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AUTOMORPHISMS OF FREE LEFT-SYMMETRIC ALGEBRAS OF RANK 2

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Abstract:

We prove that the group of automorphisms of a free left-symmetric algebra of rank two admits an amalgamated free product structure.

Keywords: free left-symmetric algebra, automorphism, free product

The basis of free left-symmetric algebras

A vector space A over an arbitrary field k is called a left-symmetric algebra if for any $x, y, z \in A$ the identity

$$(xy)z - x(yz) = (xz)y - x(zy) \text{ holds.}$$

In other words, the associator $(x, y, z) = (xy)z - x(yz)$ is symmetric with respect to y and z, i.e.

$$(x, y, z) = (y, x, z).$$

For right-symmetric algebras the following identity is satisfied:

$$(x, y, z) = (x, z, y)$$

It is clear that the opposite algebra of a left-symmetric algebra is a right-symmetric algebra. In this sense, the study of right-symmetric algebras is completely parallel to the study of left-symmetric algebras. In L. Makar-Limanov, D. Kozybaev, U. Umirbaev [1] proved that automorphisms of free right-symmetric rank two algebras are tame. We prove that the group of automorphisms of a free left-symmetric algebra of rank two admits the structure of an amalgamated free product. Deriving, proving equations from articles gives us following results.

Let k be an arbitrary field. Through $LS\langle x_1,\ x_2 \dots, x_n\rangle$ we denote the free algebra in the variables $x_1,\ x_2 \dots, x_n$ over a field k. By deg we denote the standard degree function on $LS\langle x_1,$ $x_2 \dots, x_n\rangle$ i.e., $deg\ (x_i)\ = 1$ for i.

For any nonzero $h\ \in\ LS\langle x_1,\ x_2\rangle$, and for any nonzero f ∈ LS$\langle x\rangle$ we have [2]

$$deg\ (f\ (h))\ =\ deg\ (f)\ \cdot\ deg\ (h).$$

Let X = $\{x_1,\ x_2 \dots, x_n\}$ be a finite alphabet. By X^* we denote the set of all non-associative words in the alphabet X. By $deg(u)$ we also denote the function degree on X^* such that $deg(x_i)\ =\ 1$ for all i. Each non-associative word u of degree $\geq\ 2$ is uniquely represented in the form $u\ =\ u_1u_2$, $deg(u_1), deg(u_2) <\ deg(u)$.

Put $x_1 <\ x_2 \dots <\ x_n$. Let u and v be arbitrary elements of X^*. Put $u < v$ if $deg(u) <$ $deg\ (v)$. Let $deg(u)\ =\ deg(v) \geq 2$, $u\ = u_1u_2$, $v\ = v_1v_2$, then let $u < v$ if $u_1 < v_1$ or $u_1 = v_1$ and $u_2 < v_2$.

A word is called reduced if it contains a sub-word of the form $r(st) \in X^*$, where $deg(r), deg(s), deg(t)\ \geq 1$ and $s >\ t$. A word is called *good* if it is not reduced. We denote by W the set of all right words in the alphabet X.

By $LS_n =\ LS\langle x_1,\ x_2 \dots, x_n\rangle$ we denote the free left-symmetric algebra of the variables $x_1, x_2 \dots, x_n$ over the field k. According to [23], the set of all good words W forms a linear basis LS_n. Every nonzero element f of LS_n is uniquely represented as

$$f = \lambda_1 w_1 + \lambda_2 w_2 + \cdots +\ \lambda_m w_m$$

where $w_i \in W$, $0 \neq \lambda_i \in\ k$ for all i and $w_1 <\ w_2 < \ldots <\ w_m$. The word w_1 is called is also called the lowest term of f and is denoted by $\bar{f}$.

For each $f \in LS_n$, by L_f we denote the operator of left multiplication by f acting in LS_n, i.e. $uL_f = u_f$ for all $u\ \in\ LS_n$. In particular, if $w, w_1, w_2 \dots\ w_m \in\ X^*$, then

$$L_{w_1}L_{w_2} \dots L_{w_m}w\ =\ (w_m \dots (\dots w_2(w_1w))).$$

Lemma 1[1] Every reduced word $w\ \in\ W$ can be uniquely represented in the form

$$w\ =\ L_{w1}L_{w2} \dots\ L_{wm}x_i, \tag{1.1.1}$$

where $w_j \in W$ for all j and $w_1 \geq w_2 \geq \ldots \geq w_m$.

