ҚАЗАҚСТАН РЕСПУБЛИКАСЫ ҒЫЛЫМ ЖӘНЕ ЖОҒАРЫ БІЛІМ МИНИСТРЛІГІ

«Л.Н. ГУМИЛЕВ АТЫНДАҒЫ ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ» КЕАҚ

Студенттер мен жас ғалымдардың
«ǴYLYM JÁNE BILIM - 2023»
XVIII Халықаралық ғылыми конференциясының
БАЯНДАМАЛАР ЖИНАҒЫ

СБОРНИК МАТЕРИАЛОВ
XVIII Международной научной конференции
студентов и молодых ученых
«ǴYLYM JÁNE BILIM - 2023»

PROCEEDINGS
of the XVIII International Scientific Conference
for students and young scholars
«ǴYLYM JÁNE BILIM - 2023»

2023
Астана

УДК 001+37
ББК 72+74
G99

«ǴYLYM JÁNE BILIM – 2023» студенттер мен жас ғалымдардың XVIII Халықаралық ғылыми конференциясы = XVIII Международная научная конференция студентов и молодых ученых «ǴYLYM JÁNE BILIM – 2023» = The XVIII International Scientific Conference for students and young scholars «ǴYLYM JÁNE BILIM – 2023». – Астана: – 6865 б. - қазақша, орысша, ағылшынша.

ISBN 978-601-337-871-8

Жинаққа студенттердің, магистранттардың, докоранттардың және жас ғалымдардың жаратылыстану-техникалық және гуманитарлық ғылымдардың өзекті мәселелері бойынша баяндамалары енгізілген.

The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

УДК 001+37
ББК 72+74

ISBN 978-601-337-871-8

We make a conclusion that there is an right inverse of the operator $(L_\lambda)'$, which operates in the space $L_{p'}(R)$ during the $\lambda \geq \lambda_1$ by applying Lemma 3. A right inverse is defined on $L_{p'}(R)$. So, $\ker((L_\lambda)')^* = \{0\}$, here $((L_\lambda)')^*$ is conjugate operator of $(L_\lambda)'$. Hence, we get that $\ker L_\lambda = \{0\}$, $\lambda \geq \tilde{\lambda} = \max(\lambda_0, \lambda_1)$ due to $((L_\lambda)')^*$ is an extension of the operator L_λ. So, L_λ is bounded invertible operator in the space $L_{p'}(R)$. Actually, we obtain that

$$(L_\lambda)^{-1} = M_3(\lambda)G^{-1}(\lambda), \qquad \lambda \geq \tilde{\lambda} = \max(\lambda_0, \lambda_1) \tag{7}$$

Suppose that the equation (1) has a solution, and solution is y. Here $\lambda \geq \tilde{\lambda} = \max(\lambda_0, \lambda_1)$. We should prove the estimate (2) by applying (7), Lemma 1 and all conditions of the Theorem. We obtain that

$$\left\|(q+\lambda+ir)(L_\lambda)^{-1}\right\|_{L_p \to L_p} = \left\|(q+\lambda+ir)M_3(\lambda)G^{-1}(\lambda)\right\|_{L_p \to L_p} \leq c \sup_{\eta \in R} \int_{\eta-1}^{\eta+1} b_\lambda^3(\eta) b_\lambda^{-2}(x) \exp[-\sigma|x-\eta| b_\lambda(x)]dx \leq$$

$$\leq c \sup_{\eta \in R} b_\lambda(\eta) \int_{\eta-1}^{\eta+1} \exp[-\sigma|x-\eta| b_\lambda(x)]dx \prec \infty.$$

Due to this and (1) we make a conclusion that $\left\|\sqrt{5+x^2}\left(\sqrt{5+x^2}\,y'\right)''\right\|_p \leq c\left(\|f\|_p + \|y\|_p\right)$. Eventually, by combining the last two estimates we get (2). The Theorem is completely proved.

References

1. R.D. Akhmetkaliyeva, L.-E. Persson, K.N. Ospanov, P. Wall, Some new results concerning a class of third-order differential equations // Applicable Analysis, 2015, Vol. 94, No. 2, 419-434.
2. A.E. Muslim, About maximum regularity of the solution of singular odd order differential equation, 2022.
3. R.D. Akhmetkaliyeva, Coercive estimate of the solution of the singular differential equation and its applications, 2013.

