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БАЯНДАМАЛАР ЖИНАҒЫ

Республикалық ғылыми-практикалық конференция
«Математикалық және компьютерлік модельдеудің заманауи мәселелері Қазақстанның цифрлы индустриясының дамуы жағдайында»
3-5 мамыр 2018 жыл, Астана, Қазақстан

СБОРНИК МАТЕРИАЛОВ

Республиканская научно-практическая конференция
«Современные проблемы математического и компьютерного моделирования в условиях развития цифровой индустрии Казахстана»
3-5 мая 2018 года, Астана, Казахстан

ӘОЖ 004+519+316
КБЖ 22
М 49

В подготовке Сборника принимали участие:
Адамов А.А., Нугманова Г.Н., Сергибаев Р.А., Байдавлетов А.Т.

Математикалық және компьютерлік моделдеудің заманауи мәселелері Қазақстанның цифрлы индустриясының дамуы жағдайында: Республикалық ғылыми-практикалық конференциясының БАЯНДАМАЛАР ЖИНАҒЫ = Современные проблемы математического и компьютерного моделирования в условиях развития цифровой индустрии Казахстана: СБОРНИК МАТЕРИАЛОВ Республиканской научно-практической конференции. Қазақша, орысша, ағылшынша. – Астана, 2018, 161 б.

ISBN 978-601-337-014-9

Жинаққа студенттердің, магистранттардың, докоранттардың және ғалымдардың механика, математика, математикалық және компьютерлік моделдеу, математиканы оқыту әдістемесінің өзекті мәселелері бойынша баяндамалары енгізілген.

В Сборник вошли доклады студентов, магистрантов, докторантов и ученых по актуальным вопросам механики, математики, математического и компьютерного моделирования и методики преподавания математики.

Тексты докладов представлены в авторской редакции

ISBN 978-601-337-014-9 ӘОЖ 004+519+316
КБЖ 22.1

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A NOTE ON PRACTICAL OUTPUT TRACKING FOR A CLASS OF UNCERTAIN HIGH ORDER NONLINEAR SYSTEMS

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Abstract

This paper considers the problem of global practical tracking via output feedback control for a class of more general uncertain high-order nonlinear systems. By introducing sign function and necessarily modifying the homogeneous domination approach and under a weaker growth condition, this paper proposes a new control scheme to achieve the global practical tracking. It is shown that the designedcontroller guarantees that the state of the resulting closed-loop system is globally bounded and the tracking error converges to a prescribed arbitrarily small neighborhood of the origin after a finite time.

Keywords.output feedback control, practical output tracking, homogeneous domination, nonlinear system

1. Introduction

The problem of global output tracking control of nonlinear systems is one of the most important and challenging problems in the field of nonlinear control and lots of efforts have been made during the last decades, see [1-12], as well as the references therein. With the help of the nonlinear output regulator theory [3], [4] and the method of adding a power integrator [13-15], series of research results have been obtained [5-7]. For details, in [8], practical output tracking via smooth state feedback for nonlinear systems was considered. Compared with state feedback control, the theory of output control developed slower, because there is no general and effective method to design a nonlinear observer.

This paper deals with the problem of global practical output tracking by output feedback for a class of more general high-order nonlinear systems described by

$$\begin{aligned}\dot{x}_i &= \theta_i x_{i+1}^{p_i} + \phi_i(t,x,u),\ i=1,\dots,n-1,\\ \dot{x}_n &= \theta_n u + \phi_n(t,x,u),\\ y &= \theta_0 x_1 - y_r\end{aligned} \tag{1}$$

where $x=(x_1,\dots,x_n)^{\mathrm{T}}\in R^n$ and $u\in R$ are the system state and the control input, respectively. For $i=1$, $\dots, n$, $\phi_i(t,x,u)$ are unknown continuous functions and $p_i\in R_{odd}^{\geq 1}:=\{p/q\in[0,\infty): p$ and q are odd integers, $p\geq q\}$ $(i=1,\dots,n-1)$ are said to be the high orders of the system, with p_n obviously equal to one (which is not a limitation since we can easily set $v:=u^{p_n}$ in the case of non-unity p_n). The parameters $\theta_i,\ i=0,1,\dots,n$ are unknown constants and y_r is a reference signal to be tracked. Although in the usual tracking problem the reference signal $y_r(t), t\in[0,\infty)$ as well as its derivates are assumed to be known, but in our problem only the error $y=\theta_0 x_1 - y_r$ between the output $\theta_0 x_1$ and the reference signal y_r are assumed to be measureable. Hence only y is allowed to use in the design of the control. There are two reasons torestriction the only measurement to be the error signal. One, in some practice control applications, is inevitable that the error signal is the one to be directly measured. For example, in a missile guidance system, instead of measuring the absolute position of the moving target, that is signal y_r, the onboard radar keeps measuring the distance/error between the missale and the target [1]. The other one is assuming only error signal also makes the actuator design simple, as the controller does not depend on the signal to be tracked explicitly. In this way, the controller is more adaptive to different reference signals [2].

