Improvement of Algorithms of the Topological Model of the Steady Mode of Electric Power Systems

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Abstract. The work improves the algorithms for the formation of a steady state mode of electric power systems, developed on the basis of a topological model. The topological model is developed on the basis of the theory of directed graphs using a matrix of current distribution coefficients. Algorithms for the formation of a steady state provide for conducting calculations with respect to all independent nodes of a complex network of a power system, which leads to an increase in the number of operations of iterative processes. The paper proposes a transformation of the topological model of the system, in which, in the process of solving the transformed model, iteration is performed only over nodes that have non-zero load or generation powers. The rest of the nodal stresses are simply calculated through the stresses of the above nodes. The resulting transformation makes it possible to reduce not only the number of operations in each iteration, but also the number of iterations itself. In particular, when studying a 5-node test circuit [1], instead of four complex equations of the original system, it is sufficient to iterate only one equation (reduction by 25%), and for the New England test 39-node circuit [2], instead of 38 equations It is enough to iterate over 26 equations (32% reduction).

Introduction

Methodological problems of the analysis of electric power systems (EPS) cover a wide range of issues and tasks related to the control of its modes, first of all, this is a study on the formation of equations of steady-state modes of EPS. The complexity of solving the problem of controlling the EPS, taking into account the restrictions that take into account economic and reliability indicators, is explained by the large dimension of the nonlinear equations that describe its mode and composition of telemetry and the variety of mathematical models of modes associated with this [3]. The application of the matrix calculation method in the theory of electric networks, developed in [4] and further developed in [5, 6], unambiguously determined the direction of research on the formation of the equations of the steady state mode of the EPS based on the generalized parameters of its equivalent circuit. In some papers, it is proposed to reduce complexity by replacing the nonlinear model with linear models of integer programming [7]. The topological content of the $\underline{Z}$-nodal generalized parameters made it possible to take it as the basis of mathematical models and methods for calculating complex EPS circuits.

The foundations of the topology of the equivalent circuit of electrical networks were laid down in the classical works of Kirchhoff and Maxwell. They first introduced the concepts of trees and obtained topological expressions for the determinants of the node conductance matrix) $\underline{Y_y}$ and the circuit resistance matrix $\underline{Z_k}$.

In modern literature, these topological expressions for the determinants $\underline{\Delta_y}$ and $\underline{\Delta_k}$, written in terms of the values of undirected trees and complements, are known as derivatives of Kirchhoff's laws [8].

The practical application and development of topological methods in the analysis of electrical circuits became possible after the publication of the works of Percival, Sechu, Mason, Coates and others. In the works [9,10] of Percival V.S. the concepts of a common branch representing a group of trees and a pair of generalized branches representing a group of 2 trees of a graph are derived, and theorems are given for determining groups of trees of a graph of a circuit composed of subcircuits connected by a tree in parallel and in the form of a contour. This theorem served as the basis for the development, Ionkin P.A., Sokolov A.A. [11,12], ways to find trees by decomposing the original diagram graph into node pairs, by branches and by node. The further development of the topological method, in relation to the problems of analyzing steady-state modes of EPS, was received in the works of O.T. Geraskin [13, 14].

To find the current distribution coefficients of electrical networks, the ratios of the sum of the weights of specific trees to the sum of the weights of all possible trees of the graph are calculated [15]. The complexity of the formation of matrices of current distribution coefficients lies in determining the numerators of

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topological expressions by dividing the network into two parts in order to find two graph trees. Akhmetbaev D.S. proposed an analytical approach to determining the topological content of the matrix of current distribution coefficients based on the properties of possible graph trees, without dividing the network into two parts [16,17]. In [18], an efficient algorithm for directed search and determination of the weights of possible graph trees is implemented without involving previously defined trees.

Studies conducted to analyze the methods and algorithms for searching for possible trees of a complex graph led to the development of optimal algorithms based on the principles of diakoptics [18]. The basis of the idea of optimization is the construction of special classes of trees. At the same time, in the process of grouping, parts of future graphs that are "parent" for groups of graphs are allocated and the corresponding graphs of a significantly lower dimension are constructed.

Based on the topological algorithm of the current distribution coefficients, a matrix equation of the steady state is formed.