Lemma 2 [1] Let $w\ \in$ LS$_n$ and

$$w\ =\ L_{w1}L_{w2} \dots\ L_{wm}x_i \tag{1.1.2}$$

where $w_j \in W$ for all j and $w_1 \geq w_2 \geq \ldots \geq w_m$. Then w is a reduced word.

Lemma 3 [1] Let u and v be reduced words, assume that $v = L_{v_1}L_{v_2} \dots L_{v_m}x_i$, Then

$$\overline{uv} = L_{v1}L_{v2} \dots L_{vs}L_uL_{v_{s+1}} \dots L_{v_m}x_i, \qquad (1.1.3)$$

where $v_1 \geq \dots v_s \geq u \geq v_{s+1} \geq \dots \geq v_m$.

Corollary 1[1] If $u, v, w \in W$, then $(\overline{uv})w = (\overline{uw})v$.

Lemma 4 [1] Let u, v, w be arbitrary reduced words. If $u > v$, then $(\overline{wu}) > (\overline{wv})$ and $(\overline{uw}) > (\overline{vw})$.

Corollary 2 [1] If $f, g \in \mathrm{LS}_n$, then $\overline{fg} = \overline{\bar{f}\bar{g}}$.

Lemma 5 [1] Let v, u be reduced words of the algebra $LS\langle y\rangle$, w be a reduced word of the algebra $LS\langle x, y\rangle$ and $v > u$. Then

1) $v(w) \in W$;
2) $v(w) > u(w)$.

Corollary 3 [1] If $f \in LS\langle x, y\rangle, g \in LS\langle x, y\rangle$, then $\overline{f(g)} = \overline{\bar{f}(\bar{g})}$.

Amalgamated Free Product.

Let k be an arbitrary field. Let $A = SL\langle x_1, x_2\rangle$ be free left-symmetric algebras of rank two over k. Let also $Aut(A)$ be the automorphism group of the algebra A. By $\varphi = (f_1, f_2)$ we denote an automorphism of the algebra A such that $\varphi(x) = f_1, \varphi(y) = f_2$. Automorphisms of the form

$$\sigma(1, a, f) = (ax + f(y), y),$$

$$\sigma(2, a, g) = \big(x, ay + g(x)\big),$$

where $0 \neq a \in k, f(y) \in Q\langle y\rangle, g(x) \in Q\langle x\rangle$ are called *elementary*. The subgroup $T(A)$ of $Aut(A)$ generated by all elementary automorphisms is called *the subgroup of tame automorphisms*. If the automorphism is not tame it is called *wild.*

For an automorphism $\theta = (f_1, f_2) \in Aut(A)$ we define a degree, a general degree, and a bidegree, setting, respectively

$$\deg(\theta) = max\{\deg(f_1), \deg(f_2)\},$$

$$t\deg(\theta) = \deg(f_1) + \deg(f_2),$$

$$\mathrm{bideg}(\theta) = (\deg(f_1), \deg(f_2)).$$

If

$$\theta = (f_1, f_2), \varphi = (g_1, g_2),$$

then the product in $Aut(A)$is defined by the following formula:

$$\theta \circ \varphi = (g_1(f_1, f_2), g_2(f_1, f_2)).$$

Let $Af_2(A)$ be the group of affine automorphisms of the algebra A, i.e. automorphism group of the form

$$(a_1x + b_1y + c_1,\ a_2x + b_2y + c_2),$$

where $a_i, b_i, c_i \in \mathrm{k}, \mathrm{a}_1, \mathrm{b}_2 \neq \mathrm{a}_2\mathrm{b}_1$, $Tr_2(A)$ is the group of triangular automorphisms of the algebra A, i.e. automorphism group of the form

$$(ax + f(y), by + c),$$

where $0 \neq a, b \in k, c \in k, f(y) \in SL\langle y\rangle$, and let $C = Af_2(A) \cap Tr_2(A)$.

