**Eurasian Journal of Physics and Functional Materials**

**2023, 7(3), 155-165**

# Computational scheme for breakup of $^{11}$Be on light targets

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DOI: **10.32523/ejpfm.2023070303**
Received: 24.09.2023 - after revision

We investigate the Coulomb breakup of the $^{11}$Be halo nuclei on a light (carbon) target within non-perturbative time-dependent approach including the low-lying resonance $5/2^+$ of $^{11}$Be (E= 1.232 MeV). We had found considerable contribution of the low-lying resonances ( $5/2^+$ , $3/2^-$ and $3/2^+$ ) to the breakup cross section of $^{11}$Be on a heavy ( $^{208}$Pb) target at our previous calculations. The developed computational scheme is extended to study the breakup of $^{11}$Be on a light target. This work is the initial step, where the convergence and accuracy of the computational scheme is tested.

**Keywords:** light halo nucleus; Coulomb breakup; breakup cross section; low-lying resonances, computational scheme

## Introduction

Exotic nuclei with neutron halos are of great interest due to unusual properties such as the increased radius of matter density distributions which is significantly larger than that of their isobars, exceptionally low binding energy for one or two neutrons. The unusual properties of such nuclei with neutron halos manifest themselves in direct nuclear reactions. The unusual size of these nuclei is now understood as a consequence of their exceptionally low binding energy for one or two neutrons [1, 2].

Due to the weak coupling, valence neutrons can tunnel far from the nuclear core and, with a high probability, be in the classically forbidden region [3, 4]. Thus, they form a sort of neutron halo around the nucleus, which exhibits the same characteristics (e.g. size, density, etc.) as stable nuclei. The second type of nuclear halo is the proton halo. They are also possible, though less probable due to the existence of a Coulomb barrier, which prevent the formation of a long tail in the nuclear density. Far from stable, halo nuclei cannot be studied by conventional spectroscopic methods and may rely on indirect methods such as nuclear reactions to obtain information about their exotic structure [5].

Typically, the structure of weakly bound exotic nuclei is studied using elastic scattering [6], breakup reactions [7], knockout [8], and transfer reactions [9]. In the case of weakly bound nuclei with a neutron halo, the breakup reaction is more preferable.

In this type of reaction, the nucleus being explored is sent to a target and the events in which the projectile breaks up into its component parts, in our case for example: halo-neutrons and the core-nucleus, are analyzed, thereby revealing its structure. To obtain valuable information about nuclear structure from reaction data, an accurate reaction model combined with a realistic description of the projectile is needed.

Various theoretical approaches have been developed to analyse breakup reactions: the coupled-channel technique with a discretised continuum (CDCC) [10, 11], the time-dependent model (TD) [12-16], methods based upon the eikonal approximation [17, 18] and others.

In this paper, the time-dependent Schrödinger equation is integrated with a non-perturbative algorithm on a three-dimensional spatial grid, where it is assumed that the projectile moves along a classical trajectory and its interaction with the target appears due to the difference between the Coulomb and nuclear interactions around the target. In our previous work [12], the time-dependent approach developed in [13, 14], was extended and successfully applied to the breakup of the $^{11}$Be halo nuclei [13, 14] on a heavy target ($^{208}$Pb) in a wide beam energy range (70-5 MeV/nucleon) including low-lying resonant states of $^{11}$Be. Here, we extend this approach for description of halo nuclear breakup on light targets.

An attractive feature of this method is its flexibility in choosing the interaction between the halo-nucleus, core and target, and the projectile trajectory is simultaneously classically described in the Schrödinger equation for a weakly coupled projectile halo-nucleus, which makes it possible to expand the applicability of the computational scheme to the breakup of the halo nuclei on a light target taking into account bound and resonance states in different partial and spin states of $^{11}$Be.

In present work the convergence of the computational scheme within the time-dependent model, the demanded accuracy and the convergence, over the selected value of the parameters is discussed, which is an important element for numerical calculations of breakup cross sections in this approach.