1 Topological algorithm for the formation of a steady state

Nodal voltages are determined by the expression obtained by analytical transformation of the known equations of the electrical state of networks [19,20]:

$$U = U_0 + C^T Z_B C \overline{U_D^{-1}} \overline{S(U)}, \tag{1}$$

where C is a rectangular complex matrix of current distribution coefficients; Z_B is the diagonal matrix of branch resistances; U_D is the diagonal matrix of nodal conjugate stresses; $S(U)$ is a column vector depending on the power of nodal loads and generators and conductivities of transmission lines; T - matrix transposition sign.

The resulting system of nonlinear equations for the steady state of electric power systems is developed on the basis of the topological model of the matrix of current distribution coefficients. This system was proposed and applied by the authors and represents $(n-1)$ complex nonlinear equations with respect to $(n-1)$ complex values of nodal stresses. Where n is the total number of nodes in the network. The resulting system is solved by iterative methods. As the design scheme of the EPS becomes more complex, the number of operations of the iterative process increases significantly, which reduces the efficiency of obtaining calculation results. An improved algorithm for organizing the iterative process is proposed below.

2 Improving the algorithms for calculating the steady state.

System (1) is solved by iterative methods. In this paper, a transformation of the system under consideration is proposed, in which, in the process of solving the transformed system, iteration is performed only over nodes that have non-zero load or generation powers. The rest of the nodal voltages are simply calculated through the voltages of the above nodes.

In system (1), the column vector $S(U)$ is expressed by the formula:

$$S(U) = S_0 - j \cdot A \cdot |U|^2, \tag{2}$$

where S_0 is a column vector of given (constant) load and generation powers, A is a diagonal matrix with real elements, calculated through given conductivities b.

Denote by $M = C^T Z_B C$. Substitute expression (2) into (1)

$$U = U_0 + M\overline{U_D^{-1}}(S_0 - j \cdot A \cdot |U|^2).$$

Let's open the brackets and carry out transformations:

$$U = U_0 + M\overline{U_D^{-1}}S_0 - j \cdot M \cdot A \cdot U,$$

$$(E + j \cdot M \cdot A)U = U_0 + M\overline{U_D^{-1}}S_0,$$

$$U = (E + j \cdot M \cdot A)^{-1}\left(U_0 + M\overline{U_D^{-1}}S_0\right). \tag{3}$$

System (3) can be rewritten as

$$U = V_0 + G\overline{U_D^{-1}}\overline{S_0}, \tag{4}$$

where $V_0 = (E + j \cdot M \cdot A)^{-1}U_0, G = (E + j \cdot M \cdot A)^{-1}M$.

Component wise, it can be written as:

$$U_k = (V_0)_k + \sum_l G_{kl} \cdot {(\bar{S}_0)_l}/{\overline{U_l}}. \tag{5}$$

From the system of equations (4-5), we see that when solving by iterative methods, in the right part of the system, only those values of nodal voltages are used, in which there are non-zero generation or load powers. That is, system (4-5) can be divided into two subsystems: equations with non-zero generation and/or load powers and equations without them. Only the first part is solved by iterative methods. And the second part is only formulas for calculating the remaining nodal stresses.

The resulting transformation makes it possible to reduce not only the number of operations in each iteration, but also the number of iterations itself. In particular, when studying a 5-node test circuit, instead of four complex equations of the original system, it is sufficient to iterate only one equation (75% reduction).

3 Implementation of the algorithm

Consider a 5-node test circuit.

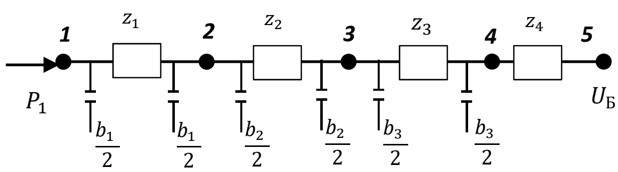

Fig.1. Design scheme

This scheme is a test circuit rucase_5_4 and the source data for is taken from open sourcesx [1].

Table 1. Initial data by nodes.

