ҚАЗАҚСТАН РЕСПУБЛИКАСЫ ҒЫЛЫМ ЖӘНЕ ЖОҒАРЫ БІЛІМ МИНИСТРЛІГІ

«Л.Н. ГУМИЛЕВ АТЫНДАҒЫ ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ» КЕАҚ

Студенттер мен жас ғалымдардың
«ǴYLYM JÁNE BILIM - 2023»
XVIII Халықаралық ғылыми конференциясының
БАЯНДАМАЛАР ЖИНАҒЫ

СБОРНИК МАТЕРИАЛОВ
XVIII Международной научной конференции
студентов и молодых ученых
«ǴYLYM JÁNE BILIM - 2023»

PROCEEDINGS
of the XVIII International Scientific Conference
for students and young scholars
«ǴYLYM JÁNE BILIM - 2023»

2023
Астана

УДК 001+37
ББК 72+74
G99

«ǴYLYM JÁNE BILIM – 2023» студенттер мен жас ғалымдардың XVIII Халықаралық ғылыми конференциясы = XVIII Международная научная конференция студентов и молодых ученых «ǴYLYM JÁNE BILIM – 2023» = The XVIII International Scientific Conference for students and young scholars «ǴYLYM JÁNE BILIM – 2023». – Астана: – 6865 б. - қазақша, орысша, ағылшынша.

ISBN 978-601-337-871-8

Жинаққа студенттердің, магистранттардың, докоранттардың және жас ғалымдардың жаратылыстану-техникалық және гуманитарлық ғылымдардың өзекті мәселелері бойынша баяндамалары енгізілген.

The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

УДК 001+37
ББК 72+74

ISBN 978-601-337-871-8

$$\leq \left(\sum_{i=-\infty}^{+\infty} c_i^{\alpha p} |z_i|^{(1+\gamma)p} \right)^{\frac{1}{p}} \left(\sum_{i=-\infty}^{+\infty} |f_i|^{p'} \right)^{\frac{1}{p'}}, \qquad \left(\frac{1}{p} + \frac{1}{p'} = 1,\ 1 \prec p \prec \infty \right).$$

Take p, α, γ satisfying conditions: $p\alpha = 1, \quad (1+\gamma)p = 2+\gamma, \quad p = \frac{1}{\alpha}, \quad \gamma = \frac{2-p}{p-1}$. Then the inequalities are eliminated: $1 < p < \infty, \gamma > -1$,

$$\sum_{i=-\infty}^{+\infty} c_i |z_i|^{2+\frac{2-p}{p-1}} \leq \left(\sum_{i=-\infty}^{+\infty} c_i |z_i|^{2+\frac{2-p}{p-1}} \right)^{\frac{1}{p}} \left(\sum_{i=-\infty}^{+\infty} |f_i|^{p'} \right)^{\frac{1}{p'}}.$$

Therefore, there is an assessment

$$\left(\sum_{i=-\infty}^{\infty} c_i |z_i|^{p'} \right)^{\frac{1}{p'}} \leq \left(\sum_{i=-\infty}^{+\infty} |f_i|^{p'} \right)^{\frac{1}{p'}} \qquad (\alpha + \frac{1}{p'} = 1).$$

Let $\{f_i\}$ be a finite sequence. Then equation (1), by virtue of condition $c_i \geq \varepsilon > 0$ in l_2 has a solution $\{y_i\}$. Using inequality (2), it is not difficult to make sure that this solution belongs to $l_{p'}$ at $\frac{1}{p'} + \alpha = 1$. Passing α to 0 or 1, we get that the above is true at $\alpha \in [0,1]$, and the inequality (2) holds.

If now $\{f_j\}$ belongs to $l_{p'} (1 \leq p' \leq \infty)$, but is not a finite sequence, then, approximating $\{f_j\}$ in $l_{p'}$ finite sequences, we get that equation (1) has a solution satisfying inequality (3).

Literature

1. Otelbayev M., On coercive estimates of the solution of difference equations//Proceedings of the Mathematical Institute of the USSR Academy of Sciences, –1988. –Vol.181. –P. 241-249.
2. Zulkazhav A., Coercive estimates and their application for second-order difference equations, 2016.
3. Ospanov K.N., Zulkhazhav A., Coercive solvability of degenerate system of second order difference equations // AIPConf. Proc. – 2016. – Vol. 1759. – P. 1-5.