# Theoretical description of the model

The neutron halo effect is due to the presence of weakly bound states of neutrons located near the continuum. One of the interesting quantum systems with a simple structure is the $^{11}$Be halo nucleus. Indeed, their bound states can be described as a core of $^{10}$Be with which the neutron is weakly bound. In order to correctly describe the breakup process of breakup reaction $^{11}$Be+$^{12}$C $\rightarrow^{10}$Be+n+$^{12}$C, it is necessary to formulate the problem in a non-perturbative way. In this work, the time-dependent Schrödinger equation is integrated with a non-perturbative algorithm on a three-dimensional spatial grid, where it is assumed that the projectile moves along a classical trajectory and its interaction develops due to the difference between the Coulomb and nuclear interactions around the target.

The time-dependent Schrödinger equation describing the dynamics of the neutron motion relative to $^{10}$Be core in the process of collision with a light target ($^{12}$C)

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = H(\mathbf{r},t)\Psi(\mathbf{r},t) = [H_0(\mathbf{r}) + V_C(\mathbf{r},t)]\Psi(\mathbf{r},t) \tag{1}$$

where $\Psi(\mathbf{r},t)$ is the wave packet relative the $^{10}$Be-core. In this equation the Hamiltonian

$$H_0(\mathbf{r}) = -\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}} + V(r) \tag{2}$$

describes the relative motion of halo nucleon and core with reduced mass $\mu = m_n m_c/M$, which consist of neutron ($m_n$), $^{10}$Be-core ($m_c$) and $^{11}$Be (M=$m_n + m_c$) masses. The *V(r)* interaction potential between the $^{10}$Be-core and neutron consists of the spherically symmetric part $V_l(r)$ and the spin-orbit interaction $V_l^s(r)(\boldsymbol{ls})$, which was described in detail at [12-14].

The time-dependent Coulomb potential

$$V_C(\mathbf{r},t) = \frac{Z_c Z_t e^2}{|m_n\mathbf{r}/M + \mathbf{R}(t)|} - \frac{Z_c Z_t e^2}{R(t)}, \tag{3}$$

is written in the center-of-mass system associated with $^{11}$Be. Here $Z_c$ and $Z_t$ are charge numbers of the core and target, respectively, $\mathbf{R}(t)$ represents the position of the target relative to the center of mass of the projectile, which is determined by the initial velocity and impact parameter as $\mathbf{R}(t) = \mathbf{b} + \mathbf{v}_0 t$ [13, 19].

When $^{11}$Be is far from the target ($t \longrightarrow \pm\infty$), the Coulomb potential (3) is zero. It acquires a maximum value, as seen in formula (3), if the target and the projectile approach at a minimum distance ($t$=0). After the collision, this time-dependent interaction $V_C(r,t)$ vanishes. Our task is to integrate the equation (1) from the initial moment of time ($t = T_{in}$), where the target and the projectile are at a large distance, and then they approach each other and again scatter over large distances ($t = T_{out}$). We write down the system of two bodies $^{10}$Be + n, between them there is a nuclear interaction $V_n(r,t)$, which is described below.

When a system of $^{10}$Be + n approach the target ($^{12}$C), the interaction $V_C(r,t)$ between the target and the projectile must also be taken into account (Eq.(3)).

As it has been shown in our previous studies of Coulomb breakup of $^{11}$Be at a heavy ($^{208}$Pb) target at a higher beam energies (70 MeV/nucleon), the time-dependent interaction between the target and projectile was accepted as a purely Coulombic. On the other hand, for lower collision energies the nuclear effect of the projectile-target interaction in the breakup cross sections is significant [12]. In this work we evaluate this effect in the case of light target ($^{12}$C) taking into account the nuclear part $\Delta V_N(\mathbf{r},t) = V_{cT}(r_{cT}) + V_{nT}(r_{nT})$ of the interaction between the target and projectile:

$$V(\mathbf{r},t) = V_C(\mathbf{r},t) + \Delta V_N(\mathbf{r},t) \tag{4}$$

In the present study, when the breakup of $^{11}$Be halo nucleus is induced by a light target ($^{12}$C), the dissociation reaction is dominated by the nuclear interactions [14]. Here optical potentials $V_{cT}$ and $V_{nT}$ with the core-target $\mathbf{r}_{cT}(t) = \mathbf{R}(t) + m_n\mathbf{r}/M$ and neutron-target $\mathbf{r}_{nT}(t) = \mathbf{R}(t) - m_c\mathbf{r}/M$ relative variables, have the form:

$$V_{xT}(r_{xT}) = -V_x f(r_{xT}, R_R, a_R) - iW_x f(r_{xT}, R_I, a_I) \tag{5}$$

with Woods-Saxon form factors $f(r_{xT}, R_R, a_R) = 1/(1 + exp(r_{xT} - R)/a)$, where $x$ stands for either core or neutron. More details of construction of the optical potential (5) are given in [12].