Nodes №	U nom	Voltage		Weighting mode, S, MVA		Generation power	
	kV	d, degree	Modulus, kV	P, MW	Q, MVA	P, MW	Q, MVA
1	500	68.73	484.53	0.00	0.00	5400	0.00
2	500	45.05	482.05	0.00	0.00	0.00	0.00
3	500	21.97	489.35	0.00	0.00	0.00	0.00
4	500	0.12	499.90	0.00	0.00	0.00	0.00
5	500	0.00	500.00	0.00	0.00	0.00	457

Table 2. Line design parameters

Lines №	Complex resistances, Z, Ohm	Conductivity b, cm
1	0.7000 + 17.5000i	8.3200
2	0.7000 + 17.5000i	8.3200
3	0.7000 + 17.5000i	8.3200
4	0.0000 + 0.1000i	0.0000

The matrices M and the column vector S_0 in this case have the form:

$$M = \begin{pmatrix} 2.1 + 52.6j & 1.4 + 35.1j & 0.7 + 17.6j & 0.1j \\ 1.4 + 35.1j & 1.4 + 35.1j & 0.7 + 17.6j & 0.1j \\ 0.7 + 17.6j & 0.7 + 17.6j & 0.7 + 17.6j & 0.1j \\ 0.1j & 0.1j & 0.1j & 0.1j \end{pmatrix},$$

$$A = \begin{pmatrix} 4.16 & 0 & 0 & 0 \\ 0 & 8.32 & 0 & 0 \\ 0 & 0 & 8.32 & 0 \\ 0 & 0 & 0 & 4.16 \end{pmatrix} \cdot 10^{-3},$$

$$S_0 = \begin{pmatrix} 5400 \\ 0 \\ 0 \\ 0 \end{pmatrix}, (E + j \cdot M \cdot A)^{-1} U_0 =$$

$$= \begin{pmatrix} 1232 - 64.767j \\ 01142 - 56.465j \\ 886.8 - 33.289j \\ 502.4 - 0.102j \end{pmatrix},$$

$$(E + j \cdot M \cdot A)^{-1} M \cdot diag(\overline{S_0}) =$$

$$= \begin{pmatrix} (0.4754 + 5.676j) \times 10^5 & 0 & 0 & 0 \\ (0.3865 + 4.319j) \times 10^5 & 0 & 0 & 0 \\ (0.2161 + 2.335j) \times 10^5 & 0 & 0 & 0 \\ 69.948 + 1.33j \times 10^3 & 0 & 0 & 0 \end{pmatrix}.$$

Thus, the first part of system (5) in this case is one complex equation for the voltage of the first node, which we write in the form:

$$U_1 = (V_0)_1 + G_{11} \cdot {(\bar{S}_0)_1}/{\overline{U_1}}. \quad (6)$$

If we write equation (6) component by component, using the following notation,

$$U_1 = x + j \cdot y, \ (V_0)_1 = x_0 + j \cdot y_0 = 1232 - 64.767j,$$

$$G_{11} \cdot (\bar{S}_0)_1 = \alpha + j \cdot \beta = (0.4754 + 5.676j) \times 10^5,$$

then we get the system of equations

$$\begin{cases} x = x_0 + \dfrac{\alpha x - \beta y}{x^2 + y^2}, \\ y = y_0 + \dfrac{\alpha y + \beta x}{x^2 + y^2}, \end{cases}$$

which reduces to a quadratic equation with respect to, for example, the variable x, solving which we find two solutions:

$$x_1 = 175.64, x_2 = 1.105 \times 10^3,$$

the corresponding imaginary parts are equal:

$$y_1 = 451.48, y_2 = 402.64$$

The second solution is eliminated due to the constraints $|U_1| \le 500$. The corresponding phase and modulus will be: $\arg(U_1) = 68.74, \ |U_1| = 484.44$.

Table 3. The total results for all nodes will have the following values:

№	Generation power		Voltage				Voltage Relative difference, %	
			Test's results		Results for the simplified algorithm			
	P, MW	Q, MVA	d, degree	Modulus, kV	d, degree	Modulus, kV	d, %	Modulus,%
1	5400	0.00	68.74	484.53	68.73	484.44	0,0%	-0,02%
2	0.00	0.00	45.05	482.05	45.73	481.86	1,5%	-0,04%
3	0.00	0.00	21.98	489.35	21.98	489.23	0,01%	-0,02%
4	0.00	0.00	0.12	499.90	0.12	499.90	0,0%	0,00%
5	0.00	457.0	0.00	500.00	0.00	500.00	0	0

Thus, in the problem under consideration, there is no need to apply iterative methods of solution, and the solution is found exactly.

Conclusion

The proposed transformation significantly reduces the number of calculations performed in the calculations of the steady state.

Studying the question of conversion efficiency in a more general case is of interest and is the subject of further research.

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