$$\frac{1}{|A|^{1/2}}\left|\int_A f(x)dx\right| \leqq 5\left(\frac{2p(r-2)}{r(p-2)}\right)^{\frac{1}{p}\left(1-\frac{2}{r}\right)} p^{\left(1-\frac{1}{r}\right)\left(1-\frac{2}{p}\right)} N^{\frac{1}{2}-\frac{1}{p}} \|a\|_{l_{2,r}}.$$

Теорема 3

Натурал сандар жиынынан алынған кез келген n үшін G_n жиынын қарастырайық.

$$G_n = \{A \subset [0,1] : A = \bigcup_{k=1}^{n} [a_k, b_k] : 1 \leqq k \leqq n\},$$

онда $l_{2,r}$ кеңістігінен алынған кез келген $\{a_k\}_{k=1}^{\infty}$ тізбегі мен $f \sim \sum_{k\in Z} a_k \sin kx$ функциясы және $2 < r < \infty$ параметрі үшін келесі теңсіздік орындалады:

$$\sup_{A\subset G_n} \frac{1}{|A|^{\frac{1}{2}} \log_2(1+n)^{\frac{1}{2}-\frac{1}{r}}} \left|\int_A f(x)dx\right| \leqq 20\|a\|_{l_{2,r}}.$$

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UDC 51-7

A FRACTIONAL q-DIFFERENTIAL EQUATION WITH THE FRACTIONAL q-DERIVATIVE

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Let $0 < q < 1$. Then the q-analogue differential operator $D_q f(x)$ is [1]:

$$D_q f(x) := \frac{f(x)-f(qx)}{x(1-q)}.$$

and the q-derivatives $D_q^n(f(x))$ of higher order are defined inductively as follows:

$$D_q^0(f(x)) := f(x), \quad D_q^n(f(x)) := D_q\left(D_q^{n-1} f(x)\right), (n = 1,2,3, \dots)$$

The q-integral (or Jackson integral) $\int_a^b f(x) d_q x$ is defined by

$$\int_0^a f(x)d_qx := (1-q)a\sum_{m=0}^{\infty} q^m f(aq^m).$$

Let $n-1<\alpha\le n; n\in\mathbb{N}$. The Riemann-Liouville q-fractional integrals $I_{a+}^{\alpha}f$ of order $\alpha>0$ are defined by
$$\left(I_{q,a+}^{\alpha}f\right)(x) := \frac{1}{\Gamma_q(\alpha)}\int_a^x (x-qt)_q^{\alpha-1}f(t)d_qt.$$

We define the fractional q-derivative $D_{q,a+}^{\alpha}f$ as follows:

$$\left(D_{q,a+}^{\alpha}f\right)(x) := \left(D_q^n\left(I_{q,a+}^{n-\alpha}f\right)\right)(x).$$

In this work we give conditions for a unique global solution to the Cauchy type problem

$$\left(D_{q,a+}^{\alpha}y\right)(x) = f(x,y(x)), \quad n-1<\alpha\le n; n\in\mathbb{N}, \tag{1}$$

$$\lim_{x\to a+}\left(D_q^{k-\alpha}y\right)(x) = b_k, b_k\in\mathbb{R}, k=0,1,2,\dots n-1, \tag{2}$$

in the space $L_q^p[a,b] := \left\{f : \left(\int_a^b |f(x)|^p d_qx\right)^{\frac{1}{p}} < \infty, 1\le p<\infty\right\}$ which are the q-extensions of the main results given in [2, Proposition 2 and Theorem 1] (see also [3, Proposition 3.1, Proposition 3.2 and Theorem 3.1]).

Our main result reads:

Theorem. *Let $a>0$, $G\subset\mathbb{R}$ be an open set and $f(.,.) : [a,b]\times G\to\mathbb{R}$ be a function such that $f(x,y(x))\in L_q^1[a,b]$ for any $y\in G$ and satisfying the condition*

$$|f(x,y_1(x)) - f(x,y_2(x))| \le C|y_1(x)-y_2(x)|.$$

If $n-1<\alpha\le n, n\in\mathbb{N}$, $0\le\beta\le 1$, $\gamma=(n-\alpha)(1-\beta)$, $I_{q,a+}^{\gamma}y\in AC_q^n[a,b]$, then there exists a unique solution $y(x)\in L_{\alpha,\beta,q}^1[a,b]$ to the Cauchy type problem (1)-(2).

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