

Classical color fields as a dark matter candidate

Vladimir Dzhunushaliev*

*Departement of Physics and Microelectronics Engineering,
Kyrgyz-Russian Slavic University, Bishkek,
Kievskaya Str. 44, 720021, Kyrgyz Republic*

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Abstract: A model of Dark Matter is proposed where the Dark Matter is a classical color field. The color fields are invisible as they may interact with colored elementary particles like the 't Hooft–Polyakov monopole only. Comparison with the Universal Rotation Curve is carried out. © Versita Warsaw and Springer-Verlag Berlin Heidelberg. All rights reserved.

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1 Introduction

In astrophysics and cosmology, Dark Matter (DM) is matter of unknown composition that does not emit or reflect enough electromagnetic radiation to be observed directly, but whose presence can be inferred from gravitational effects on visible matter. According to present observations of structures larger than galaxy-sized as well as Big Bang cosmology, DM accounts for the vast majority of mass in the observable universe. Among the observed phenomena consistent with DM observations are the rotational speeds of galaxies and orbital velocities of galaxies in clusters, gravitational lensing of background objects by galaxy clusters such as the Bullet cluster, and the temperature distribution of hot gas in galaxies and clusters of galaxies. DM also plays a central role in structure formation and galaxy evolution, and has measurable effects on the anisotropy of the cosmic microwave background. All these lines of evidence suggest that galaxies, clusters of galaxies, and the universe as a whole contain far more matter than that which interacts with electromagnetic radiation: the remainder is called the “dark matter component”.

The first to provide evidence and infer the existence of the phenomenon that has come

* E-mail: dzhun@krsu.edu.kg

to be called “dark matter” was F. Zwicky [1]. He applied the virial theorem to the Coma cluster of galaxies and obtained evidence of unseen mass.

According to results published in [2], dark matter has been observed separately from ordinary matter through measurements of the Bullet Cluster, actually two nearby clusters of galaxies that collided about 150 million years ago. Researchers analyzed the effects of gravitational lensing to determine the total mass distribution in the pair and compared that to X-ray maps of hot gases, thought to constitute the large majority of ordinary matter in the clusters. The hot gases interacted during the collision and remain closer to the center. The individual galaxies and DM did not interact and are farther from the center. Recently the Bullet cluster data has been offered as evidence of dark matter

Numerous astrophysical observations, e.g., Doppler measurements of rotation velocities in disk galaxies, have established the failure of the classical Newtonian theory, if only visible matter is taken into account [3]. Historically, theoretical concepts addressing this problem can be subdivided into two categories. The first category comprises the DM theories [4], whereas the second group assumes that Newton’s gravitational law requires modification [5].

DM theories are based on the hypothesis that there exist significant amounts of invisible (non-baryonic) matter in the universe, interacting with ordinary visible matter only via gravity. Since it is empirically very successful, DM has become a widely accepted cornerstone of the contemporary cosmological standard model [6]. Nevertheless, it must also be emphasized that until now DM has been detected only indirectly by means of its gravitational effects on visible matter or light.

In order to explain the existence of regions where the velocity of stars is $\approx const$ it is necessary (in the DM framework) to have a large enough region with DM. For the classical SU(3) gauge theory this is the ordinary situation: the point is that the classical Yang–Mills equations ordinarily give us solutions with an infinite mass [7], i.e. the mass density is such that $\rho \geq r^{-2}$. By indirection this problem is connected with the confinement problem in quantum chromodynamics which claims that the field distribution between quark and antiquark is a flux tube with a finite linear energy density. Such flux a tube can not be obtained in the framework of the classical Yang–Mills theory. It is possible in the framework of the quantum Yang–Mills theory only.

The classical gauge theories are a nonlinear generalization of Maxwell electrodynamics. The non-Abelian gauge theories were invented by Yang and Mills in the 1950s. For most of this period it was not known whether any of the interactions observed in nature can be described by a non-Abelian gauge theory. Nevertheless, the elegance of these theories attracted interest. The Weinberg–Salam model (SU(2) \times U(1) gauge theory) and quantum chromodynamics (SU(3) gauge theory) are the two existing Yang–Mills theories of real phenomenological importance. These theories can be formulated in terms of Feynman path integrals, i.e. functional integrals over all classical field configurations weighted by a factor $\exp(-\text{action})$. If one knew everything about classical field configurations, then in principle all questions concerning the quantum theory could be answered. Partial information about classical fields might yield, at least, some insight into the quantum

theory. For review of classical solutions of Yang–Mills theories see Ref. [8].

