that the HOODO can guarantee a stable estimation error with amplitude less than  $0.5 \text{ rad/s}$  and  $0.7 \text{ rad/s}$  for the speed and aerodynamic estimation error, respectively.

It can be revealed from the results that the use of HOODO-based LQR can significantly improve a system performance by reducing the tracking error comparing to the HOO-based LQR in both cases. It means the designed HOODO-based LQR method is more robust to changes in the system and parameters uncertainties. Moreover, by using of the designed HOODO-based control scheme, the estimation of  $\omega$  and  $T_a$  in the PMSG-based WECS shows better results even in the cases of changing some parameters than the results reported in [35,36,40].

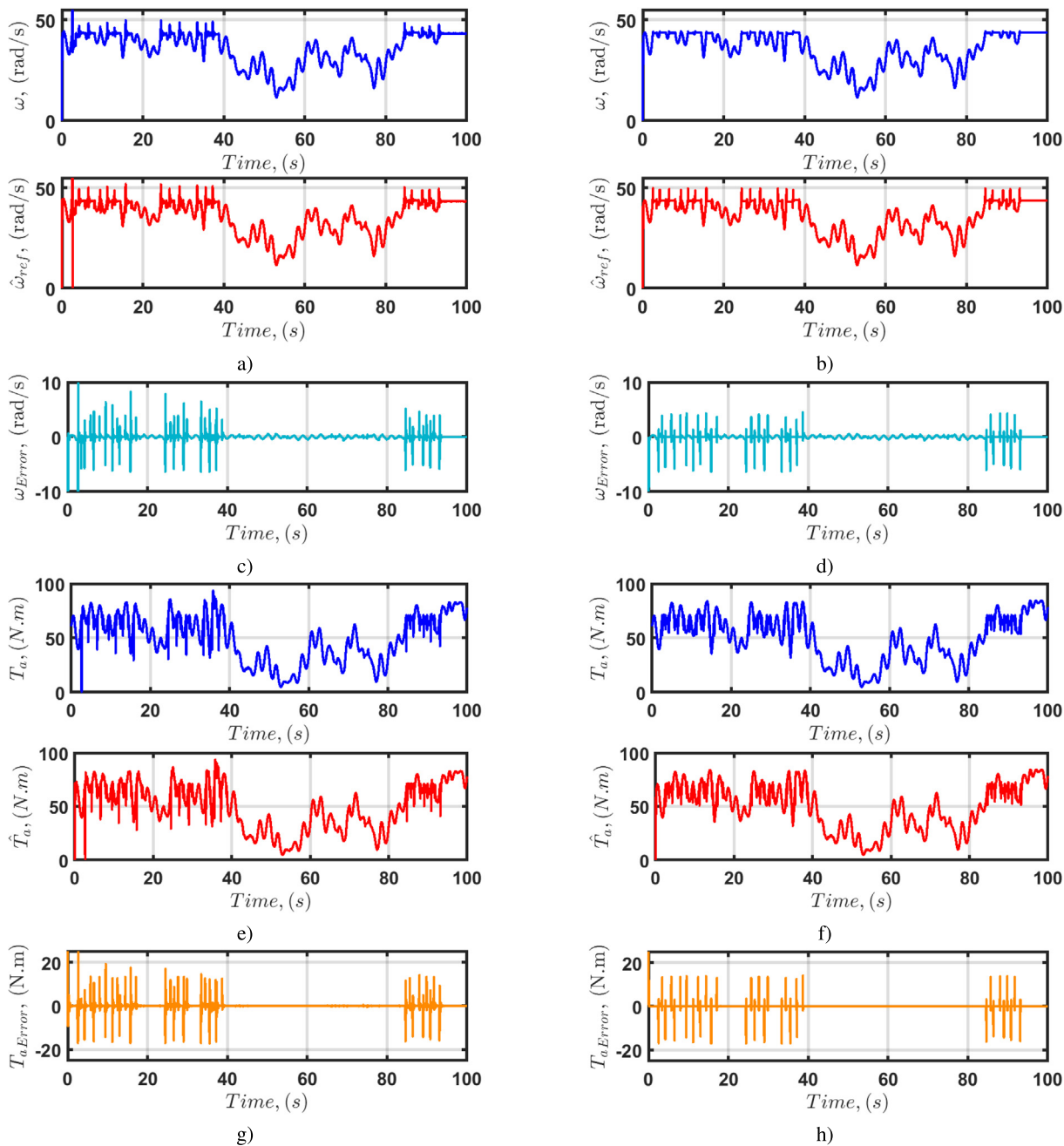


**Fig. 18.** Scenario 2. Tracking of reference wind speed and aerodynamic torque estimation performance for  $r = 1$ : a) Speed estimation with HOO; b) Speed estimation with HOODO; c) Speed estimation error with HOO; d) Error of the speed estimation with HOODO; e) Estimation of  $T_a$  with HOO; f) Estimation of  $T_a$  with HOODO; g) Estimation error of  $T_a$  with HOO; h) Estimation error of  $T_a$  with HOODO.