The analytical expression of such potentials is obtained by selecting the parameters of general form factors so as to fit the calculated scattering cross sections onto experimental data. A compilation of optical potentials for different projectiles and targets can be found in Ref. [20-23]. We use here the parameters of the optical potentials (5) from the table III of the work [24], which are given in Table I of the present paper. The potential of the core-target interaction is proposed by Al-Khalili, Tostevin, and Brooke [25] consistent with the elastic scattering of $^{10}$Be on $^{12}$C (denoted as ATB in the following) [24]. For the n-$^{12}$C interaction, commonly used parametrization of Becchetti and Greenlees [21] (BG) is considered. The expediency of using this parameterization of the nuclear part of the interaction was analyzed in the work [24].

Table 1.
Parameters of the core-target [25] and neutron-target [21] optical potentials at 67 MeV/nucleon.

| $V_{l,even}(MeV)$ | $V_x(MeV)$ | $W_x(MeV)$ | $R_R(fm)$ | $R_I(fm)$ | $a_R(fm)$ | $a_I(fm)$ |
|---|---|---|---|---|---|---|
| $^{10}$Be | 123.0 | 65.0 | 3.33 | 3.47 | 0.80 | 0.80 |
| $n$ | 34.54 | 13.4 | 2.68 | 2.88 | 0.75 | 0.58 |

The total breakup cross section is calculated as a function of the relative energy $E$ between the emitted neutron and the core nucleus including neutron interaction with the core in the final state of the breakup process [12, 14, 26] by the formula as in [13]

$$\frac{d\sigma_{bu}(E)}{dE} = \frac{4\mu k}{\hbar^2}\int_{b_{min}}^{b_{max}} \sum_{j=l+s} \sum_{lm} | \int \phi_{lj}(kr) Y_{lm}(\hat{r}) \Psi(\mathbf{r}, T_{out}) d\mathbf{r}|^2 bdb \quad (6)$$

Here $\phi_{ljm}(kr)$ is the radial part of the eigenfunction of the Hamiltonian $H_0(\mathbf{r})$ (2) in the continuum spectrum ($E = k^2\hbar^2/(2\mu) > 0$), normalized to spherical Bessel function $j_l(kr)$ as $kr \to \infty$ if if $V(r) = 0$. To find the states of the continuous spectrum of problem $H_0(\mathbf{r})\phi_{ljm}(\mathbf{r})(E, \mathbf{r}) = E\phi_{ljm}(E, \mathbf{r})$, we used the method of reducing the scattering problem to a boundary value problem, described in the work [27]. Summation over ($l, m$) in (6) includes all 16 partial waves up to $l_{max}$ = 3 inclusive, as in [12].

Since the wave functions $\phi_{lj}(\mathbf{r})$ of the continuum spectrum of the Hamiltonian Eq.(2) are orthogonal to the states of the discrete spectrum of the same Hamiltonian, the elimination $\Psi_{bu}(\mathbf{r}, t)\rangle = (1 - \sum_{ljm \in bound} |\phi_{ljm}(\mathbf{r})\rangle\langle\phi_{ljm}(\mathbf{r})|)|\Psi(\mathbf{r}, t)\rangle$ of the bound states from the neutron wave packet after collision with the target [13] is not required here.

A more detailed description of the stationary Schrödinger solution, as well as the expansion in angular variables, the transition to a quasi-uniform radial grid, and all the details of numerical integration can be found in previous works [12-14, 26]. The parameterization of potential between the neutron and $^{10}$Be core and how the resonant states were included in the analysis of the breakup reaction were discussed at [28] in detail.

# Convergence of the computational scheme and accuracy of the approach

This work is the initial stage in the study of the breakup of the halo nucleus on a light target ($^{12}$C) by the time-dependent approach and one of the objectives of this work is to investigate the convergence of the numerical scheme. In this section, the convergence and accuracy of the numerical technique is discussed.