In this paper we use the solutions of the classical SU(3) gauge theory for an explanation of the rotational velocity of stars outside the core of the galaxy.

2 Initial equations for the SU(3) gauge field

We consider the classical SU(3) Yang–Mills gauge field A_μ^a . The field equations are

$$D_\nu F^{a\mu\nu} = 0, \quad (1)$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$ is the field strength tensor; f^{abc} are the SU(3) structural constants; $a, b, c = 1, 2, \dots, 8$ are color indices; g is the coupling constant. We use the following ansatz [9]

$$A_0^2 = -2\frac{z}{gr^2}\chi(r), \quad A_0^5 = 2\frac{y}{gr^2}\chi(r), \quad A_0^7 = -2\frac{x}{gr^2}\chi(r), \quad (2)$$

$$A_i^2 = 2\frac{\epsilon_{3ij}x^j}{gr^2}[h(r) + 1], \quad (3)$$

$$A_i^5 = -2\frac{\epsilon_{2ij}x^j}{gr^2}[h(r) + 1], \quad (4)$$

$$A_i^7 = 2\frac{\epsilon_{1ij}x^j}{gr^2}[h(r) + 1] \quad (5)$$

for the SU(2) \in SU(3) components of the gauge field and

$$(A_0)_{\alpha\beta} = 2\left(\frac{x^\alpha x^\beta}{r^2} - \frac{1}{3}\delta^{\alpha\beta}\right)\frac{w(r)}{gr}, \quad (6)$$

$$(A_i)_{\alpha\beta} = 2\left(\epsilon_{is\alpha}x^\beta + \epsilon_{is\beta}x^\alpha\right)\frac{x^s}{gr^3}v(r), \quad (7)$$

for the coset components; $i = 1, 2, 3$ are space indices; ϵ_{ijk} is the absolutely antisymmetric Levi–Civita tensor; the functions $\chi(r), h(r), w(r), v(r)$ are unknown functions. The coset components $(A_\mu)_{\alpha\beta}$ in the matrix form are written as

$$(A_\mu)_{\alpha\beta} = \sum_{a=1,3,4,6,8} A_\mu^a (T^a)_{\alpha,\beta} \quad (8)$$

where $T^a = \frac{\lambda^a}{2}$ are the SU(3) generators, λ^a are the Gell-Mann matrices. The corresponding equations are

$$x^2 w'' = 6w(h^2 + v^2) - 12hv\chi, \quad (9)$$

$$x^2 \chi'' = 2\chi(h^2 + v^2) - 4hvw, \quad (10)$$

$$x^2 v'' = v^3 - v + v(7h^2 - w^2 - \chi^2) + 2hw\chi, \quad (11)$$

$$x^2 h'' = h^3 - h + h(7v^2 - w^2 - \chi^2) + 2vw\chi, \quad (12)$$

here the dimensionless radius $x = r/r_0$ is introduced, r_0 is a constant.

3 Numerical investigation

In this section we present a typical numerical solution of Eqs. (13, 14). We will investigate the case $\chi = h = 0$

$$x^2 w'' = 6wv^2, \quad (13)$$

$$x^2 v'' = v^3 - v - vw^2. \quad (14)$$

For the numerical investigation we have to start from the point $x = \delta \ll 1$. Here we have the following approximate solution

$$v \approx 1 + v_2 \frac{x^2}{2}, \quad w \approx w_3 \frac{x^3}{2}, \quad x \ll 1 \quad (15)$$

where v_2, w_3 are arbitrary constants. The typical behavior of the functions $v(x)$ and $w(x)$ is presented in Fig. 1.

The mass density $\rho(x)$ is

$$\begin{aligned} \rho &= \frac{1}{2c^2} \left(-F_{0i}^a F^{a0i} + \frac{1}{4} F_{ij}^a F^{aij} \right) \\ &= \frac{1}{g^2 c^2 r_0^4} \left[4 \frac{v'^2}{x^2} + \frac{2}{3} \frac{(xw' - w)^2}{x^2} + 2 \frac{(v^2 - 1)^2}{x^4} + 4 \frac{v^2 w^2}{x^4} \right] \\ &= \frac{1}{g^2 c^2 r_0^4} \varepsilon(x) \end{aligned} \quad (16)$$

where c is the speed of light. The profile of the dimensionless energy density $\varepsilon(x)$ is presented in Fig. 2.

