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The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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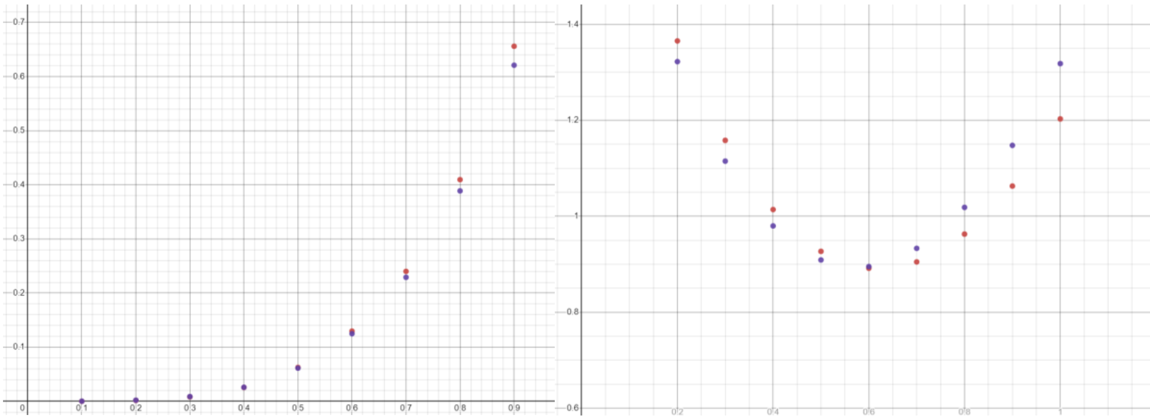

a) 1-мысал b) 2-мысал

1-сурет. Екі мысалдың жалпы және жуық шешімдерінің графикалық көрінісі.

Қорытындылай келе, дифференциалдық теңдеулердің сандық шешімінің дәлдігін бақылау және қатені бақылау маңызды. Бұған дәлірек сандық әдістерді қолдану, уақыт қадамын азайту немесе бастапқы жағдайларды жақсарту арқылы қол жеткізуге болады.

Шектеулерге қарамастан, дифференциалдық теңдеулердің сандық шешімі ғылыми және инженерлік қосымшаларда маңызды құрал болып қала береді және аналитикалық жолмен шешілмейтін күрделі жүйелерді модельдеуге және талдауға мүмкіндік береді.

Қолданылған әдебиеттер тізімі

1. Турчак, Л.И. Плотников П.В. Основы численных методов: учебное пособие / Л.И. Турчак – 2-е изд., перераб. и доп. – М.: ФИЗМАТЛИТ, 2003 – 304 стр
2. Нұрахметов Д. Б. «Модельдеудің сандық әдістері: жаттығулар мен есептер», Астана, С. Сейфуллин атындағы Қазақ агротехникалық университеті, 2015, 130 б.
3. Филлипов А.Ф. Сборник задач по дифференциальным уравнениям. – Ижевск: НИЦ «Регулярная и хаотическая динамика», 2000, 176 стр

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SOME ESTIMATES OF NON-INCREASING REARRANGEMENT OF GENERALIZED FRACTIONAL MAXIMAL FUNCTION AND CONES GENERATED BY THEM

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Let $L_0 = L_0(R^n)$ be the set of all Lebesgue measurable functions $f : R^n \to C$; $\tilde{L}_0 = \tilde{L}_0(R^n)$ is the set of functions $f \in L_0$, for which the non-increasing rearrangement of the f^* is not identical to infinity. Non-increasing rearrangement f^* defined by the equality:

$$f^*(t) = \inf\left\{y \in [0,\infty) : \lambda_f(y) \le t\right\}, \quad t \in R_+ = (0,\infty),$$

where

$$\lambda_f(y) = \mu_n\left\{x \in R^n : \left|f(x)\right| > y\right\}, \quad y \in [0,\infty)$$

is the Lebesgue distribution function [1].