Time evolution in the computing the breakup cross section (6) starts at initial time $T_{in}$ and stops at final time $T_{out}$ by iteration over $N_T$ time steps $\Delta$t as explained in [14]. The initial (final) time $T_{in}$ ($T_{out}$) has to be sufficiently big $|T_{in}|, T_{out} \to +\infty$, fixed from the demand for the time-dependent potential $V_C(\mathbf{r}, t)$ to be negligible at the beginning (end) of the time evolution at $t = T_{in}$ ($T_{out}$).

The unitarity of the evolution operator of the computational approach for integration of the time-dependent Schrödinger equation (1) was shown in [13, 14]. It ensures that the normalization of the neutron wave-packet to unity is preserved with the required accuracy at chosen $-T_{in} = T_{out}$ = 20 $\hbar$/MeV, the time step $\Delta$t was 0.01 $\hbar$/MeV following the investigation performed in Ref. [12-14, 27, 29] for the breakup of halo nucleus of $^{11}$Be on a heavy target. In this work for the breakup reaction of $^{11}$Be on a light ($^{12}$C) target, the time interval chosen for this analysis is fixed by $-T_{in} = T_{out}$ = 10 $\hbar$/MeV. As it is illustrated in Fig.1 the step of integration over the time variable $\Delta$t = 0.01 $\hbar$/MeV and initial (final) time

$-T_{in} = T_{out}$ = 10 $\hbar$/MeV in computing the breakup cross sections on a carbon target keeps the demanded order of accuracy (of a few percents).

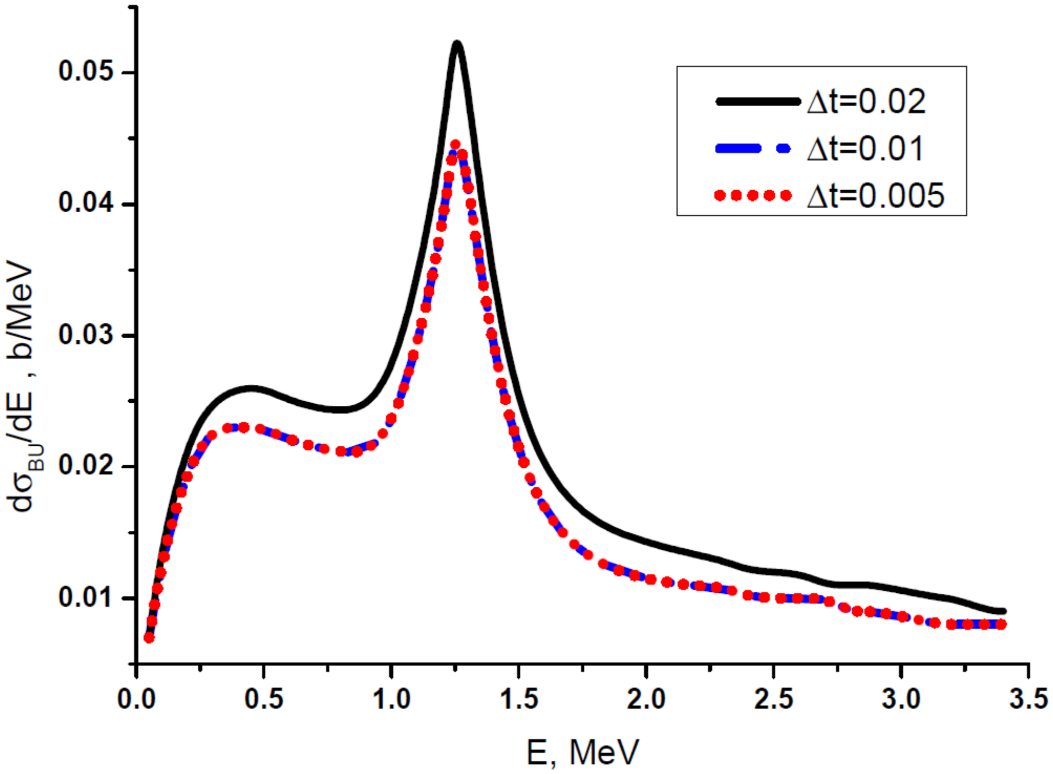


Figure 1. The convergence by the time step ( Δ t= 0.005, 0.01, 0.02 $\hbar$ /MeV) and boundary of the integral (6) $-T_{in} = T_{out}$ computing for $^{11}$Be+ $^{12}$C $\rightarrow^{10}$ Be+n+ $^{12}$C with including one resonance $5/2^+$ and two bound states $1/2^+$ and $1/2^-$ . Calculations were performed for 67 MeV/nucleon initial energy, $r_m$ =600 fm, N= 225.

