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Жинаққа студенттердің, магистранттардың, докоранттардың және жас ғалымдардың жаратылыстану-техникалық және гуманитарлық ғылымдардың өзекті мәселелері бойынша баяндамалары енгізілген.

The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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For $M_\phi f_2(y), y \in B(x,r)$ we have

$$M_\phi f_2(y) \le C \sup_{t \succ 2r} \phi(t) \int_{B(x,t)} |f(z)| dz \le C \sup_{t \succ 2r} \|f\|_{L_\rho(B(x,t))} \phi(t) t^p .$$

Then for all $y \in B(x,r)$ we get

$$M_\phi f(y) \le C\phi(t)t^n Mf(x) + \sup_{t>r} \|f\|_{L_p(B(x,t))} \phi(t)t^n \le C\phi(t)t^n Mf(x) + \|f\|_{\phi,\varphi^{\frac{1}{p}}} \sup_{t>r} \varphi(x,t)\phi(t)t^n .$$

Then we obtain

$$M_\phi f(y) \le C \min\left\{ \varphi(x,t)^{\frac{p}{q}-1} Mf(x), \varphi(x,t)^{\frac{p}{q}} \|f\|_{M\rho,\varphi^{\frac{1}{\rho}}} \right\} \le C \sup_{s \succ 0} \min\left\{ s^{\frac{\xi}{q}-1} Mf(x), s^{\frac{p}{q}} \|f\|_{M\, p,\varphi^{\frac{1}{p}}} \right\} =$$

$$= (Mf(x))^{\frac{p}{q}} \|f\|_{M_{p,\varphi^{\frac{1}{p}}}}^{1-\frac{p}{q}} ,$$

where we have used that the supremum is achieved when the minimum parts are balanced. Hence for all $y \in B(x,r)$, we have

$$M_\phi f(y) \le C(Mf(x))^{\frac{p}{q}} \|f\|_{M_{\rho,\varphi^{\frac{1}{p}}}}^{1-\frac{\rho}{q}} .$$

Consequently the statement of the theorem follows in view of the boundedness of the maximal operator M in $LM_{p,\theta,\varphi}$ provided by Theorem in virtue of condition.

$$\|M_\phi f\|_{GM_{q,\theta,\varphi^{\frac{1}{q}}}} = \sup_{x\in R^n, t\succ 0} \varphi(x,t)^{-\frac{1}{q}} t^{-\frac{n}{q}} \|M_\phi f\|_{L_q(B(x,t))} \le C\|f\|_{GM_{p,\theta,\varphi^{\frac{1}{p}}}}^{1-\frac{p}{q}} \sup_{x\in R^n, t\succ 0} \varphi(x,t)^{-\frac{1}{q}} t^{-\frac{n}{q}} \|Mf\|_{L_p(B(x,t))}^{\frac{p}{q}} =$$

$$= \|f\|_{GM_{p,\theta,\varphi^{\frac{1}{p}}}}^{1-\frac{p}{q}} \left(\sup_{x\in R^n, t>0} \varphi(x,t)^{-\frac{1}{p}} t^{-\frac{n}{p}} \|Mf\|_{L_p(B(x,t))} \right)^{\frac{p}{q}} = \|f\|_{GMp,\theta,\varphi^{\frac{1}{p}}}^{1-\frac{p}{q}} \|Mf\|_{M_{p,\theta,\varphi}}^{\frac{p}{q}} \le C\|f\|_{GM\, p,\theta,\varphi^{\frac{1}{p}}} ,$$

if $1 < p < q < \infty$.

In the case $\phi(t)t = t^\alpha$ we get the Adams type result on generalized Morrey spaces.

Similar statements for the classical fractional-maximal operator were obtained in $[1]-[3]$.

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ON DISCRETIZATION OF ONE INTEGRAL

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Denote by $\mathrm{M}^{+}(0,\infty)$ the set of all positive measurable functions on $(0,\infty)$.

Let $1 \le p < \infty$, $0 < q < \infty$ and u, υ and ω are weights, (i.e. locally integrable non-negative functions on $(0,\infty)$), φ is strictly increasing function on $(0,\infty)$, and $\frac{\varphi}{U}$ is decreasing on $(0,\infty)$, where

$$U(s) = \int_0^s u(t)dt.$$

Our goal is characterize the following inequality

$$\left(\int_0^\infty \left(\sup_{t<s<\infty} \frac{1}{\varphi(s)} \int_0^s \left(\int_\tau^\infty h(t)dt\right) u(\tau)d\tau\right)^q \omega(z)dz\right)^{\frac{1}{q}} \le C\left(\int_0^s h^p(\tau)\upsilon(\tau)d\tau\right)^{\frac{1}{p}} \qquad (1)$$

for all $\mathrm{M}^{+}(0,\infty)$.

