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The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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where

$$\Delta_h^\alpha(f) = \sum_{v=0}^{\infty} (-1)^v \binom{\alpha}{v} f(x + (\alpha - v))$$

difference of the fractional order  $\alpha, \alpha > 0$ , of the function  $f \in L_p$  at a point  $x$  with step  $h$ .

We have obtained an Ulyanov-type inequality for moduli of smoothness of fractional order with MVBVS sequences.

**Theorem 1.** Let  $f \in L_p$ ,  $1 < p < q < \infty$ ,  $\theta = 1/p - 1/q$ ,  $\rho > 0$  and  $\lambda = \{\lambda_n\}_{n=1}^\infty \in MVBVS$ . Then for any  $\alpha > 0$ ,

$$\begin{aligned} \omega_\alpha\left(\varphi, \frac{1}{2^n}\right)_q &\leq C \left( \sum_{k=\frac{2^n}{\mu}}^{\infty} |\lambda_k|^q k^{\theta q - 1} \omega_{\alpha+\theta+\rho}\left(f, \frac{1}{k}\right)_p \right)^{1/q} \\ &+ 2^{n(\theta+\rho)} \omega_{\alpha+\theta+\rho}\left(f, \frac{1}{2^n}\right)_p \max_{\substack{k=\frac{2^l}{\mu} \\ 1 \leq l \leq n}} \sum_{k=\frac{2^l}{\mu}}^{\mu 2^l} \frac{|\lambda_k|}{k^{\rho+1}} \end{aligned}$$

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## ON THE SOLUTION OF FRACTIONAL $q$ -DIFFERENCE EQUATION

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First we recall some elements of  $q$ -calculus, for more information see e.g. the books [1] and [2]. Throughout this paper, we assume that  $0 < q < 1$  and  $0 \leq a < b < \infty$ .

Let  $\alpha \in \mathbb{R}$ . Then a  $q$ -real number  $[\alpha]_q$  is defined by

$$[\alpha]_q := \frac{1-q^\alpha}{1-q},$$

where  $\lim_{q \rightarrow 1} \frac{1-q^\alpha}{1-q} = \alpha$ .

We introduce for  $k \in \mathbb{N}$ :

$$(a; q)_0 = 1, (a; q)_n = \prod_{k=0}^n (1 - q^k a), (q; a)_\infty = \lim_{n \rightarrow \infty} (a, q)_q^n, \text{ and } (a; q)_\alpha = \frac{(a; q)_\infty}{(q^\alpha a; q)_\infty}.$$

The  $q$ -analogue of the power function  $(a - b)_q^\alpha$  is defined by

$$(a - b)_q^\alpha := a^\alpha \frac{(\frac{a}{b}; q)_\infty}{(q^{\alpha} \frac{a}{b}; q)_\infty}.$$

$$\text{Notice that } (a - b)_q^\alpha = a^\alpha (\frac{a}{b}; q)_\alpha.$$

The  $q$ -analogue of the binomial coefficients  $[n]_q!$  are defined by

$$[n]_q ! := \begin{cases} 1, & \text{if } n = 0, \\ [1]_q \times [2]_q \times \cdots \times [n]_q, & \text{if } n \in \mathbb{N}, \end{cases}$$

The gamma function  $\Gamma_q(x)$  is defined by

$$\Gamma_q(x) := \frac{(q; q)_\infty}{(q^{x}; q)_\infty} (1 - q)^{1-x},$$

for any  $x > 0$ . Moreover, it yields that

$$\Gamma_q(x)[x]_q = \Gamma_q(x + 1).$$

**Definition 1.** The Riemann-Liouville  $q$ -fractional integrals  $I_{q,a+}^\alpha f$  of order  $\alpha > 0$  are defined by

$$(I_{q,a+}^\alpha f)(x) := \frac{1}{\Gamma_q(\alpha)} \int_a^x (x - qt)_q^{\alpha-1} f(t) d_q t.$$