UDC 517.927.2

ONE CLASS OF PERTURBATION PROBLEMS FOR THE LAPLACE OPERATOR

Kudaibergenov Serik
serik4648@gmail.com
L. N. Gumilyov Eurasian National University
2nd year doctorant in mathematics, Astana, Kazakhstan.
Supervisor – Turdebek B.

Perturbation theory is the most used techniques in quantum mechanics, atomic physics and especially in the study of dynamical systems. The classical method is for one perturbated problem obtaining complete system of eigenfunctions. (If system is complete, it is a basis of the space.) Main purpose of this work to defining does singular perturbated Laplace operators has a complete system of eigenfunctions or not. In this work for one class of perturbation problems we selected all problems with complete eigenfunction system. Respectively we will know in advance which perturbation problem has solution or not.

Let consider Laplace operator $Lu = -\Delta u = f$ in a Hilbert space $L_2(\Omega)$, where Ω – is a bounded domain in R^n with an infinitely smooth boundary $\partial\Omega$. As a fixed operator we can take Dirichlet problem

$$\begin{cases} \Delta u = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \cdots + \frac{\partial^2 u}{\partial x_n^2} = -f(x) \\ \qquad\qquad u|_{\partial u} = 0 \end{cases} , x \in \Omega \subset R^n$$

which is correct restriction of the maximal operator $\hat{L}u = f$ with the domain $D(\hat{L}) = \{u \in L_2(\Omega): \quad \hat{L}u = -\Delta u \in L_2(\Omega)\} = W_2^2(\Omega)$. Let operator $u = L_D^{-1} f$ be the inverse operator of the Dirichlet operator with the domain $D(L_D) = \{u \in W_2^2(\Omega): \quad u|_{\partial u} = 0\}$ and operator the Kf be any linear bounded operator that maps Hilbert space into kernel of the maximal operator. Then according to correct restriction theory [2], all correct restrictions of the maximal operator have an inverse in the following form $L_K^{-1} f = L_D^{-1} f + Kf$.

In our case operator $K: L_2(\Omega) \to ker\hat{L} = \{u \in W_2^2(\,\Omega\,): -\Delta u = 0\,\}$ has the following form $Kf(x) = \varphi(x) \iint_\Omega f(\tau)\overline{g(\tau)}d\tau$ where $\varphi(x)$ is a harmonic function in $L_2(\Omega)$ and $g(x)$ is any function from $L_2(\Omega)$. Then all correct restriction of the maximal operator is an operator $L_K u = -\Delta u = f$ with the domain $D(L_K) = \{u \in W_2^2(\Omega): \ (I - KL_K)u \in D(L_D)\}$.

Operator KL_D bounded if and only if operator K satisfies the condition $R(K^*) \subset D(L_D^*)$. Adjoint operator of the Kf has the form $K^* f(x) = g(x) \iint_\Omega f(\tau)\overline{\varphi(\tau)}d\tau$ and the condition $R(K^*) \subset D(L_D^*)$ equivalent to the $g(x) \in \mathrm{D}(L_D^*) = D(L_D) = \{g \in W_2^2(\Omega): \quad g|_{\partial u} = 0\}$. Note that Dirichlet operator is positive operator ($L_D \geq 0$) , it is also self-adjoint operator ($L_D^* = L_D$).

By using Green's formula, we express the operator KL_D in the following form:

$$KL_D u(x) = -\varphi(x) \iint_\Omega \Delta u(\tau)\overline{g(\tau)}d\tau = -\varphi(x) \int\limits_{\partial\Omega} \left[\frac{\partial u(\tau)}{\partial n}\overline{g(\tau)} - u(\tau)\frac{\partial \overline{g(\tau)}}{\partial n}\right] dS -$$

$$-\varphi(x) \iint\limits_\Omega u(\tau)\Delta\overline{g(\tau)}d\tau = -\,\varphi(x) \iint\limits_\Omega u(\tau)\Delta\overline{g(\tau)}d\tau$$

In the work [1] was proven that operator $A_K u = Lu - \overline{KL_K}Lu$ with the domain $D(A_K) = D(L)$ is the similar to the correct restriction of L_K if $\overline{D(L_k)} = H$ and $R(K^*) \subset D(L^*) \cap D(L_K^*)$.