The rotation curve is defined as

$$V^2 = G \frac{m(r)}{r} = \frac{4\pi G}{c^2} \frac{1}{r} \int_0^r r^2 \rho(r) dr = \frac{G\hbar}{c} \frac{1}{g'^2 r_0^2} \frac{1}{x} \int_0^x x^2 \varepsilon(x) dx = \frac{G\hbar}{c} \frac{1}{g'^2 r_0^2} \frac{m(x)}{x}, \quad (17)$$

where $m(r)$ is the mass of the color fields A_μ^a inside a sphere with the radius $r = xr_0$, $m(x)$ is the dimensionless mass, $g'^2 = g^2 c \hbar$ is the dimensionless coupling constant, and G is the Newton gravitational constant.

The asymptotic behavior of the solution $x \gg 1$ is

$$v(x) \approx A \sin(x^\alpha + \phi_0), \quad (18)$$

$$w(x) \approx \pm \left[\alpha x^\alpha + \frac{\alpha - 1 \cos(2x^\alpha + 2\phi_0)}{4 x^\alpha} \right], \quad (19)$$

$$3A^2 = \alpha(\alpha - 1) \quad (20)$$

with $\alpha > 1$.

4 The comparison with a Universal Rotation Curve for spiral galaxies

Unfortunately we have not any analytical solution and therefore it is very difficult to carry out the numerical investigation as the coefficient $\frac{G\hbar}{c}$ in Eq. (17) is very small and

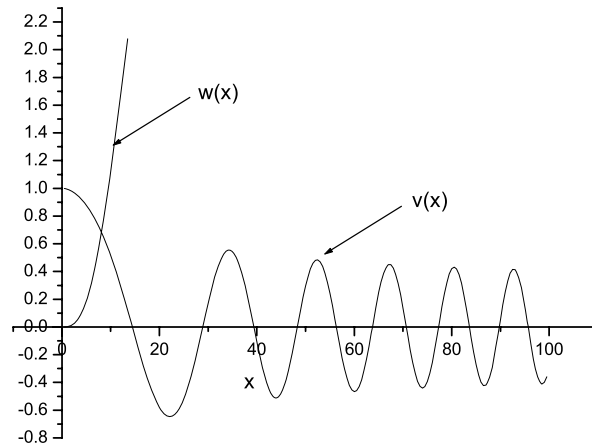


Fig. 1 The profile of the functions $w(x), v(x), v_2 = 10^{-2}, w_3 = 10^{-2}$.

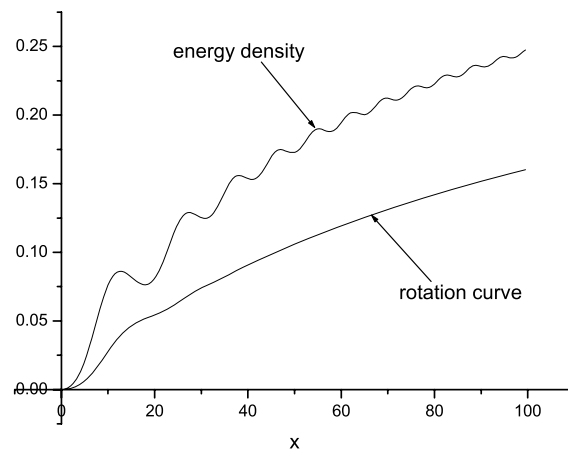


Fig. 2 The profile of the dimensionless energy density $\varepsilon(x)$ and the dimensionless rotation curve $\frac{cg^2 r_0^2}{G\hbar} V^2(x)$.

asymptotically we have strongly oscillating function $v(x)$ in Eq. (18) (if r_0 is very small). Therefore in this section we will investigate the rotation curve of gauge DM close to the center and far away from the center.

In Ref. [10] a Universal Rotation Curve of spiral galaxies is offered which describes any rotation curve at any radius with a very small cosmic variance

$$V_{URC} \left(\frac{r}{R_{opt}} \right) = V(R_{opt}) \left[\left(0.72 + 0.44 \log \frac{L}{L_*} \right) \frac{1.97x^{1.22}}{(x^2 + 0.78^2)^{1.43}} + 1.6 e^{-0.4(L/L_*)} \frac{x^2}{x^2 + 1.5^2 \left(\frac{L}{L_*} \right)^{0.4}} \right]^{1/2} \text{ km s}^{-1} \quad (21)$$

where $R_{opt} \equiv 3.2 R_D$ is the optical radius and R_D is the disc exponential length-scale;

$x = r/R_{opt}$; L is the luminosity. We would like to compare the rotation curve for the color fields Eq. (17) with the Universal Rotation Curve (23) where, for example, $L/L_* = 1$

$$V_{URC} \left(\frac{r}{R_{opt}} \right) = V(R_{opt}) \left[\frac{1.4184 x^{1.22}}{(x^2 + 0.782)^{1.43}} + \frac{1.07251 x^2}{x^2 + 1.5^2} \right]^{1/2} \text{ km s}^{-1}. \quad (22)$$