The function $f^{**} : (0,\infty) \to [0,\infty]$ is defined as

$$f^{**}(t) = \frac{1}{t}\int_0^t f^*(\tau)d\tau; \quad t \in R_+$$

We define the following class of function.

Definition 1. Let $R \in (0,\infty]$. A function $\Phi : (0,R) \to R_+$ belongs to the class $B_n(R)$ if the following conditions hold:

(1) Φ decreases and is continuous on $(0,R)$;

(2) There exists a constant $C \in R_+$ such that

$$\int_0^r \Phi(\rho)\rho^{n-1}d\rho \le C\Phi(r)r^n, \ r \in (0,R).$$

For example,

$$\Phi(\rho) = \rho^{\alpha-n} \in B_n(R), \ (0<\alpha<n); \quad \Phi(\rho) = \ln\frac{eR}{\rho} \in B_n(R), \ R \in R_+.$$

For $\Phi \in B_n(R)$ the following estimate also holds:

$$\int_0^r \Phi(\rho)\rho^{n-1}d\rho \ge n^{-1}\Phi(r)r^n, \ r \in (0,R).$$

Therefore

$$\int_0^r \Phi(\rho)\rho^{n-1}d\rho \cong \Phi(r)r^n, \ r \in (0,R).$$

$$\Phi(\rho) \in B_n(R) \Rightarrow \{0 \le \Phi \downarrow; \ \Phi(r)r^n \uparrow, \ r \in (0,R)\}.$$

Definition 2. Let $\Phi : R_+ \to R_+$, E is rearrangement invariant space. The *generalized fractional-maximal function* $M_\Phi f$ is defined for the function $f \in E(R^n) \cap L_1^{loc}(R^n)$ by the equality

$$(M_\Phi f)(x)=\sup_{r>0}\Phi(r)\int_{B(x,r)} f(y)dy\,,$$

where $B(x,r)$ is a ball with the center at the point x and radius r. That is, consider the operator $M_\Phi : L_1^{loc}(R^n)\to L_0(R^n)$.

In the case $\Phi(r)=r^{\alpha-n}$, $\alpha\in(0,n)$ we obtain the classical fractional-maximal function $M_\alpha f$ [3]:

$$(M_\alpha f)(x)=\sup_{r>0}\frac{1}{r^{n-\alpha}}\int_{B(x,r)}|f(y)|dy$$

We denote by $M_E^\Phi=M_E^\Phi(R^n)$ the set of the functions u, for which there is a function $f\in E(R^n)$ such that

$$u(x)=(M_\Phi f)(x)\,,$$

$$\|u\|_{M_E^\Phi}=\inf\left\{\|f\|_E : f\in E(R^n),\, M_\Phi f=u\right\}.$$

Theorem 1. Let $\Phi\in B_n(\infty)$. Then there exist a positive constant C depending from n such that

$$(M_\Phi f)^*(t)\le C\sup_{t<s<\infty} s\Phi(s^{1/n})f^{**}(s)\,,\ \ t\in(0,\infty)$$

for every $f\in L_1^{loc}(R^n)$. This **i**nequality is sharp in the sense that for every $\varphi\in L_0^+(0,\infty;\downarrow)$ there exists a function $f\in L_1(R^n)$ such that $f^*=\varphi$ a.e. on $(0,\infty)$ and

$$(M_\Phi f)^*(t)\ge C\sup_{t<s<\infty} s\Phi(s^{1/n})f^{**}(s)\,,\ \ t\in(0,\infty)\,.$$

Theorem 2. Let $\Phi\in B_n(\infty)$. Then there exist a positive constant C depending from n such that

$$(M_\Phi f)^{**}(t)\le C\sup_{t<s<\infty} s\Phi(s^{1/n})f^{**}(s)\,,\ \ t\in(0,\infty)$$

for every $f\in L_1^{loc}(R^n)$.