**Definition 2.** The Riemann-Liouville fractional  $q$ -derivative  $D_{q,a+}^\alpha f$  of order  $\alpha > 0$  is defined by

$$(D_{q,a+}^\alpha f)(x) := \left( D_{q,a+}^{[\alpha]} I_{q,a+}^{[\alpha]-\alpha} f \right) (x).$$

The  $q$ -analogue differential operator  $D_q f(x)$  is

$$D_q f(x) := \frac{f(x) - f(qx)}{x(1-q)},$$

and the  $q$ -derivatives  $D_q^n(f(x))$  of higher order are defined inductively as follows:

$$D_q^0(f(x)) := f(x), \quad D_q^n(f(x)) := D_q(D_q^{n-1} f(x)), (n = 1, 2, 3, \dots)$$

Our main result:

**Theorem 3.** Assume that the following conditions holds true.

C1. Assume that  $f(t, u(t))$ . In addition, assume that be increasing function respect to the second variable, means where  $[0, 1] \times [-\tilde{r}, \infty)$ .

C2. Let  $0 < \lambda < 1$  be a constant and  $0 < y < \tilde{r}$ , then there exist  $\varphi(\lambda) > \lambda$  such that  
 $f(t, \lambda x + (\lambda - 1)y) \geq \varphi(\lambda)f(t, x)$ .

C3. Last condition is positivity of  $f(t, 0)$ , means  $f(t, 0) > 0$ , specifically  $f(t, 0) \neq 0$  for all possible  $0 \leq t \leq 1$ .

Then following q-fractional differential equation problem with given initial values has a unique solution. In addition, following sequence shows successive approximation approach for the solution.

$$\begin{cases} (D_q^\alpha u)(t) + f(t, u(t)) = b & t \in (0, 1), \\ u(0) = (D_q u)(0) = 0 & (D_q u)(1) = \beta(D_q u)(\xi) \end{cases} \quad \alpha \in (2, 3)$$

$$v_n(t) = \int_0^1 G(t, qs) f(s, v_{n-1}(s)) d_q s + \frac{\beta t^{\alpha-1}}{(1 - \beta \zeta^{\alpha-2}) ([\alpha - 1]_q)} \int_0^1 H(\zeta - qs) f(s, v_{n-1}(s)) d_q s - r(t).$$

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## AUTOMORPHISMS OF THE UNIVERSAL MULTIPLICATIVE ENVELOPING ALGEBRAS OF FINITE DIMENSIONAL DUAL LEIBNIZ ALGEBRAS WITH ZERO MULTIPLICATION

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Let  $K$  be an arbitrary field. An algebra  $A$  over  $K$  is called *dual Leibniz algebra* if it satisfies the identity

$$(xy)z = x(yz) + x(zy). \quad (1)$$

In [1] J.-L. Loday proved that any dual Leibniz algebra with respect to multiplication

$$x \circ y = xy + yx$$

is an associative commutative algebra. A linear basis of free dual Leibniz algebras is also given in [1].

Let  $A$  be an arbitrary dual Leibniz algebra over  $K$ . Let

$$L_A = \{L_x | x \in A\}$$

and

$$R_A = \{R_x | x \in A\}$$

be two isomorphic copies of the vector space  $A$  with the fixed isomorphisms  $A \rightarrow L_A (x \mapsto L_x)$  and  $A \rightarrow R_A (x \mapsto R_x)$ . The universal (multiplicative) enveloping algebra  $U(A)$  is an associative algebra with the identity 1 generated by the two vector spaces  $L_A$  and  $R_A$  satisfying the defining relations

$$\begin{aligned} R_x R_y &= R_{xy+yx}, \\ (2) \end{aligned}$$

$$R_x L_y = L_y R_x + L_x, \quad (3)$$

$$L_{xy} = L_x L_y + L_x R_y, \quad (4)$$

for all  $x, y \in A$ . Recall that every dual Leibniz  $A$ -bimodule  $M$  can be regarded as a left  $U(A)$ -module with respect to the action

$$L_a m = am, R_a m = ma, a \in A, m \in M$$