For the Dark Matter the Universal Rotation Curve is

$$V_{DM}^2 \left(\frac{r}{R_{opt}} \right) = V^2(R_{opt}) \frac{1.07251 x^2}{x^2 + 1.5^2} \text{ km s}^{-1}. \quad (23)$$

The profiles of $V_{URC}(x)$, $V_{DM}^2(x)$, $V_{LM}^2(x)$ are presented in Fig. 3. (V_{DM}^2 is the rotation curve for the Dark Matter, $V_{LM}^2(x)$ is the rotation curve for the light matter).

At the center ($r \approx 0$) the approximate solution has the form (15) and the mass density (16) approximately is

$$\rho(x) \approx \frac{6}{g^2 c^2 r_0^4} \frac{v_2^2}{r_0^4}. \quad (24)$$

Consequently the rotation curve will be

$$V^2 \approx \frac{G\hbar}{c} \frac{1}{g'^2 r_0^2} \frac{v_2^2}{r_0^2} \left(\frac{r}{r_0} \right)^2 m s^{-1}. \quad (25)$$

Comparison with Eq. (23) with $x \ll 1$ gives us

$$v_2 \approx 20 \sqrt{\frac{c}{G\hbar}} g' \frac{V_{opt}}{R_{opt}} r_0^2. \quad (26)$$

If r_0 is very small in comparison with R_{opt} then far away from the center the functions $v(x)$ and $w(x)$ are as presented in Eqs. (18-20) and the mass density is

$$\varepsilon_\infty(x) = c^2 \rho_\infty(r) \approx \frac{2}{3} \frac{1}{g^2 r_0^4} \alpha^2 (\alpha - 1) (3\alpha - 1) \left(\frac{r}{r_0} \right)^{2\alpha-4}. \quad (27)$$

In this case we can estimate the values of the square of the speed in the following way:

$$\begin{aligned} V^2 &= \frac{G\hbar}{c} \frac{1}{g'^2 r_0^2} \frac{1}{x} \left(\int_0^{x_1} x^2 \varepsilon(x) dx + \int_{x_1}^x x^2 \varepsilon(x) dx \right) \\ &\approx \left[\frac{G\hbar}{c} \frac{1}{g'^2 r_0^2} \frac{1}{x} \int_0^x x^2 \varepsilon_\infty(x) dx \right] - V_0^2, \end{aligned} \quad (28)$$

$$V_0^2 = \frac{G\hbar}{c} \int_0^{x_1} x^2 [\varepsilon_\infty(x) - \varepsilon(x)] dx, \quad (29)$$

where for the region $x > x_1$ the asymptotical Eq. (27) is valid. One can say that V_0^2 is a systematic error of the equation

$$V^2 = \frac{G\hbar}{c} \frac{1}{g'^2 r_0^2} \frac{1}{x} \int_0^x x^2 \varepsilon_\infty(x) dx \quad (30)$$

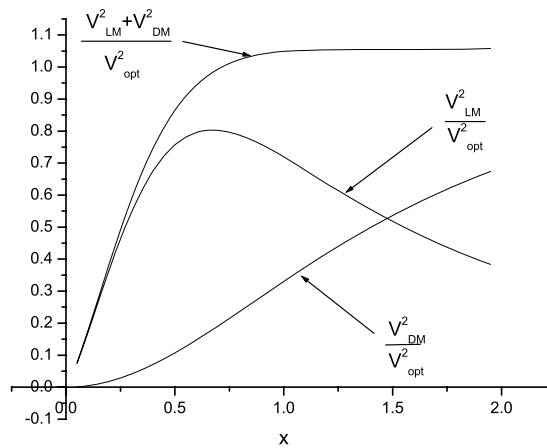


Fig. 3 Profiles of the dimensionless rotation curves for light and dark matter [10].