For discretizing with respect to the radial variable $r$, a sixth-order (seven point) finite-difference approximation on a quasi-uniform grid has been used on the interval $r \in [0, r_m]$ with fixed $r_m$ . The grid has been realized by the mapping $r \rightarrow x$ of the initial interval onto $x \in [0, 1]$ by the formula $r = r_m(e^{8x} - 1)/((e^8 - 1))$ [12, 28].

The results of the computed breakup cross section $d\sigma(E), b_{max}/dE$ of the reaction $^{11}$Be+$^{12}$C $\rightarrow^{10}$Be+n+$^{12}$C at initial beam energy 67 MeV/nucleon on different radial meshes $N_r$ =500, 1000 and 2000 are presented in Fig.2 below. The calculations are performed for different relative energies $E$ (MeV) taking into account two bound states $1/2^+$ and $1/2^-$ and the low-lying resonance $5/2^+$ of $^{11}$Be with the position of peaks at $E = 1.232$ MeV. The dotted line represents the results obtained on a quasi-uniform grid with $N_r = 500$ ( Δ x= 0.002 fm) and $r_m = 600$ fm. It is demonstrated in graph that choosing of the radial mesh points $N_r = 500$ with $r_m = 600$ fm does not reach the convergence of integration over $r$ step Δ x. The solid line indicates the results calculated with $N_r = 2000$ mesh points (generated by the step Δ x =5*10$^{-3}$ and the edge at $r_m$ = 600 fm gives the accuracy of integration of the order of about a few percents following the results of Fig. 3. It is important to note that the radial Schrödinger equation $H_0(\mathbf{r})$ (2) is investigated on the same radial grid as the TDSE and the radial wave functions of the $^{11}$Be bound states are normalized to unity.

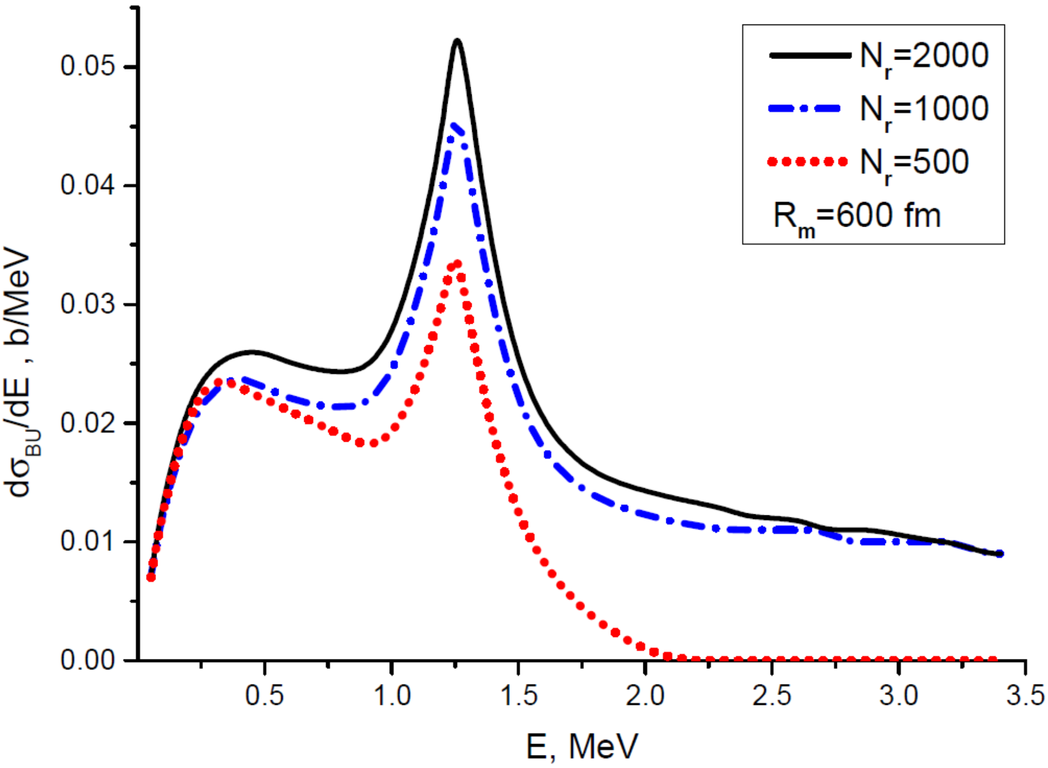


Figure 2. The convergence of the computed breakup cross section of the reaction $^{11}$Be+ $^{12}$C $\rightarrow^{10}$ Be+n+ $^{12}$C over the radial grid meshes $N_r$ = 500, 1000, 2000 with a fixed boundary of integration $r_m$ = 600 fm N=225, 67 MeV/nucleon.