Recently, in [9-12] and [2], the practical output feedback tracking problem was also investigated for a class of nonlinear systems with higher-order growing unmeasurable states, extending the results on stabilization in [16-19].

In [9-12] and [2], the following condition on the uncertain term $\phi_i(\cdot)$ is assumed:

$$|\phi_i(t,z,u)| \le C\left(|x_1|^{(r_i+\tau)/r_1} + \cdots + |x_i|^{(r_i+\tau)/r_i}\right) + C \qquad (2)$$

where $C>0$, $\tau>0$ or $\tau \ge -1\Big/\sum_{l=1}^{n} p_1\cdots p_{l-1}$ are constants and r_i's are defined as $r_1=1, r_{i+1}p_i = r_i+\tau>0$, $i=1,\ldots,n$. Nevertheless, from both practical and theoretical points of view, it is still somewhat restrictive to require system (1) satisfying such restriction. To illustrate the limitation, let us consider the following simple system:

$$\dot{x}_1 = \theta_1 x_2 + x_1^{3/5},\ \ \dot{x}_2 = \theta_2 u,\quad y = \theta_0 x_1 - y_r,\quad \theta_i \in [1,2],\quad i = 0,1,2$$

where $p_1 = p_2 = 1$, $\phi_1 = x_1^{3/5}$ and $\phi_2 = 0$. For the simple system, it is easily verified that the works [9-12] and [2] cannot lead to any output feedback tracking controller because of the presence of low-order term $x_1^{3/5}$ dissatisfied the growth condition.

In this paper, by introducing a combined homogeneous domination and sign function approach, we shall solve the above problems.

2. Mathematical Preliminaries

We collect the definition of homogeneous function and several useful lemmas.

First, we recall some important definitions regarding to homogeneous systems (For more details, see, e.g,.[20], [21], [23] and [22]). Now, let $x=(x_1,\ldots,x_n)\in R^n$ be a fixed coordinate, and $s>0$, $r_i>0$ $(i=1,\ldots,n)$ be real numbers. Then:

(i) *A dilation* $\Delta_s(x)$ is a mapping defined by

$$\Delta_s(x) = \left(s^{r_1}x_1,\cdots,s^{r_n}x_n\right),\quad \forall s>0$$

where r_i are called *the weights of the coordinate*. For simplicity of notation, the dilation weight is denoted by $\Delta=(r_1,\ldots,r_n)$.

(ii) A function $V\in C(R^n,R)$ is said to be *homogeneous of degree* τ if there is a real number $\tau\in R$ such that

$$V(\Delta_s(x)) = s^{\tau}V(x_1,\cdots,x_n),\quad \forall x\in R^n - \{0\}\ .$$

(iii) A vector field $f\in C(R^n,R^n)$ is said to be *homogeneous of degree* τ if there is a real number $\tau\in R$ such that for $i=1,\ldots,n$

$$f_i(\Delta_x(x)) = s^{\tau+r_i} f_i(x_1,\cdots,x_n),\quad \forall x\in R^n - \{0\}.$$

(iv) *A homogeneous* p*-norm* is defined as

$$\|x\|_{\Delta,p} = \left(\sum_{i=1}^{n}|x_i|^{p/r_i}\right)^{1/p},\ \forall x\in \mathrm{R}^n,\ p\ge 1.$$

For the simplicity, write $\|x\|_\Delta$ for $\|x\|_{\Delta,2}$.

Next, we introduce several technical lemmas which will play an important role and be frequently used in the later control design.

Lemma1[21]. Given a dilation weight $\Delta=(r_1,\ldots,r_n)$, suppose $V_1(x)$ and $V_2(x)$ are homogeneous of degree τ_1 and τ_2, respectively. Then, $V_1(x)V_2(x)$ is also homogeneous with respect to the same dilation Δ. Moreover, the homogeneous degree of $V_1(x)V_2(x)$ is $\tau_1+\tau_2$.

Lemma2[21]. Suppose $V: R^n\to R$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then, the following holds:

(i) $\partial V/\partial x_i$ is also homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i.