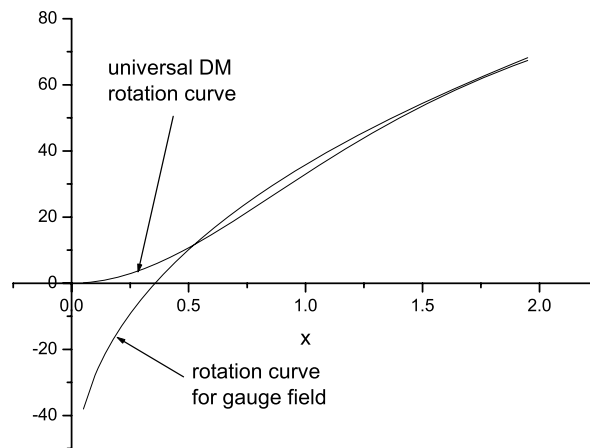


Fig. 4 Comparison of the DM rotation curve Eq. (23) with the rotation curve Eq. (31) for the SU(3) classical color field (2)–(7). $\alpha = 1.2, g' = 0.3, R_{opt} = 20 \text{ kpc} = 6 \times 10^{17} \text{ m}, V_{opt} = 100 \text{ km/s} = 10^5 \text{ m/s}, V_0 = 70 \text{ km/s} = 7 \times 10^4 \text{ m/s}$.

and the numerical value of V_0^2 is defined near to the center of galaxy where according to Eq. (29) the difference $\varepsilon_\infty(x) - \varepsilon(x)$ is maximal. Thus the asymptotical behavior of the rotation curve for the domain filled with the SU(3) gauge field is

$$V^2 \approx \frac{2G\hbar}{3c} \frac{1}{g'^2 r_0^2} \alpha^2 (\alpha - 1) (3\alpha - 1) \left(\frac{r}{r_0}\right)^{2\alpha-2} - V_0^2. \tag{31}$$

In Fig. 4 we see that it is possible to choose the parameters α, r_0, g', V_0 in such a way that we have very good coincidence of the Universal Rotation Curve Eq. (23) and the rotation curve for a spherically symmetric distribution of SU(3) gauge field Eq. (2-7).

5 Invisibility of color fields

The invisibility of classical color fields is based on the fact that only *colored* elementary particles may interact with the classical non-Abelian gauge fields. The equations describing the motion of a colored particle are Wong's equations

$$m \frac{d^2 x^\mu}{ds^2} = -g F^{a\mu\nu} M^a \frac{dx^\nu}{ds}, \quad (32)$$

$$\frac{dM^a}{ds} = -g f^{abc} \left(A^{b\mu} \frac{dx^\mu}{ds} \right) M^c, \quad (33)$$

where $x^\mu(s)$ is the 4D trajectory of the particle with the mass m , M^a is the color components of color charge of the particle, $(M^a)^2 = M^2 = \text{const}$. Now we see that ordinary elementary particles do not interact with the color fields presented here as they are colorless, in consequence of the confinement for strong interactions.

Only monopoles and dyons may interact with these fields. Another possibility to find the influence of the color fields on elementary particles is the following: some particles (proton, neutron, and so on) may have an inner structure, i.e. a color electric or magnetic dipole or quadrupole which will interact with the external inhomogeneous color field. But this interaction should be very small.

6 Conclusions and discussion

We have shown that in principle the classical SU(3) color fields have a weakly decreasing mass density that allows us to offer the corresponding gauge field distribution as a candidate for DM explaining the Universal Rotation Curve of spiral galaxies. The question about the adaptability of the presented model of DM for the explanation of galaxy movement in clusters and galaxy clusters in superclusters demands separate consideration, which will be done in our future investigations. The distinctive feature of this DM model is that it uses well established SU(3) Yang–Mills theory and there is no necessity either to modify Newton's gravitational law or to introduce weakly interacting supersymmetrical particles. It is interesting that the invisibility of gauge DM is connected with the confinement in quantum chromodynamics.

Very important questions are: How big is the SU(3) domain? Does it have an infinite or finite volume? In this connection it is necessary to make the next remark: the gauge fields presented here are strongly oscillating far away from the center (for $r_0 \ll R_{opt}$). At some distance from the center the oscillations become so strong that quantum effects begin to be essential: we should take into account the Heisenberg Uncertainty Principle. These effects appear when the quantum fluctuations of the gauge field become comparable with the magnitude of the color fields on a distance scale of about the period of the oscillations. In this case we have to apply *nonperturbative* quantization in order to describe the quantum color fields. Now we would like to note that in Ref. [11] an approximate model of a non-perturbative quantization is offered and a solution describing a ball filled with a gauge condensate is derived. The quantum fields of the ball decrease

asymptotically as $\frac{e^{-r/l_0}}{r}$ (where l_0 is a constant) and we will have a finite total mass of the SU(3) domain. Thus as a whole the picture of the color field distribution (which is DM in the presented model) looks as follows: there is a ball filled with the *classical* color field which gives us the observable rotation curve and far enough from the center the gauge field becomes the *quantum* one.

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