Using the Fubbini theorem for non-negative functions, we have

$$\int_0^s u(\tau)\int_\tau^\infty h(t)dtd\tau = \int_0^s U(\tau)h(\tau)d\tau + U(s)\int_s^\infty h(\tau)d\tau .$$

Therefore the (1) is equivalent with following inequality

$$\left(\int_0^\infty \left(\sup_{t<s<\infty} \frac{1}{\varphi(s)} \left(\int_0^s U(\tau)h(\tau)d\tau + U(s)\int_s^\infty h(\tau)d\tau\right)\right)^q \omega(z)dz\right)^{\frac{1}{q}} \le C\left(\int_0^s h^p(\tau)\upsilon(\tau)d\tau\right)^{\frac{1}{p}} \qquad (2)$$

If φ is a non-negative and monotone function on $(a,b) \subseteq R$, then by $\varphi(x)$, where $x \in (a,b)$, we mean the value $\varphi(x-) := \lim_{t\to x-} \varphi(t)$.

Lemma 1[2]. Let φ be a non-negative, non-decreasing, finite and right-continuous function on (a,b). There is a strictly increasing sequence $\{x_k\}_{k=N-1}^{M}$; $-\infty \le N \le M \le +\infty$, with elements from the closure of the interval (a,b), such that:

i) if $N > -\infty$, then $\varphi(\mathrm{x}_M) > 0$ and $\varphi(\mathrm{x}) = 0$ for every $x \in (a, x_N)$; if $M < +\infty$ then, $x_{M+1} = b$;

ii) if $N \le k \le M$, then $\varphi(x_{k+1}-) \le 2\varphi(x_k)$;

iii) if $N < k < M$, then $2\varphi(x_k-) \le \varphi(x_{k+1})$.

Definition 1[2]. Let φ be a non-negative, non-decreasing, finite and right-continuous function on (a,b). There is a strictly increasing sequence $\{x_k\}_{k=N-1}^{M}$; $-\infty \le N \le M \le +\infty$, is said to be a discretizing sequence of the function φ, if it satisfies the conditions (i)–(iii) of Lemma 1.

Definition 2[3]. Let φ be a continuous strictly increasing function on $[0,\infty)$ such that $\varphi(0) = 0$ and $\lim_{t\to\infty} \varphi(t) = \infty$. Then we say that φ is admissible.

We write A ≲ B (or A ≳ B) if $A \le C_1 B$ (or $C_2 A \ge B$) for some positive constant C independent of appropriate quantities involved in the expressions A and B, and A ≈ B if A ≲ B and A ≳ B.

Let φ be an admissible function. We say that a function h is $\varphi-quasiconcave$ if h is equivalent to a non-decreasing function on $[0,\infty)$ and $\frac{h}{\varphi}$ is equivalent to a non-increasing function on $(0,\infty)$. We say that function h is non-degenerate if

$$\lim_{t\to 0+} h(t) = \lim_{t\to\infty}\frac{1}{h(t)} = \lim_{t\to\infty}\frac{h(t)}{\varphi(t)} = \lim_{t\to 0+}\frac{\varphi(t)}{h(t)} = 0$$

The family of non-degenerate $\varphi-quasiconcave$ functions will be denoted by Ω_φ.
We say that h is quasiconcave when $h\in\Omega_\varphi$ with $\varphi(t)=t.$

Let discretize the inequality (2). To this we need the following notations: first denote by $B(x_k,x_{k+1})$ the best constant of the weighted iterated Copson and Hardy inequalities, that is,

$$B(x_k,x_{k+1}) = \sup_{h\in M^+(0,\infty)} \frac{\sup\limits_{x_k<t<x_{k+1}} \frac{G(l)}{\varphi(l)}\left(\int_{x_k}^{t} U(\tau)h(\tau)d\tau + U(l)\int_{t}^{x_{k+1}} h(\tau)d\tau\right)}{\left(\int_{x_k}^{x_{k+1}} h^p(\tau)\upsilon\right)^{\frac{1}{p}}}$$

and using the characterizations of weighted iterated Copson and Hardy inequalities, we have

$$B(x_k,x_{k+1}) \approx \begin{cases} \sup\limits_{x_k<t<x_{k+1}} \frac{G(l)}{\varphi(l)}\left(\int_{x_k}^{t} U^{p'}(\tau)\upsilon^{1-p'}(\tau)d\tau + U(l)\int_{t}^{x_{k+1}} \upsilon^{1-p'}(\tau)d\tau\right)^{1/p'}, 1<p<\infty \\ \sup\limits_{x_k<t<x_{k+1}} \frac{G(l)}{\varphi(l)}\left(\sup\limits_{x_k<s<t}\frac{u(s)}{\upsilon(s)} + U(l)\sup\limits_{t<s<x_{k+1}}\frac{1}{\upsilon(s)}\right), p=1 \end{cases}$$