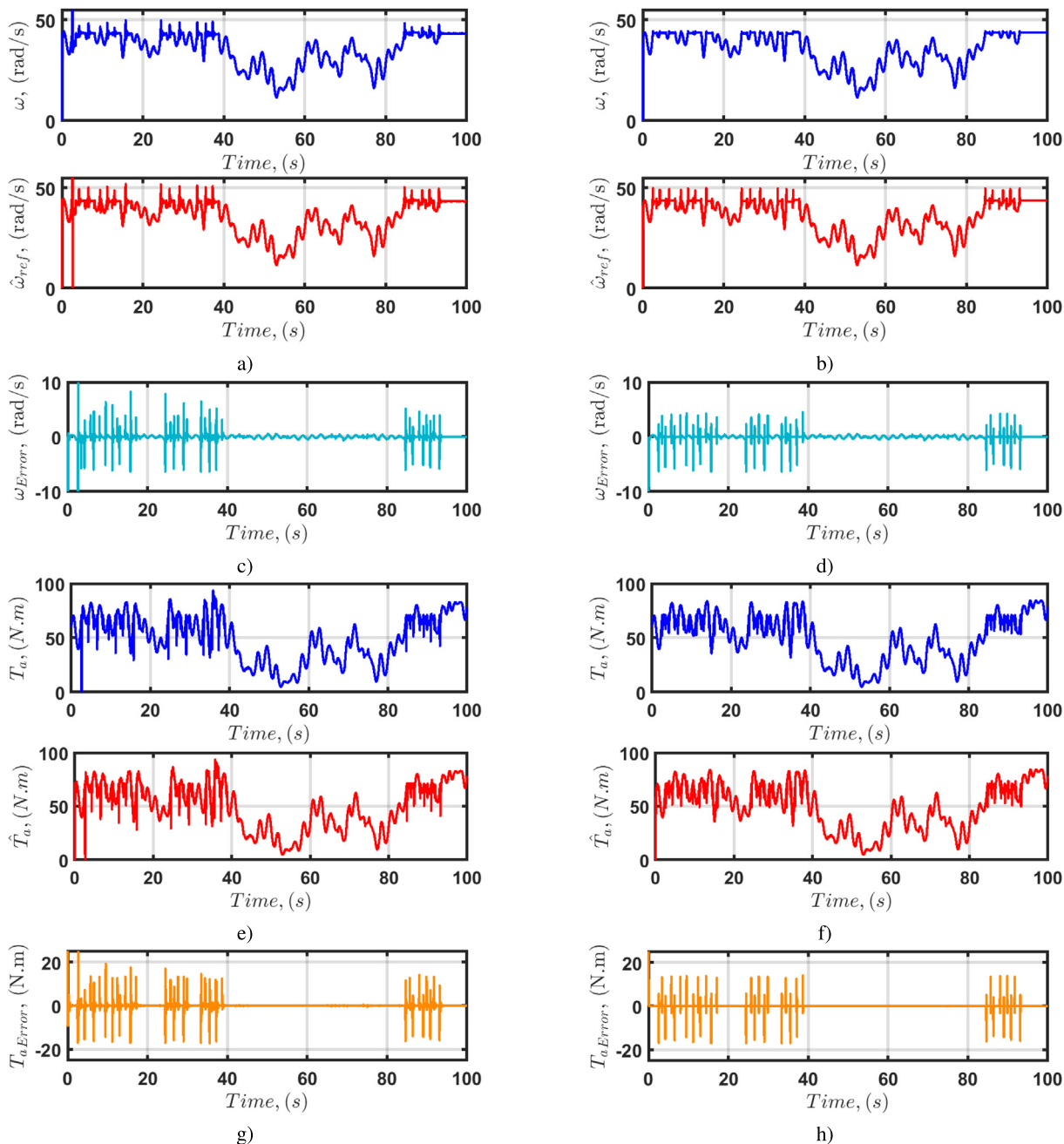
## 6. Conclusions

This work introduced the high-order optimal disturbance observer (HOODO)-based LQR for PMSG-based WECSs. The presented HOODO is able to estimate an aerodynamic torque considered as a disturbance and, hence, a speed of the wind with fast varying dynamics. Detailed analysis on the stability of the HOODO-based composite controller is carried out via the Lyapunov approach. The simulations show that the HOODO-based LQR has