Definition 3. Define $\Im_T=\mathrm{K(T)}$ for $T\in(0,\infty]$ as a set of cones considering from measurable non-negative functions on $(0,\mathrm{T})$, equipped with positive homogeneous functionals $\rho_{KM}(T)\colon \mathrm{K(T)}\to[0,\infty)$ with properties:

(1) $h\in \mathrm{K(T)}$, $\alpha\ge 0\Rightarrow \alpha\mathrm{h}\in\mathrm{K(T)}$, $\rho_{K(T)}(\alpha h)=\alpha\rho_{K(T)}(h)$;

(2) $\rho_{K(T)}(h)=0\Rightarrow h=0$ almost everywhere on $(0,\mathrm{T})$.

Definition 4 [4]. Let $\mathrm{K(T)}$, $M(\mathrm{T})\in\Im_T$. The cone $\mathrm{K(T)}$ covers the cone $M(\mathrm{T})$ (notation: $M(\mathrm{T})\prec K(\mathrm{T})$) if there exist $C_0=C_0(T)\in R_+$, and $C_1=C_1(T)\in[0,\infty)$ with $C_1(\infty)=0$ such that for each $h_1\in M(\mathrm{T})$ there is $h_2\in K(\mathrm{T})$ satisfying

$$\rho_{K(T)}(h_2) \le C_0\rho_{M(T)}(h_1), \quad h_1(t) \le h_2(t) + C_1\rho_{M(T)}(h_1), t \in (0,T)$$

The equivalence of the cones means mutual covering:

$$M(\mathrm{T}) \approx \mathrm{K(T)} \Leftrightarrow \mathrm{M(T)} \prec \mathrm{K(T)} \prec \mathrm{M(T)}$$

Let E is rearrangement-invariant space (briefly: RIS). We consider the following four cones of decreasing rearrangements of generalized fractional maximal functions equipped with homogeneous functionals, respectively:

$$K_1 \equiv K_1 M_E^{\Phi} := \{h \in L^+(\mathbb{R}_+): h(t) = u^*(t), \ \ t \in \mathbb{R}_+, \ \ u \in M_E^{\Phi}\}$$

$$\rho_{K_1}(h) = inf\left\{\|u\|_{M_E^{\Phi}}: u \in M_E^{\Phi};\ u^*(t) = h(t), \ \ t \in \mathbb{R}_+\right\}$$

$$K_2 \equiv K_2 M_E^{\Phi} := \{h \in L^+(\mathbb{R}_+): h(t) = u^{**}(t), \ \ t \in \mathbb{R}_+, \ \ u \in M_E^{\Phi}\}$$

$$\rho_{K_2}(h) = inf\left\{\|u\|_{M_E^{\Phi}}: u \in M_E^{\Phi};\ u^{**}(t) = h(t), \ \ t \in \mathbb{R}_+\right\}$$

This means that the cones K_1 and K_2 consist of non-increasing rearrangements of generalized fractional maximal functions. Consider the following cone of non-increasing rearrangements equipped with a homogeneous functional

$$K_3 \equiv K_3 M_E^{\Phi} = \left\{h \in L^+(\mathbb{R}_+): h(t) = \sup_{t<\tau<\infty} \tau\Phi(\tau^{1/n})u^{**}(\tau), \ \ t \in \mathbb{R}_+, \ \ u \in M_E^{\Phi}\right\}$$

$$\rho_{K_3}(h) = inf\left\{\|u\|_{E(\mathbb{R}^n)}, u \in E(\mathbb{R}^n): h(t) = \sup_{t<\tau<\infty} \tau\Phi(\tau^{1/n})u^{**}(\tau), \ \ t \in \mathbb{R}_+\right\}$$

Theorem 3. Let $\Phi \in B_n(R)$. Then

$$K_1 \approx K_2 \approx K_3.$$

Banach function space (BFS) $X = X(\rho),$ generated by a rearrangement-invariant functional norm $\rho,$ is called a rearrangement -invariant space.

Denote by $\tilde{X}(\mathbb{R}_+)$ the representation of Luxembourg $\|f\|_{X(\mathbb{R}^n)} = \|f\|_{\tilde{X}(\mathbb{R}_+)}$.