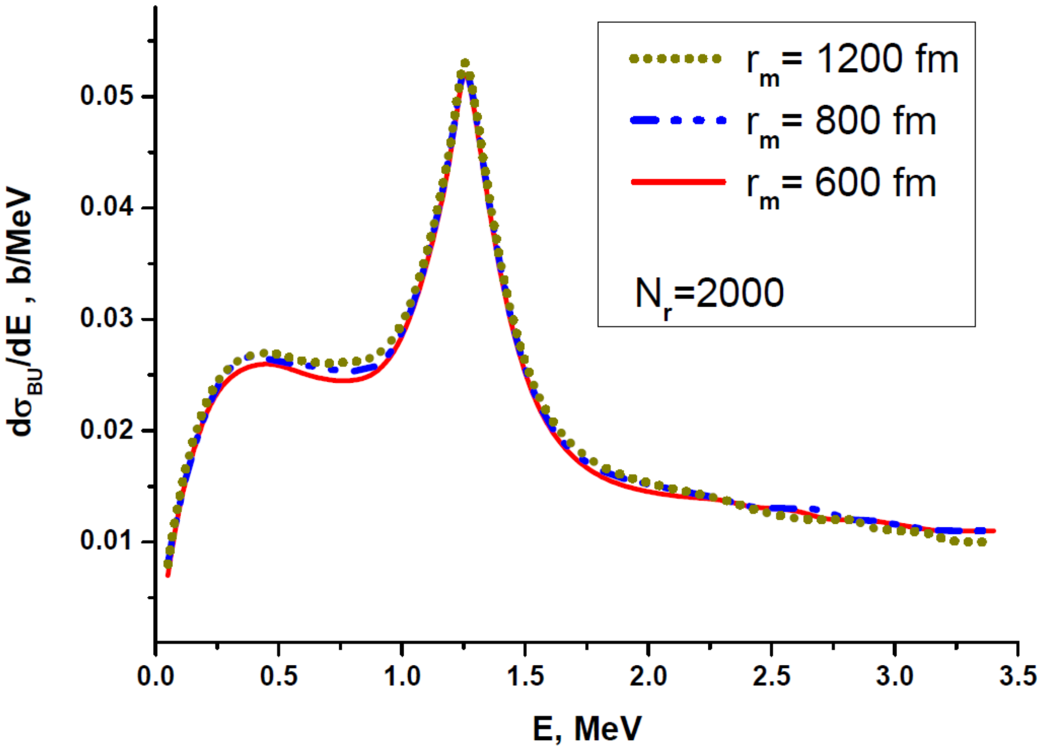


Figure 3. The convergence of the breakup cross section $d\sigma(E), b_{max}/dE$ over the boundary $r_m$ ( $r_m$ = 600, 800, 1200 fm) of the radial variable with the step $\Delta$ x =5*10 $^{-3}$ , N= 225, 67 MeV/nucleon.

In the calculation of the breakup cross section the choice of edges of integration over impact parameters $b_{min}$ and $b_{max}$ must be carefully tested. It was investigated in previous calculations at [12-14] that the integration over the interval [30 fm, 400 fm] gives about 60 % of the calculated cross section near the maximum. As it can be seen, the demanded accuracy (to be in order of one percent) in computing the integral (6) with pure Coulomb interaction (3) is achieved as $b_{min}$ = 12 fm and $b_{max}$ = 400 fm in the case of the breakup of $^{11}$Be in a heavy lead target [12-14]. Here the nuclear interaction effects were simulated by a cutoff $b_{min}$ = 12 fm of the impact parameters at intermediate beam energies. The inclusion of nuclear interaction $\Delta V_N(\mathbf{r},t)$ between the projectile and the target requires the reduction of the impact parameters cutoff to $b_{min}$ = 5 fm [12, 29].