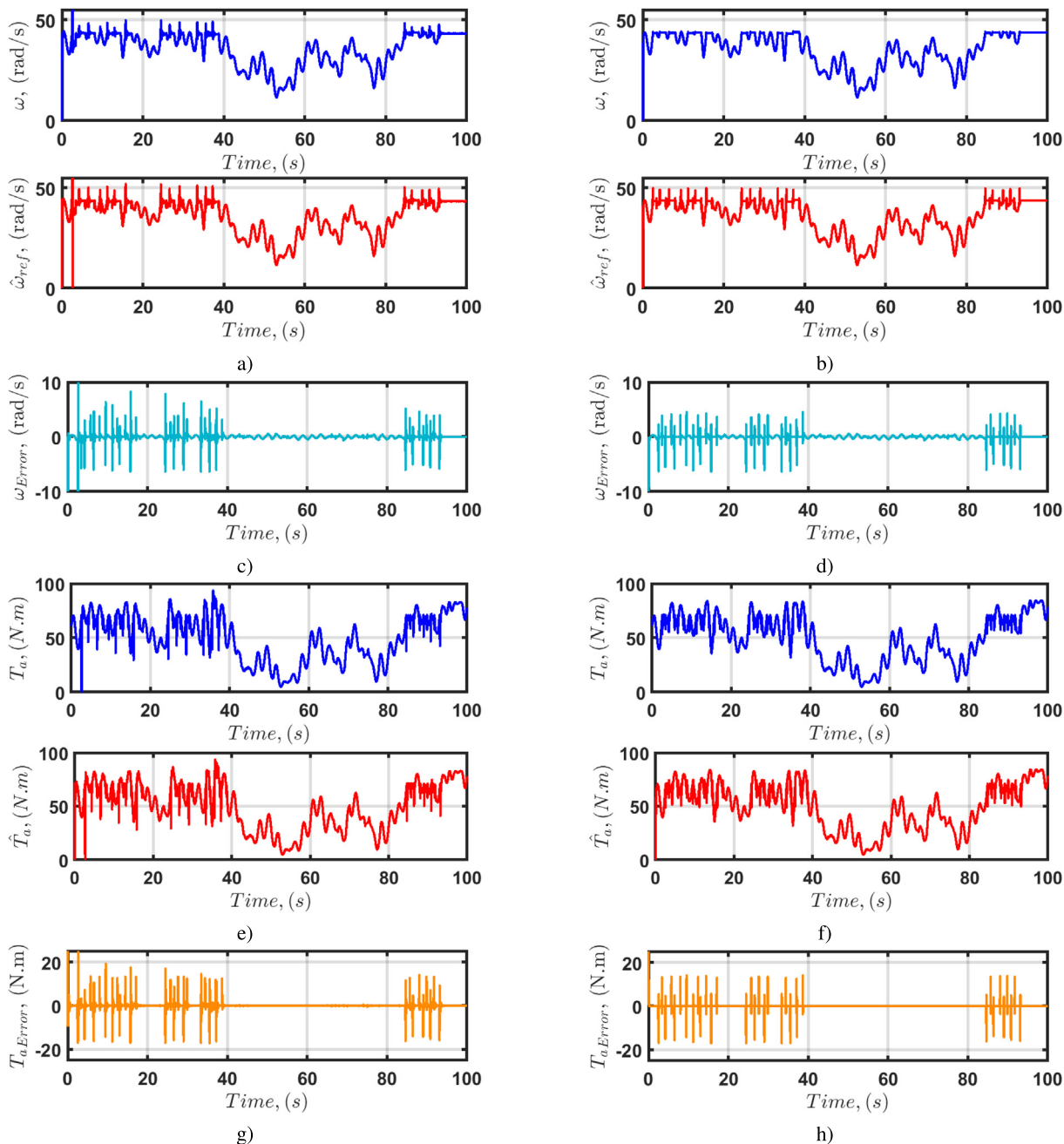
better accuracy than HOO-based LQR method. The presented controller with HOODO can guarantee smaller error and faster response time than previous observer-based control schemes reported so far. In addition, its parameter insensitivity property is significantly better than that of the HOO-based LQR. The work performed confirms the effectiveness of the proposed HOODO and its superiority compared to other types of high-order observers. Future work might focus on: 1) building an experimental setup and verify the performance of the proposed methods exper-