(ii) There is a constant $\sigma>0$ such that $V(x)\le\sigma\|x\|_\Delta^\tau$. Moreover, if $V(x)$ is positive-definite, there is a constant $\rho>0$ such that $\rho\|x\|_\Delta^\tau \le V(x)$.

Now, we introduce several technical lemmas which will play an important role and be frequently used in the later control design.

Lemma3[5]. For any real numbers $x \ge 0,\ y > 0$ and $m \ge 1$, the following inequality holds:

$$x \le y + (x/m)^m \left((m-1)/y\right)^{m-1}.$$

Lemma4[24]. For all $x, y \in R$ and a constant $p \ge 1$ the following inequalities holds:

(i) $|x+y|^p \le 2^{p-1}\left|x^p + y^p\right|$, $\left(|x|+|y|\right)^{1/p} \le |x|^{1/p} + |y|^{1/p} \le 2^{(p-1)/p}\left(|x|+|y|\right)^{1/p}$

If $p \in R_{odd}^{\ge 1}$, then

(ii) $|x-y|^p \le 2^{p-1}\left|x^p - y^p\right|$ and $\left|x^{1/p} - y^{1/p}\right| \le 2^{(p-1)/p}|x-y|^{1/p}$.

Lemma5[24]. Let c, d be positive constants. Then, for any real-valued function $\gamma(x,y) > 0$, the following inequality holds:

$$|x|^c |y|^d \le \frac{c}{c+d}\gamma(x,y)|x|^{c+d} + \frac{d}{c+d}\gamma^{-c/d}(x,y)|y|^{c+d}.$$

Lemma6[25].For $x, y \in R$ and $0 < p \le 1$ the following inequality holds:

$$\left(|x|+|y|\right)^p \le |x|^p + |y|^p.$$

When $p = a/b \le 1,$ where $a > 0$ and $b > 0$ are *odd* integers

$$\left|x^p + y^p\right| \le 2^{1-p}|x+y|^p.$$

Lemma7[26].If $p = a/b \in R_{\text{odd}}^{\ge 1}$ with $a \ge b \ge 1$ being some real numbers, then for any $x, y \in R$

$$\left|x^p - y^p\right| \le 2^{1-1/b}\left|\text{sgn}(x)|x|^a - \text{sgn}(y)|y|^a\right|^{1/b}.$$

Lemma8[8]. If $f:\ [a,b] \to R\ \ (a \le b)$ is monotone continuous and satisfies $f(a) = 0$, then

$$\left|\int_a^b f(x)dx\right| \le |f(b)| \cdot |b-a|.$$

3. Main Results

This paper deals with the practical output tracking problem by output feedback for nonlinear systems (1). Here, we give a precise definition of our practical tracking problem [9], [11].

The problemof global practical tracking by an output feedback: Consider system (1) and assume that the reference signal $y_r(t)$ is a time-varying C^1- bounded function on $[0,\infty)$. For any given $\varepsilon > 0$, designan output controllerhaving the following structure

$$\begin{cases} \dot{\zeta} = \alpha(\zeta, y),\ \ \zeta(0) \in R^{n-1} \\ u = \beta(\zeta, y), \end{cases} \tag{3}$$

where α, β are some smooth functions, such that

i) All the state $[x(t), \zeta(t)] \in R^{2n-1}$ of the closed-loop system (1) with output controller (3) is well-defined on $[0,+\infty)$ and globally bounded.

ii) For any initial state $[x(0), \zeta(0)]$, there is a finite time $T := T(\varepsilon, x(0), \zeta(0)) > 0$, such that

$$|y(t)| = |x_1(t) - y_r(t)| < \varepsilon, \quad \forall t \ge T > 0. \tag{4}$$

In order to solve the global practical output tracking problem, we made the following assumptions:

Assumption1. For $i = 1, \ldots, n$, there are constants $C_1,\ C_2$ and $\tau \ge -1\Big/\sum_{l=1}^n p_1 \cdots p_{l-1}$ such that

$$|\phi_i(t,x,u)| \le C_1\left(|x_1|^{(r_i+\tau)/r_1} + \cdots + |x_i|^{(r_i+\tau)/r_i}\right) + C_2 \tag{5}$$

where $r_1 = 1, \quad r_{i+1} = (r_i + \tau)/p_i > 0, \quad i = 1, \ldots, n$ and $\sum_{l=1}^{n} p_1 \cdots p_{l-1} = 1$ for the case of l = 1.

Remark1. Assumption1, which gives the nonlinear growth condition on the system drift terms, encompasses the assumptions in existing results [9-12] and [2]. Specifically, when $\tau \geq 0$, it reduces to Assumptions in [9-10] and [1-2]. When τ is some ratios of odd integers in $\tau \in [-1/\sum_{l=1}^{n} p_1 \cdots p_{l-1}, 0]$, it encompasses the condition used in [11].