For breakup reaction $^{11}$Be+$^{12}$C $\rightarrow^{10}$Be+n+$^{12}$C the evolution is computed from impact parameters $b_{min}$ = 0 fm up to $b_{max}$. The step $\Delta$b is chosen in order to ensure the convergence of the integral in Eq.(6). It varies from $\Delta$b= 0.25 fm at small b up to $\Delta$b= 2 fm at large b as in [24].

Table 2 illustrates the convergence of the integral (3) as a function of the upper bound $b_{max}$ for a few relative energies $E$. The total breakup cross section calculated for a collision energy of 67 MeV/nucleon taking into account bound and the low-lying resonance states $5/2^+$ of $^{11}$Be nucleus. As can be seen, the computed results with optical potential converge faster (at $b_{max}$ = 150 fm) and satisfied the demanded accuracy of the integral (6).

Table 2.
Convergence of the method for the breakup cross section $d\sigma(E), b_{max}/dE$ in (b/MeV) as a function of the impact parameter $b_{max}$ (fm) and energy $E$ (MeV).

| $b_{max}$ | $E$=0.05 | $E$=0.02 | $E$=0.4 | $E$=0.8 | $E$=1.0 | $E$=1.3 | $E$=1.6 | $E$=2.0 | $E$=2.4 | $E$=2.8 |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 0.002 | 0.004 | 0.005 | 0.004 | 0.004 | 0.007 | 0.002 | 0.001 | 0.001 | 0.001 |
| 12 | 0.004 | 0.011 | 0.015 | 0.018 | 0.022 | 0.045 | 0.014 | 0.012 | 0.011 | 0.011 |
| 30 | 0.006 | 0.018 | 0.022 | 0.023 | 0.026 | 0.048 | 0.016 | 0.014 | 0.012 | 0.011 |
| 70 | 0.007 | 0.021 | 0.025 | 0.024 | 0.027 | 0.048 | 0.016 | 0.014 | 0.012 | 0.011 |
| 100 | 0.007 | 0.022 | 0.026 | 0.024 | 0.027 | 0.048 | 0.016 | 0.014 | 0.012 | 0.011 |
| 150 | 0.008 | 0.022 | 0.026 | 0.025 | 0.027 | 0.048 | 0.016 | 0.014 | 0.012 | 0.011 |
| 200 | 0.008 | 0.022 | 0.026 | 0.025 | 0.027 | 0.048 | 0.016 | 0.014 | 0.012 | 0.011 |

Overall, the computational time is directly proportional to the numbers of angular $N$ and radial $N_r$. grid points. The splitting-up method gives a fast convergence with respect to the numbers of grid points $N$ (angular) and $N_r$ (quasi-uniform radial).

In order to investigate the convergence of the numerical scheme by the angular grid, we calculated the cross section $d\sigma(E), b_{max}/dE$ for a beam energy 67 MeV/nucleon at different angular grid points $N$ = 49, 81, 121, 169, 225 (the number of basis function $N = N_\theta \times N_\varphi$, more details see in [12-14]) with including two bound states and resonance $5/2^+$ of $^{11}$Be at 67 MeV/nucleon. As it is illustrated in fig.4, the approach achieves the convergence at $N$ = 169 ($N_\theta = N_\varphi = 13$).

One of the main task of our investigation is to extend the time-dependent approach for calculation of the breakup cross sections at low energy beams. Firstly, we investigate the convergence of computational scheme at low energies over the angular grid number $N$ at our previous works [12, 27, 29]. For this the calculation of breakup cross section $d\sigma(E), b_{max}/dE$ for a lower beam energies with demanded accuracy of the order of one percent, the basis should be extended from $N = 49$ to $N = 121$ (see [12]) due to the slowing down of convergence. For this reason we use here $N = 225$ ($N_\theta = N_\varphi = 15$) basis functions for breakup cross sections of $^{11}$Be on a $^{12}$C at lower beam energies.