UDC 517.51

BOUNDEDNESS OF THE GENERALIZED FRACTIONAL – MAXIMAL OPERATOR IN GLOBAL MORREY TYPE SPACES

Sibagatova Perizat Muratovna, Argynbekov Askhat Makhabbatuly,
sibagatova99@list.ru, a_-_-_a@list.ru
2nd year master student, 1st year doctoral student, L.N. Gumilyov Eurasian National University, Astana, Kazakhstan
Research supervisor – N.A. Bokayev

For a measurable function $\phi:(0,\infty) \to (0,\infty)$ the generalized fractional maximal operator M_ϕ is defined by

$$(M_\phi f)(x) = \sup_{t>0} \phi(t) \int_{B(x,t)} |f(y)|dy,$$

where $B(x,t)$ is the ball in R^n of radius r centered at $x \in R^n$.

Definition 1. Let $\varphi(x,r)$ be a positive measurable function on $R^n \ x(0,\infty)$ and $1 \le p, \theta < \infty$. We denote by $GM_{P,\theta,\varphi} \equiv GM_{P,\theta,\varphi}(R^n)$ the global Morrey type spaces, the space of all functions $f \in L_p^{loc}(R^n)$ with finite norm

$$\|f\|_{GM_{P,\theta,\varphi}} = \left\| \varphi(x,r)^{-1} \left|B(x,r)^{-1}\right| \|f\|_{L_p(B(x,r))} \right\|_{L\theta(0,\infty)},$$

where $|B(x,r)|$ is the volume of $B(x,r)$.

Theorem 1. Let $1 < p < \infty$ and (φ_1, φ_2) satisfy the condition

$$\sup_{r<t<\infty} t^{-\frac{n}{p}} ess \inf_{t<s<\infty} \varphi_1(x,s) s^{\frac{n}{p}} \le C\varphi_2(x,r),$$

where C does not depend on x and r. Then the operator M is bounded from GM_{P,θ,φ_1} to GM_{P,θ,φ_2}.

Theorem 2. Let $1 < p < q < \infty, \phi(t)$ be almost decreasing function and let $\phi(t)t^n$ satisfies the condition: there exists constants $C_1 > 0$ and $0 < 2k_1 < k_2 < \infty$ such that

$$\sup_{r<s\le 2r} \phi(s) \le C_1 \sup_{k_1<t<k_2 r} \phi(t), r > 0,$$

and the inequality

$$\int_0^{k_2 r} \phi(s) s^{n(1-\frac{1}{p})} ds \le C\phi(r) r^n,$$

where C does not depend on $r > 0$.

Let $\varphi(x,t)$ satisfy the condition $\sup_{r<t<\infty} t^{-n} ess \inf_{t<s<\infty} \varphi(x,s)s^n \le C\varphi(x,r)$ and

$$\phi(r) r^n \varphi(x,r) + \sup_{t>r} \phi(t) t^n \varphi(x,t) \le C\varphi(x,r)^{\frac{p}{q}},$$

where C does not depend on $x \in R^n$ and $r > 0$. Then the operator M_ϕ is bounded from $LM_{P,\theta,\varphi}$ to $LM_{q,\theta,\varphi^{\frac{1}{q}}}$.

Proof. Let $x_0 \in R^n, 1 < p < q < \infty, f \in LM_{P,\theta,\varphi^{\frac{1}{p}}}$ and $f \in M_{P,\theta,\varphi}$. Write $f = f_1 + f_2$, where $B = B(x,r), f_1 = f_{x_2 B}$ and $f_2 = f_{x_2 B}$. Then we have $M_\phi f(x) \le M_\phi f_1(x) + M_{\phi,} f_2(x)$.

For $M_\phi f_1(y), y \in B(x,r)$, following Hedberg's trick we obtain

$$M_\phi f_1(y) = \sup_{t>0} \phi(t) \int_{B(y,t)\cap B(x,2r)} |f(z)| dz \le C \sup_{t>0} \int_{B(y,t)\cap B(x,2r)} \frac{\phi(|y-z|)}{|y-z|^n} |f(z)| dz \approx$$

$$\approx \sup_{t>0} \sum_{k=-\infty}^{0} \int_{B(y,t)\cap(B(x,2^{k+1}r)\backslash B(x,2^k r))} \frac{\phi(|y-z|)}{|y-z|^n} |f(z)| dz \le C \sup_{t>0} \sum_{k=-\infty}^{0} \int_{2^k k_1 r}^{2^k k_2 r} \phi(s) \frac{1}{s} ds \int_{B(y,t)\cap B(x,2^{k-1}r)} |f(z)| dz \approx$$

$$\approx (Mf)(x) \sup_{t>0} \sum_{k=-\infty}^{0} \int_{2^k k_1 r}^{2^k k_2 r} \phi(s) ds = (Mf)(x) \int_0^{k_2 r} \phi(s) ds \le CMf(x)\phi(s)s^n.$$

$Mf(x) = \sup_{t>0} \frac{1}{|B(x,t)|} \int_{B(x,t)} |f(y)| dy$ - from Hardy-Littlewood operator.