Theorem 1. Let $q\in(0,\infty)$. Assume that U and $\frac{1}{\varphi}$ are admissible functions, ω is weight, and $h\in M^+(0,\infty)$. If $\{x_k\}$ is a discretizing sequence of G, where

$$G(t) = \left(\int_0^t \omega(s)ds + \varphi(l)^q \int_t^\infty \varphi^{-q}(s)\omega(s)ds\right)^{\frac{1}{q}},$$

then

$$\int_0^\infty \left(\sup_{t<s<\infty}\frac{1}{\varphi(s)}\left(\int_0^s U(\tau)h(\tau)d\tau + U(s)\int_s^\infty h(\tau)d\tau\right)\right)^q \omega(z)dz \approx$$

$$\approx \sum_{k\in Z} G(x_k)\left[\sup_{x_k<t<\infty}\frac{1}{\varphi(t)}\left(\int_0^t U(\tau)h(\tau)d\tau + U(t)\int_t^\infty h(\tau)d\tau\right)\right]^q \approx$$

$$\approx \sum_{k\in Z}\left[\sup_{x_k<t<x_{k+1}}\frac{G(l)}{\varphi(l)}\left(\int_{x_k}^t U(\tau)h(\tau)d\tau + U(l)\int_t^{x_{k+1}} h(\tau)d\tau\right)\right]^q$$

For proving Theorem 1 we use following propositions:

Lemma 2[3]. Let $q\in(0,\infty)$, let u be an admissible function and let ν be a non-degenerate positive Borel measure. Let h be the fundamental function of ν with respect to u^q and let f be a measurable function on $[0,\infty)$. Let $\{x_k\}$ be a discretizing sequence for h with respect to u^q. Then

$$\int_0^\infty \left(\sup_{y\in(0,\infty)} \frac{|f(y)|}{u(x)+u(y)} \right)^q d\nu(x)$$

$$\approx \sum_{k\in Z} \left(\sup_{y\in(0,\infty)} \frac{|f(y)|}{u(x_k)+u(y)} \right) h(x_k)$$

$$\approx \sum_{k\in Z} \left(u^{-1}(x_k) \sup_{x_{k-1}\le y<x_k} |f(y)| + \sup_{x_k\le y<x_{k+1}} |f(y)| u(y)^{-1} \right) h(x_k)$$

$$\approx \sum_{k\in Z} \sup_{x_k\le y<x_{k+1}} |f(y)|^q u(y)^{-q} h(y).$$

Definition 3[3]. Let $\{a_k\}$ be a sequence of the positive real numbers. We say that $\{a_k\}$ is strongly decreasing and write $a_k \downarrow\downarrow$ when $\sup_{k\in Z} \frac{a_{k+1}}{a_k} < 1$.

Lemma 3[1]. If $\tau_k \downarrow\downarrow$, then for any $q>0$,

$$\sum_{k\in Z} \left(\int_0^{x_k} h \right)^q \tau_k \approx \sum_{k\in Z} \left(\int_{x_{k-1}}^{x_k} h \right)^q \tau_k ,$$

$$\sup_{k\in Z} \left(\int_0^{x_k} h \right)^q \tau_k \approx \sup_{k\in Z} \left(\int_{x_{k-1}}^{x_k} h \right)^q \tau_k ,$$

$$\sum_{k\in Z} \left(\int_{x_k}^{\infty} h \right)^q \tau_k^{-1} \approx \sum_{k\in Z} \left(\int_{x_k}^{x_{k+1}} h \right)^q \tau_k^{-1} ,$$

$$\sup_{k\in Z} \left(\int_{x_k}^{\infty} h \right)^q \tau_k^{-1} \approx \sup_{k\in Z} \left(\int_{x_k}^{x_{k+1}} h \right)^q \tau_k^{-1} .$$

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РОБОТТЫҢ ТЕГІСТЕЛГЕН АЙНАЛЫП ӨТУ ТРАЕКТОРИЯСЫ