Let G be an arbitrary group, G_0, G_1, G_2 be subgroups of G, and $G_0 = G_1 \cap G_2$. A group G is called a *free product of subgroups* G_1 and G_2 *with a combined subgroup* G_0 and is denoted by $G = G_1 *_{G_0} G_2$, if

a) G is generated by subgroups G_1 and G_2;

b) The defining relations of G consist only of the defining relations of the subgroups G_1 and G_2.

If S_1 is a system of left representatives G_1 in G_0, S_2 is a system of left representatives G_2 in G_0, then G is a free product of subgroups G_1 and G_2. with a combined subgroup G_0 if and only if when each $g \in G$ is uniquely represented as

$$g = g_1 \dots g_k c,$$

where $g_i \in S_1 \cup S_2, i = 1, \dots, k$, $g_i, g_i + 1$ do not simultaneously belong to S_1 and S_2, $c \in G_0$.

The notation $h_i(y)$ in the proofs of the following several lemmas means that $h_i(y) \in \mathrm{Q}\langle\mathrm{y}\rangle$ is a homogeneous element of degree i with respect to a function of degree deg in one variable y. It is clear that $h_0(y) \in \mathrm{k}$.

Lemma 6. a) The system of elements

$$A_0 = \{id = (x, y), \gamma = (y, x + ay) | a \in k\}$$

is a system of representatives of left adjacent classes $Af_2(A)$ in the subgroup C.

b) Elements system

$$B_0 = \{\beta = (x + q(y), y) | q(y) = h_2(y) + \dots + h_n(y)\}$$

is a system of representatives of left adjacent classes $Tr_2(A)$ in the subgroup C.

Lemma 7 Let A_0, B_0 be the sets defined in Lemma 6. Then any tame automorphism φ of the algebra A can be decomposed into a product of the form

$$\varphi = \gamma_1 \diamond \beta_1 \diamond \gamma_2 \diamond \beta_2 \diamond \dots \diamond \gamma_k \diamond \beta_k \diamond \gamma_{k+1} \diamond \lambda, \qquad (1)$$

where $\gamma_i \in A_0, \gamma_2, \dots, \gamma_k = id, \beta_i \in B_0, \beta_1, \dots, \beta_k \neq id, \lambda \in C$.

Lemma 8 Let $\varphi = (f_1, f_2)$ be an automorphism of the algebra A, representable as a product

$$\varphi = (f_1, f_2) = \beta_1 \diamond \gamma_2 \diamond \beta_2 \diamond \ldots \diamond \gamma_k \diamond \beta_k,$$

where $id \neq \gamma_i \in A_0$, $id \neq \beta_i \in B_0$ for all i. If $\beta_i = (x + q_i(y), y), deg(q_i(y)) = n_i$ for all $1 \leq i \leq k$, then

$$deg\,(f_1) = n_1 n_2 \ldots n_{k-1} n_k,$$
$$deg\,(f_2) = n_1 n_2 \ldots n_{k-1}, if\ k > 1$$

and

$$deg\,(f_2) = 1,\ if\ k = 1.$$

Lemma 9 The decomposition (1) of the automorphism φ from Lemma 7 is unique.

Theorem 1 Let $A = SL\langle x_1, x_2 \rangle$ be free left-symmetric algebra in two variables x_1, x_2 over k. The group of tame automorphisms of A is a free product of subgroups of affine automorphisms $Af_2(A)$ and triangular automorphisms $Tr_2(A)$ with the combined subgroup $C = Af_2(A) \cap Tr_2(A)$, i.e.

$$T(A) = Af_2(A) *_C Tr_2(A).$$

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ЖОҒАРҒЫ РЕТТІ ИНТЕГРАЛДЫ ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУ ҮШІН ИНТЕГРАЛДЫҚ ШАРТЫ БАР БІР ШЕКАРАЛЫҚ ЕСЕП

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$C[0,1]$ кеңістігінде төмендегідей есепті қарастырайық:

$$\begin{cases} y^{(n)} - \sum_{i=0}^{n-1} g_i(x) \int_0^1 h_i(t) y^{(n-1)-i}(t) dt = f(x) \\ \sum_{j=1}^{n} \left[\alpha_{ij} y^{j-1}(0) + \beta_{ij} y^{j-1}(1) + \int_0^1 b_{ij}(x) y^{j-1}(x) dx \right] = 0, (i = \overline{1,n}) \end{cases} \quad (1)$$