For $M_\phi f_2(y), y \in B(x,r)$ we have

$$M_\phi f_2(y) \le C \sup_{t \succ 2r} \phi(t) \int_{B(x,t)} |f(z)| dz \le C \sup_{t \succ 2r} \|f\|_{L_\rho(B(x,t))} \phi(t) t^p .$$

Then for all $y \in B(x,r)$ we get

$$M_\phi f(y) \le C\phi(t)t^n Mf(x) + \sup_{t > r} \|f\|_{L_p(B(x,t))} \phi(t)t^n \le C\phi(t)t^n Mf(x) + \|f\|_{\phi,\varphi^{\frac{1}{p}}} \sup_{t>r} \varphi(x,t)\phi(t)t^n .$$

Then we obtain

$$M_\phi f(y) \le C \min\left\{ \varphi(x,t)^{\frac{p}{q}-1} Mf(x), \varphi(x,t)^{\frac{p}{q}} \|f\|_{M\rho,\varphi^{\frac{1}{\rho}}} \right\} \le C \sup_{s \succ 0} \min\left\{ s^{\frac{\xi}{q}-1} Mf(x), s^{\frac{p}{q}} \|f\|_{M\, p,\varphi^{\frac{1}{p}}} \right\} =$$

$$= (Mf(x))^{\frac{p}{q}} \|f\|^{1-\frac{p}{q}}_{M_{p,\varphi^{\frac{1}{p}}}} ,$$

where we have used that the supremum is achieved when the minimum parts are balanced. Hence for all $y \in B(x,r)$, we have

$$M_\phi f(y) \le C(Mf(x))^{\frac{p}{q}} \|f\|^{1-\frac{\rho}{q}}_{M_{\rho,\varphi^{\frac{1}{p}}}} .$$

Consequently the statement of the theorem follows in view of the boundedness of the maximal operator M in $LM_{p,\theta,\varphi}$ provided by Theorem in virtue of condition.

$$\|M_\phi f\|_{GM_{q,\theta,\varphi^{\frac{1}{q}}}} = \sup_{x \in R^n, t \succ 0} \varphi(x,t)^{-\frac{1}{q}} t^{-\frac{n}{q}} \|M_\phi f\|_{L_q(B(x,t))} \le C \|f\|^{1-\frac{p}{q}}_{GM_{p,\theta,\varphi^{\frac{1}{p}}}} \sup_{x \in R^n, t \succ 0} \varphi(x,t)^{-\frac{1}{q}} t^{-\frac{n}{q}} \|Mf\|^{\frac{p}{q}}_{L_p(B(x,t))} =$$

$$= \|f\|^{1-\frac{p}{q}}_{GM_{p,\theta,\varphi^{\frac{1}{p}}}} \left(\sup_{x \in R^n, t > 0} \varphi(x,t)^{-\frac{1}{p}} t^{-\frac{n}{p}} \|Mf\|_{L_p(B(x,t))} \right)^{\frac{p}{q}} = \|f\|^{1-\frac{p}{q}}_{GMp,\theta,\varphi^{\frac{1}{p}}} \|Mf\|^{\frac{p}{q}}_{M_{p,\theta,\varphi}} \le C \|f\|_{GM\, p,\theta,\varphi^{\frac{1}{p}}} ,$$

if $1 < p < q < \infty$.

In the case $\phi(t)t = t^\alpha$ we get the Adams type result on generalized Morrey spaces.

Similar statements for the classical fractional-maximal operator were obtained in [1]–[3].

References

1. V. Burenkov, A. Gogatishvili, V.S. Guliyev and R. Mustafayev, Boundedness of the fractonal maximal operator in local Morrey-type spaces // Complex Variables and Elliptic Equations 55:8-10, 2010, p. 739-758.

2. Y. Sawano, S. Sugano and H. Tanaka, Generalized fractional integral operators and fractional maximal operators in the framework of Morrey spaces // Transactions of the American Society 363, 2011, p. 6481-6503.

UDK 517.52

ON DISCRETIZATION OF ONE INTEGRAL

Yerniyashova Zhadyra Izgalievna

zhadira-90@mail.ru, yerniyashova.z@gmail.com