Assumption2. The reference signal $y_r(t)$ is continuously differentiable. Moreover, there is a known constant D>0, such that

$$|y_r(t)| + |\dot{y}_r(t)| \leq D, \quad \forall t \in [0, \infty). \tag{6}$$

Assumption3. For $i = 0, 1, \ldots, n$, there exist positive constants $\Gamma_i, \ \mathrm{H}_i$ such that

$$\Gamma_i \leq \theta_i \leq \mathrm{H}_i. \tag{7}$$

Now, we state the main result of this paper as follows:

Theorem1. Under assumptions1-3 on system (1), the global practical output tracking problem stated aboveis solvable by output controller of the form (3).

*Proof.*With the help of these Lemmaes1-8and [27], we can prove of the main results. First, we introduce the following change of coordinates

$$\vartheta_1 = \theta_0 x_1, \ \vartheta_i = \theta_0^{1/(p_0 \cdots p_{i-1})} \theta_1^{1/(p_1 \cdots p_{i-1})} \cdots \theta_{i-1}^{1/p_{i-1}} x_i, \ i = 2, \ldots, n \tag{8}$$

where $p_0 = 1$, under which system (1) can be written as

$$\begin{aligned} \dot{\vartheta}_i &= \vartheta_{i+1}^{p_i} + \varphi_i(t, \vartheta, u), \quad i = 1, \ldots, n-1 \\ \dot{\vartheta}_n &= \theta u + \varphi_n(t, \vartheta, u) \\ y &= \vartheta_1 - y_r(t) \end{aligned} \tag{9}$$

where $\varphi_1(t, \vartheta, u) = \theta_0 \phi_1(t, x, u), \quad \varphi_i(t, \vartheta, u) = \theta_0^{1/(p_0 \cdots p_{i-1})} \theta_1^{1/(p_1 \cdots p_{i-1})} \cdots \theta_{i-1}^{1/p_{i-1}} \phi_i(t, x, u), \quad i = 2, \ldots, n$ and $\theta = \theta_0^{1/(p_0 \cdots p_{i-1})} \theta_1^{1/(p_1 \cdots p_{i-1})} \cdots \theta_{n-1}^{1/p_{n-1}} \theta_n$. Via Assumption 3, it can be verified that $\varphi_i(t, \vartheta, u)$ will satisfy Assumption 2 with a new growth rate $\tilde{C}_1, \ \tilde{C}_2$. That is

$$|\varphi_i(t, \vartheta, u)| \leq \tilde{C}_1 \left(|\vartheta_1|^{(r_i+\tau)/r_1} + \cdots + |\vartheta_i|^{(r_i+\tau)/r_i} \right) + \tilde{C}_2, \quad i = 1, \ldots, n. \tag{10}$$

Due to remaining of the prove is very similar to [12] and pages limit, so omitted here.

4. Conclusions

In this paper, an output feedback tracking controller for a class of high-order uncertain nonlinear systems was presented under weaker condition. It was shown that the global practical tracking problem is solvable using the homogenous observer and controller, which can be explicitly constructed. First, we designed an output feedback controller for the nominal system without the perturbing nonlinearties. Then, we utilized the homogeneous domination approach by introducing an adjustable scaling gain into the output feedback controller obtained for the nominal system. Further, it was also shown that an appropriate choice of gain will enable us to globally track for a class of uncertain nonlinear systems in finite time. Finally, the proposed approach can also widen the applicability to a broader class of systems.

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КОМБИНАЦИЯЛЫҚ ЦИФРЛІК ҚҰРЫЛҒЫЛАРДЫ VHDL ТІЛІНДЕ МОДЕЛДЕУ

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Қазіргі заманғы цифрлік құрылғылар мен жүйелердің элементтік базасы цифрлік интегралды сұлбалар болып табылады. Цифрлік құрылғылар шартты түрде 4 топқа бөлінеді (Сурет 1).

Базалық логика құрылғыларына: инвертор, конъюнктор, дизъюнктор, «Шеффер штрихы» және «Пирс сілтемесі» функциялары мен буфер элементтері жатады.

Тізбектелген құрылғылардың құрамында әртүрлі триггерлер, ал комбинациялық құрылғылардың қатарына: кодер, декодер, мультиплексер мен демультиплексер кіреді.

Цифрлік құрылғылардың ішіндегі ең көпфункциялды күрделі құрылғылар: тура және кері есептеуіштер, регистрлер, компараторлар мен АЛҚ элементтері.