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The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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AUTOMORPHISMS OF THE UNIVERSAL MULTIPLICATIVE ENVELOPING ALGEBRAS OF FINITE DIMENSIONAL DUAL LEIBNIZ ALGEBRAS WITH ZERO MULTIPLICATION

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Let K be an arbitrary field. An algebra A over K is called $dual\,Leibniz\,algebra$ if it satisfies the identity

$$(xy)z = x(yz) + x(zy). \quad (1)$$

In [1] J.-L. Loday proved that any dual Leibniz algebra with respect to multiplication

$$x \circ y = xy + yx$$

is an associative commutative algebra. A linear basis of free dual Leibniz algebras is also given in [1].

Let A be an arbitrary dual Leibniz algebra over K. Let

$$L_A = \{L_x | x \in A\}$$

and

$$R_A = \{R_x | x \in A\}$$

be two isomorphic copies of the vector space A with the fixed isomorphisms $A \to L_A (x \longmapsto L_x)$ and $A \to R_A (x \longmapsto R_x)$. The universal (multiplicative) enveloping algebra $U(A)$ is an associative algebra with the identity 1 generated by the two vector spaces L_A and R_A satisfying the defining relations

$$R_x R_y = R_{xy+yx}, \quad (2)$$

$$R_x L_y = L_y R_x + L_x, \quad (3)$$

$$L_{xy} = L_x L_y + L_x R_y, \quad (4)$$

for all $x, y \in A$. Recall that every dual Leibniz A-bimodule M can be regarded as a left $U(A)$ −module with respect to the action

$$L_a m = am, R_a m = ma, a \in A, m \in M$$

Conversely, every left $U(A)$-module can be considered as a dual Leibniz A-bimodule [4].

Given definition of the universal enveloping algebra is good for algebras without identity element. If the identity element 1 is fixed in the signature then we have to add the relations $L_1 = R_1 = Id = 1$ and consider only unital modules. It is easy to see that the dual Leibniz algebra is an algebra without identity element.

Theorem 1. *Let A be a dual Leibniz algebra over K and let*

$$e_1, e_2, \dots, e_m, \dots \tag{5}$$

be a linear basis of A. Then the set of all associative words of the form

$$1, L_{e_i}, R_{e_j}, L_{e_i}R_{e_j} \tag{6}$$

where $i, j \geq 1$ is a linear basis for $U(A)$.

Let A a finite dual Leibniz algebra over a field K with zero multiplication. Let $e_1, e_2, \dots, e_n$ linear basis of algebra $U(A)$. By Theorem 1, the basis of universal multiplicative enveloping algebras consists of words of the form

$$1, L_{e_i}, R_{e_j}, L_{e_i}R_{e_j}, \qquad i, j \geq 1$$

It follows from $e_i e_j = 0$ for all $i, j \geq 1$, that the defining relations of the algebra $U(A)$ have the form

$$R_{e_i}R_{e_j} = R_{e_i}L_{e_j} = 0 \tag{7}$$

$$L_{e_i}L_{e_j} = -L_{e_i}R_{e_j} \tag{8}$$

Lemma 1. *Let u, v be basis words of $U(A)$. If $deg u \geq 2$ or $deg v \geq 2$, then $uv = 0$.*

Theorem 2. *Let A be the finite dimensional dual Leibniz algebra with zero multiplication over arbitrary field K. φ is affine automorphism of universal multiplicative enveloping algebra $U(A)$ of A, then*

$$\varphi(L_{e_i}) = \sum_{j=1}^{n} \alpha_{ij} L_{e_i} + \sum_{j=1}^{n} \beta_{ij} R_{e_j},$$

$$\varphi(R_{e_i}) = \sum_{j=1}^{n} (-\alpha_{ij} + \beta_{ij}) R_{e_j},$$

for all $1 \leq i \leq n$, where the determinants of matrices (α_{ij}) and $(-\alpha_{ij} + \beta_{ij})$ are not zero.

We describe the automorphisms of the universal multiplicative enveloping algebras of finite dimensional dual Leibniz algebras with zero multiplication over arbitrary field K.

Theorem 3. *Let A be the finite dimensional dual Leibniz algebra with zero multiplication over arbitrary field K. φ is the automorphisms of the universal multiplicative enveloping algebra $U(A)$ of A.*

$$\varphi(L_{e_i}) = f_2 + \sum_{j=1}^{n} \alpha_{ij} L_{e_j} + \sum_{j=1}^{n} \beta_{ij} R_{e_j}$$

$$\varphi(R_{e_i}) = g_2 + \sum_{j=1}^{n}(-\alpha_{ij} + \beta_{ij})R_{e_j}$$

for all $1 \leq i \leq n$, *where the determinants of matrices* (α_{ij}) *and* $(-\alpha_{ij} + \beta_{ij})$ *are not zero,* f_2, g_2 *are any homogeneous elements of degree 2 of the algebra* $U(A)$.

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WEIGHTED ESTIMATES OF DISCRETE BILINEAR HARDY TYPE OPERATORS

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Let $1 < p, s, q < +\infty$; let $u = \{u_n\}, v = \{v_n\}, w = \{w_n\}, n \in N$ be positive sequences of real numbers. Let $f = \{f_n\}_{n=1}^{\infty}, g = \{g_n\}_{n=1}^{\infty}$ be arbitrary sequences of nonnegative numbers. In this work we study the characterization problem for the bilinear discrete Hardy type operators of the following form

$$\left(\sum_{n=1}^{\infty} u_n^q \left(\sum_{i=n}^{\infty} a_{in} f_i\right)^q \left(\sum_{i=n}^{\infty} g_i\right)^q\right)^{\frac{1}{q}} \leq C \left(\sum_{i=1}^{\infty} |v_i f_i|^p\right)^{\frac{1}{p}} \left(\sum_{i=1}^{\infty} |w_i g_i|^s\right)^{\frac{1}{s}}, \qquad (1)$$

where C is the best constant in (1) and does not depend on f and g; (a_{ij}) is non-negative matrix with elements $a_{ij} \geq 0$, when $i \geq j \geq 1, a_{ij} = 0$, when $i < j$ and which satisfy the Oinraov's condition: there exists $d \geq 1$ such that

$$\frac{1}{d}(a_{ik} + a_{kj}) \leq a_{ij} \leq d(a_{ik} + a_{kj}), \forall i \geq k \geq j \geq 1 \qquad (2)$$