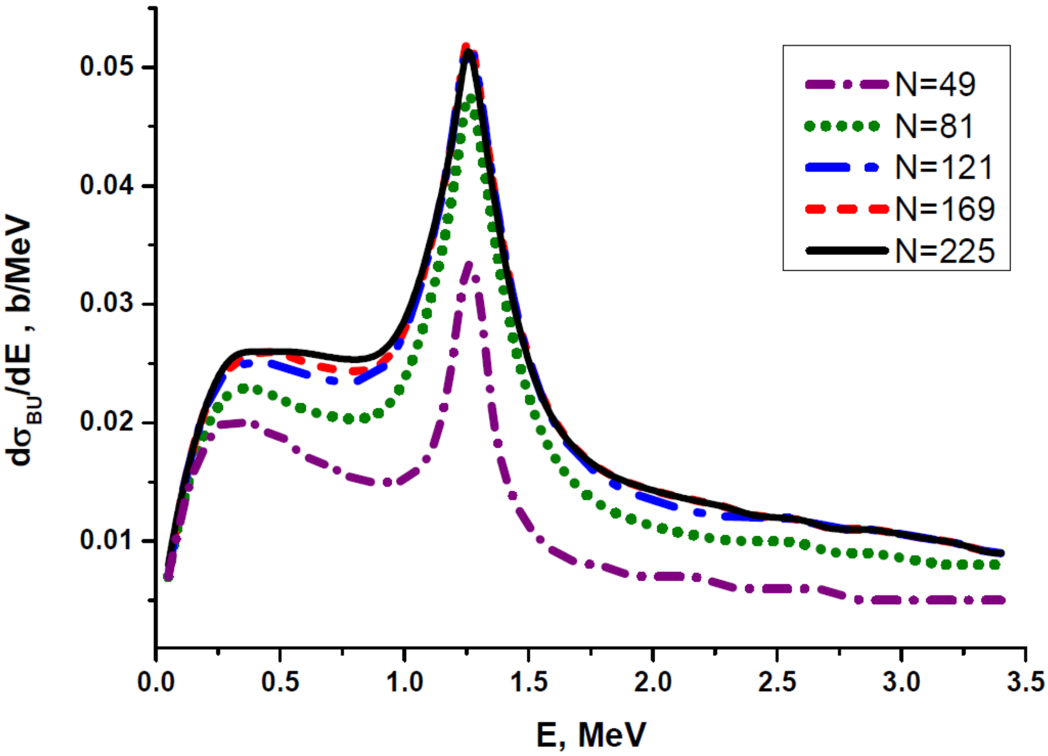


Figure 4. The convergence of the breakup cross section $d\sigma(E), b_{max}/dE$ over the number $N$ of angular grid points $N$ ($N = 49, 81, 121, 169, 225$) including resonant state $5/2^+$ of $^{11}$Be at 69 MeV/nucleon, for N=225, $N_r = 2000$, $r_m = 600$ fm.

Thus, the convergence of the computational scheme and accuracy of the method are demonstrated over all parameters of the integral (6) at 67 MeV/nucleon including two bound and low-lying resonance $5/2^+$ states of $^{11}$Be.

# Conclusion

In this work the breakup of the $^{11}$Be halo nuclei on light target in intermediate (67 MeV/nucleon) beam energy within non-perturbative time-dependent approach is investigated. In this approximation, the projectile is seen as evolving in a time-dependent potential which simulates its interaction with the light target $^{12}$C. The following time-dependent Schrödinger equation is solved numerically.

The parameterization of potential between the neutron and $^{10}$Be core is here adjusted not only to reproduce the two physical bound states of the nucleus, but also its low-lying resonant state ($5/2^+$) (for more details see [12, 29]). This resonance plays a significant role in the nuclear-induced breakup reaction. Its presence indeed leads to the occurrence of a narrow peak in the breakup cross section, which was first measured at RIKEN at 67 MeV/nucleon by Fukuda et al. [7].

Additionally, in this paper we dwelled in detail on the study of the convergence of the computational scheme. The optimal choice of convergence in terms of radial, and angular grids, time evolution and edges of the integral is shown as applied to the breakup cross section of a halo nuclei on a light target.

This work is part of a project to study the breakup of $^{11}$Be on a light target at low beam energies. A detailed analysis of the nuclear dominated breakup of $^{11}$Be on a light target halo nuclei including low-lying resonances ($5/2^+$, $3/2^-$ and $3/2^+$) of $^{11}$Be with penetrating into not-investigated so far region of low beam energies is planned.

## Acknowledgments

This research is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP13067742).

Additionally, authors note that the convergence of the computational scheme was analyzed with the algorithm obtained with support of the Russian Science Foundation under the Grant No. 20-11-20257.

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