Theorem. Let L be boundary correct extension of the minimal operator L_0, that is $L_0 \subset L \subset \hat{L}$. If L_K is densely defined in H and satisfies the condition $R(K^*) \subset D(L^*) \cap D(L_K^*)$ then $\overline{KL_K}$ is bounded in H and a correct restriction L_K is similar to the correct operator $A_K u = Lu - \overline{KL_K}Lu$ with the domain $D(A_K) = D(L)$.

Proof. Transforming the equations $L_K^{-1} = L^{-1} + K = (I + KL)L^{-1}$ we get $L_K^{-1} = (I + KL)L^{-1}$. The condition $R(K^*) \subset D(L^*) \cap D(L_K^*)$ guarantees boundedness of the operators KL and KL_K. $I + KL$ is invertible operator with $(I + KL)^{-1} = I - KL_K$. Then we have

$$A_K^{-1} = (I + KL)^{-1} L_K^{-1} (I + KL) = (I + KL)^{-1}(I + KL)L^{-1}(I + KL)$$

$$A_K^{-1} = L^{-1}(I + KL)$$

Hence by corollary of [3] (page 259) we have $D(A_K) = D(L)$ and

$$A_K = (I + KL)^{-1} L = L - KL_K L$$

Proof is complete.

Density of the domain $D(L_K)$ in whole Hilbert space is equivalent to the

$$\iint\limits_\Omega \Delta\overline{g(\tau)}\varphi(\tau)d\tau \neq -1$$

This result can be obtained by $0 = \langle u, \phi\rangle = \langle L_K^{-1} f, \phi\rangle = \langle f, (L_K^{-1})^* \phi\rangle$. Since this holds for all functions $(x) \in L_2(\Omega)$, the function $(L_K^{-1})^* \phi = 0$. Another method is by the criterion of the density of the domain (L_K) , i.e., $\overline{D(L_K)} = L_2(\Omega)$ is equivalent to the $ker(I + K^* L_D^*) = 0$. Since the domain $D(L_K)$ is dense condition $R(K^*) \subset D(L^*) \cap D(L_K^*)$ becomes $R(K^*) \subset D(L^*)$.

So, our next aim is to find the operator KL_K. To calculate the operator KL_K we will use the fact that $(I + KL_D)^{-1} = I - KL_K$.

$$(I + KL_D)u(x) = u(x) + \varphi(x) \iint_\Omega u(\tau)\Delta\overline{g(\tau)}d\tau = v(x)$$

By multiplying both side with the function $\Delta\overline{g(x)}$ and integrating over the domain $\Omega \subset R^n$ we get

$$\iint_\Omega \Delta\overline{g(\tau)}u(\tau)d\tau \left[1 + \iint_\Omega \Delta\overline{g(\tau)}\varphi(\tau)d\tau\right] = \iint_\Omega \Delta\overline{g(\tau)}v(\tau)d\tau$$

So, the operator

$$u(x) = (I + KL_D)^{-1}v(x) = (I - KL_K)v(x)$$

$$u(x) = v(x) - \frac{\varphi(x)}{1 + \iint_\Omega \Delta\overline{g(\tau)}\varphi(\tau)d\tau} \iint_\Omega \Delta\overline{g(\tau)}v(\tau)d\tau$$

Operator KL_K has the following form

$$KL_K u(x) = \frac{\varphi(x)}{1 + \iint_\Omega \Delta\overline{g(\tau)}\varphi(\tau)d\tau} \iint_\Omega \Delta\overline{g(\tau)}v(\tau)d\tau$$

Note that $\iint_\Omega \Delta\overline{g(\tau)}\varphi(\tau)d\tau + 1 \neq 0$ is criterion of the $\overline{D(L_K)} = L_2(\Omega)$.
Then operator

$$A_K u(x) = -\Delta u(x) + \frac{\varphi(x)}{1 + \iint_\Omega \Delta\overline{g(\tau)}\varphi(\tau)d\tau} \iint_\Omega \Delta\overline{g(\tau)}\Delta u(\tau)d\tau = f(x)$$

with the domain $D(A_K) = \{u \in W_2^2(\Omega):\ \ u|_{\partial u} = 0\}$ is similar to the correct restriction operator $L_K u = -\Delta u = f$ with the domain $D(L_K) = \{u \in W_2^2(\Omega):\ (I - KL_K)u \in D(L_D)\}$.