Theorem 4. Let $\Phi \in B_n(\infty)$. The embedding $M_E^{\Phi}(\mathbb{R}^n) \hookrightarrow X(\mathbb{R}^n)$ is equivalents to embedding $KM_E^{\Phi}(\mathbb{R}_+) \mapsto \tilde{X}(\mathbb{R}_+)$.

Similar questions for the space of generalized Riesz potentials are considered in [2] and [3].

References

3. Bennett C., Sharpley R., Interpolation of operators. Pure and applied mathematics, Volume 129. Boston, MA: Acad. Press Inc., 1988.

4. Goldman M.L. On the cones of rearrangements for generalized Bessel and Riesz potentials // Complex Variables and Elliptic Equations 55:8-10, 2010, p.817-832.

5. Goldman M.L. Optimal embeddings of generalized Bessel and Riesz potentials // Trudy Matematicheskogo Instituta imeni V.A. Steklova, 2010, Vol. 269, pp. 91–111.

6. Bokayev N. A., Goldman M. L., Karshygina G. Zh. Cones of functions with monotonicity conditions for generalized Bessel and Riesz potentials. Math.Notes. 2018. Vol. 104, No. 3, pp. 356–373.

UDC 517

MODULI OF SMOOTHNESS AND LIOUVILLE-WEYL DERIVATIVES

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Let $L_p = L_p[0,2\pi]$ $(1 \le p < \infty)$ be a space of 2π periodic measurable functions, for which $|f|^p$ is integrable, and $L_\infty = C[0,2\pi]$ is a space of 2π periodic continuous functions with $\|f\|_\infty = \max\{|f(x)|, 0 \le x \le 2\pi|\}$.

Let the function $f \in L_1$ have the following Fourier series

$$f(x) \approx \sigma(f) := \frac{a_0(f)}{2} + \sum_{\nu=1}^{\infty}\left(a_\nu(f)\cos\nu x + b_\nu(f)\sin\nu x\right) \tag{1}$$

The transformed Fourier series of (1) is given by

$$\sigma(f,\lambda,\beta) := \sum_{\nu=1}^{\infty}\lambda_\nu\left(a_\nu(f)\cos\left(\nu x + \frac{\pi\beta}{2}\right) + b_\nu(f)\sin\left(\nu x + \frac{\pi\beta}{2}\right)\right),$$

where $\beta \in \Re$ and $\lambda = \{\lambda_n\}$ is a sequence of positive numbers.

The function whose series coincides with $\sigma(f,\lambda,\beta)$ we will call (λ,β) derivative of the function f and denote it by $f^{(\lambda,\beta)}$. If $\lambda_n = n^r, r > 0, \beta = r$, then $f^{(\lambda,\beta)} = f^{(r)}$ -fractional derivatives in the sense of Weyl and for $\lambda_n = n^r, r > 0, \beta = r+1$, $f^{(\lambda,\beta)} \equiv \bar{f}^{(r)}$, where $\overline{f}$ is conjugate function f.

The main task is to find estimates for the modulus of smoothness of a function with a transformed Fourier series in terms of the moduli of smoothness of the original function for different parameters $1 < p < q \le \infty$, for the average value of a bounded variational sequence.

Definition 1. A sequence $\lambda := \{\lambda_n\}_{n=1}^{\infty}$ belongs to the MVBVS class (mean value of a bounded variational sequence) if there exists $\mu \ge 2$ and the following condition is satisfied

$$\sum_{k=n}^{2n}|\lambda_k - \lambda_{k+1}| \le C\left(\frac{1}{n}\sum_{k=\frac{n}{\mu}}^{\mu n}|\lambda_k|^p\right)^p,$$

for all integers *n*, where the constant *C* does not depend on *n*.
Let $\omega_k(f,\delta)_p$ be the moduli of smoothness of the natural order $k \in N$ of the function , i.e.,