**Fig. 19.** Scenario 2. Tracking of reference wind speed and aerodynamic torque estimation performance for  $r = 2$ : a) Speed estimation with HOO; b) Speed estimation with HOODO; c) Speed estimation error with HOO; d) Error of the speed estimation with HOODO; e) Estimation of  $T_a$  with HOO; f) Estimation of  $T_a$  with HOODO; g) Estimation error of  $T_a$  with HOO; h) Estimation error of  $T_a$  with HOODO.



**Fig. 20.** Scenario 2. Tracking of reference wind speed and aerodynamic torque estimation performance for  $r = 3$ : a) Speed estimation with HOO; b) Speed estimation with HOODO; c) Speed estimation error with HOO; d) Error of the speed estimation with HOODO; e) Estimation of  $T_a$  with HOO; f) Estimation of  $T_a$  with HOODO; g) Estimation error of  $T_a$  with HOO; h) Estimation error of  $T_a$  with HOODO.



**Fig. 21.** Scenario 2. Tracking of reference wind speed and aerodynamic torque estimation performance for  $r = 4$ : a) Speed estimation with HOO; b) Speed estimation with HOODO; c) Speed estimation error with HOO; d) Error of the speed estimation with HOODO; e) Estimation of  $T_a$  with HOO; f) Estimation of  $T_a$  with HOODO; g) Estimation error of  $T_a$  with HOO; h) Estimation error of  $T_a$  with HOODO.

**Table 5**  
Tracking error of the reference speed for 10 simulations with random parameter variations in the defined ranges.

Parameters		Simulations									
		1	2	3	4	5	6	7	8	9	10
$\Delta R_s, \%$		+22	+7	+25	+11	+27	+30	+19	+35	+17	+33
$\Delta L_s, \%$		-12	-15	-2	-9	-8	-6	-3	-14	-4	-10
$\Delta \psi_m, \%$		-1.9	-0.3	-1.7	-1.1	-2	-0.9	-0.2	-0.5	-1.5	-1.3
$\tilde{\omega}_{LQR+HOODO}, \text{rad/s}$	$r = 1$	11.1011	12.1692	8.7026	10.8419	9.6300	9.7866	9.5824	11.1827	9.3662	10.4607
	$r = 2$	0.4717	0.4711	0.4728	0.4717	0.4723	0.4722	0.4723	0.4717	0.4724	0.4720
	$r = 3$	0.4815	0.4809	0.4825	0.4815	0.4821	0.4820	0.4820	0.4815	0.4821	0.4818
	$r = 4$	0.4705	0.4699	0.4717	0.4706	0.4712	0.4711	0.4712	0.4706	0.4713	0.4709

**Table 6**  
Tracking error of  $T_a$  for 10 simulations with random parameter variations in the defined ranges.

Parameters		Simulations									
		1	2	3	4	5	6	7	8	9	10
$\Delta R_s, \%$		+22	+7	+25	+11	+27	+30	+19	+35	+17	+33
$\Delta L_s, \%$		-12	-15	-2	-9	-8	-6	-3	-14	-4	-10
$\Delta \psi_m, \%$		-1.9	-0.3	-1.7	-1.1	-2	-0.9	-0.2	-0.5	-1.5	-1.3
$\tilde{T}_{a_{LQR+HOODO}}, N \cdot m$	$r = 1$	1.5862	1.5946	1.5090	1.5843	1.5221	1.5259	1.5216	1.5866	1.5179	1.5803
	$r = 2$	0.6275	0.6276	0.6273	0.6275	0.6274	0.6274	0.6274	0.6275	0.6274	0.6275
	$r = 3$	0.6899	0.6901	0.6895	0.6899	0.6897	0.6897	0.6897	0.6899	0.6897	0.6898
	$r = 4$	0.6253	0.6253	0.6251	0.6252	0.6252	0.6252	0.6252	0.6253	0.6251	0.6252

imentally. 2) Developing a finite time stability theory for closed-loop observer-based control for WECSs.

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