Example: For clarity take $= 2$, then Dirichlet problem is

$$\begin{cases} \Delta u(x_1, x_2) = \frac{\partial^2 u(x)}{\partial x_1^2} + \frac{\partial^2 u(x)}{\partial x_2^2} = -f(x_1, x_2) \\ u|_{\partial\Omega} = 0 \end{cases}, \quad x \in \Omega \subset R^2$$

As defined above, by taking $g(x) \in L_2(\Omega)$ as

$$\begin{cases} \Delta g(x) = \ln|x - x_0| \\ g|_{\partial\Omega} = 0 \end{cases}, x, x_0 \in \Omega \subset R^2$$

where x_0 is the arbitrary point from the domain $\Omega \subset R^2$ and $|\cdot|$ is norm of the Euclidean space R^2. Note that function $g(x)$ satisfies the condition $g(x) \in D(L_D) = \{g \in W_2^2(\Omega):\ \ g|_{\partial u} = 0\}$. Then operator A_K form is following well-known valued at point $x_0 \in \Omega$ perturbation operator

$$A_K u(x) = -\Delta u(x) + \frac{\varphi(x)}{1 + \iint_\Omega \Delta\overline{g(\tau)}\varphi(\tau)d\tau} \iint_\Omega \ln|\tau - \tau_0|\Delta u(\tau)d\tau = f(x)$$

Then by Green's formula, operator

$$A_K u(x) = -\Delta u(x) + \frac{\varphi(x)}{1 + \iint_\Omega \ln|\tau - \tau_0|\varphi(\tau)d\tau} \left[u(x_0) + \int_\partial \frac{\partial u}{\partial n} \ln|\tau - \tau_0|dS\right]$$

with the domain $D(A_K) = \{u \in W_2^2(\Omega):\ \ u|_{\partial u} = 0\}$ is similar to correct operator L_K. (Operator K completely defined by the function $g(x)$.) Since the Dirichlet problem has complete system of eigenfunctions, this perturbation problem also has complete system of eigenfunctions. So, we can take this result for any harmonic functions φ and for any point $x_0 \in \Omega$.

So, the completeness of the eigenfunctions of the perturbation problem can be replaced with easier problem completeness of the eigenfunctions of Laplace operators. Since they similar operators their spectrum coincides. By changing the operator $Kf(x) = \varphi(x) \iint_{\Omega} f(\tau)\overline{g(\tau)}d\tau$ (choosing the functions φ and g) we will get completely new type of perturbation problem. In practice, if we confronted with such are perturbation problems which are defined on arbitrary domain $\Omega \subset R^n$, we can always (by calculating the functions φ and g) we bring the question of the completeness of the eigenfunctions with the completeness of the correct Laplace operators.

Literature

1. B.N. Biyarov. Similar transformation of one class of correct restriction. arXiv, 2021.
2. B.K. Kokebaev, M. Otelbaev, A.N. Shynibekov. About expansions and restrictions of operators in Banach space. Uspekhi Matem. Nauk 37, no. 4, 116-123, 1982..
3. P. Fillmore , J.Williams, On operator ranges. Advanced in Math. 7, 1971.

UDC 517

CONVERGENCE OF SERIES WITH GENERALIZED MOTONE COEFFICIENTS

Ospanov Aidar
aidar.ospan@list.ru
Eurasian National University named after L.N. Gumilyov
Astana, Kazakhstan
Scientific adviser: A.A. Jumabayeva

Definition 1 Suppose, $\{a_{i,j}\}_{i=1,j=1}^{\infty}$ - consider a sequence of personal sums of a series along with a given sequence, and a numerical sequence

$$\{S_{k,l}\}_{k=1,j=1}^{\infty}.$$

Each of its elements forms the sum of some members of the original sequence

$$\{S_{k,l}\} = \sum_{i=1,j=1}^{i=k,j=l} a_{i,j}.$$

As a rule, the following symbol is used to designate series:

$$\sum_{i=1,j=1}^{\infty} a_{i,j}.$$

This shows the initial sequence of elements of the series, as well as the summation rule. According to this, it is said about the convergence of double numerical series:

• A double number series converges if its sequence of personal sums converges, i.e. the series $\{a_{i,j}\}$ converges and the sum is equal to s if

$$\forall \varepsilon > 0 : \exists m_o, n_o : m > m_o, n > n_o : \left||s_{mn} - s|\right| < \varepsilon.$$

It is also possible to write down the condition for convergence to the sum of s dual series as follows:

$$\lim_{n,m \to \infty} s_{mn} = s$$