Differential geometry of surfaces and Heisenberg ferromagnets

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Abstract

The relation between differential geometry of surfaces and some Heisenberg ferromagnet models is considered.

1 Introduction

The deep interrelation between many partial differential equations (PDE) of the classical differential geometry (DG) of surfaces and modern soliton equations is well established now [2-8]. Generally speaking, the interaction between DG and PDE has been studied since the 19^{th} century. For example, it can be found in the classical works of Lie, Darboux, Goursat, Bianchi, Backlund, E.Cartan. In particular, there are many very interesting PDE originating from DG. The famous sine-Gordon equation, for example, first appeared in DG. Some of the other soliton equations have been known in DG for a long time [9-11].

One of interesting subclass of soliton equations, useful both from the mathematical and physical point of view, is Heisenberg ferromagnets (HF) or spin systems (SS) [1,12-27]. In the present paper, we will construct some classes of surfaces corresponding several SS in two dimensions.

2 Some fundamental formulas on the theory of surfaces

Let M^2 be a surface in the 3-dimensional space E^3 parametrized by the coordinates x, y and defined by the position vector $\mathbf{r}(x, y)$. The unit normal vector to the surface defines as usual

$$\mathbf{n} = \frac{\mathbf{r}_x \wedge \mathbf{r}_y}{|\mathbf{r}_x \wedge \mathbf{r}_y|}. \tag{1}$$

Now we cite a few formulas for the tangent vectors which induce some surfaces.

2.1 The Rodrigues formula

The simplest example is the Rodrigues formula (RF) [11]

$$\mathbf{r}_x = -\rho_1 \mathbf{N}_x, \quad \mathbf{r}_y = -\rho_2 \mathbf{N}_y \tag{2}$$

where $\mathbf{N}$ is in general some vector-function, ρ_i is the scalar function.

2.2 The Lelieuvre formula

Our second example is the Lelieuvre formula (LF) [4, 7, 9-11, 19-23]

$$\mathbf{r}_x = \rho \mathbf{N}_x \wedge \mathbf{N}, \quad \mathbf{r}_y = \rho \mathbf{N} \wedge \mathbf{N}_y. \tag{3}$$

2.3 The Schief formula

One of interesting generalizations of the LF (3) is the Schief formula (SF) [11]

$$\mathbf{r}_x = \rho \mathbf{N}_x \wedge \mathbf{N} + \mu \mathbf{N}_x, \quad \mathbf{r}_y = \rho \mathbf{N} \wedge \mathbf{N}_y + \mu \mathbf{N}_y. \tag{4}$$

2.4 The Myrzakulov formula

At last, we consider the Myrzakulov formula (MF) [27-28]

$$\mathbf{r}_x = a_1 \mathbf{N} \wedge \mathbf{N}_x + a_2 \mathbf{N} \wedge \mathbf{N}_y + a_3 \mathbf{N}_x + a_4 \mathbf{N}_y + a_5 \mathbf{N} \tag{5a}$$

$$\mathbf{r}_y = b_1 \mathbf{N} \wedge \mathbf{N}_x + b_2 \mathbf{N} \wedge \mathbf{N}_y + b_3 \mathbf{N}_x + b_4 \mathbf{N}_y + b_5 \mathbf{N} \tag{5b}$$

where $\mathbf{N}$ is some vector function, a_i, b_i are in general some real functions. We note that the RF (2), the LF (3) and the SF (4) are the particular cases of the MF (5).

3 Surfaces and N-systems

The compatibility condition of the equations (5) gives [28]

$$(a_{1y} - b_{1x})\mathbf{N} \wedge \mathbf{N}_x + (a_{2y} - b_{2x})\mathbf{N} \wedge \mathbf{N}_y + (a_1 + b_2)\mathbf{N}_y \wedge \mathbf{N}_x +$$

$$+\mathbf{N} \wedge [a_2 \mathbf{N}_{yy} + (a_1 - b_2)\mathbf{N}_{xy} - b_1 \mathbf{N}_{xx}] + (a_{5y} - b_{5x})\mathbf{N} + (a_{3y} - b_5 - b_{3x})\mathbf{N}_x +$$

$$+(a_5 + a_{4y} - b_{4x})\mathbf{N}_y + [(a_3 - b_4)\mathbf{N}_{xy} + a_4 \mathbf{N}_{yy} - b_3 \mathbf{N}_{xx}] = 0. \tag{6a}$$

Hence we obtain

$$b_{5x} - a_{5y} = \frac{1}{\mathbf{N} \cdot \mathbf{N}}\{(a_{3y} - b_5 - b_{3x})\mathbf{N} \cdot \mathbf{N}_x + +(a_5 + a_{4y} - b_{4x})\mathbf{N} \cdot \mathbf{N}_y$$

$$+(a_1 + b_2)\mathbf{N} \cdot (\mathbf{N}_y \wedge \mathbf{N}_x) + \mathbf{N} \cdot [(a_3 - b_4)\mathbf{N}_{xy} + a_4 \mathbf{N}_{yy} - b_3 \mathbf{N}_{xx}]\}. \tag{6b}$$

Equations (6) we call the $\mathbf{N}$-equations or $\mathbf{N}$-systems [28].

4 Surfaces and n-systems or spin systems

Let us now we consider the case when

$$\mathbf{N} = \mathbf{n} \equiv \mathbf{S} \tag{7}$$

where $\mathbf{S}$ is the spin vector $\mathbf{S} = (S_1, S_2, S_3)$, $\quad \mathbf{S}^2 = S_1^2 + S_2^2 + S_3^2 = 1$. In this case, the equations (6) we call the **n**-equations or spin systems (SS). Let us consider the MF [28]

$$\mathbf{r}_x = a_1 \mathbf{S} \wedge \mathbf{S}_x + a_2 \mathbf{S} \wedge \mathbf{S}_y + a_3 \mathbf{S}_x + a_4 \mathbf{S}_y + a_5 \mathbf{S} \tag{8a}$$

$$\mathbf{r}_y = b_1 \mathbf{S} \wedge \mathbf{S}_x + b_2 \mathbf{S} \wedge \mathbf{S}_y + b_3 \mathbf{S}_x + b_4 \mathbf{S}_y + b_5 \mathbf{S} \tag{8b}$$

which is the spin form of the formulas (5). At the same time, after (7) the equations (6) take the form [28]

$$(a_{1y} - b_{1x}) \mathbf{S} \wedge \mathbf{S}_x + (a_{2y} - b_{2x}) \mathbf{S} \wedge \mathbf{S}_y + (a_1 + b_2) \mathbf{S}_y \wedge \mathbf{S}_x +$$

$$+ \mathbf{S} \wedge [a_2 \mathbf{S}_{yy} + (a_1 - b_2) \mathbf{S}_{xy} - b_1 \mathbf{S}_{xx}] + (a_{5y} - b_{5x}) \mathbf{S} + (a_{3y} - b_5 - b_{3x}) \mathbf{S}_x +$$

$$+ (a_5 + a_{4y} - b_{4x}) \mathbf{S}_y + [(a_3 - b_4) \mathbf{S}_{xy} + a_4 \mathbf{S}_{yy} - b_3 \mathbf{S}_{xx}] = 0 \tag{9a}$$

$$b_{5x} - a_{5y} = (a_1 + b_2) \mathbf{S} \cdot (\mathbf{S}_y \wedge \mathbf{S}_x) + \mathbf{S} \cdot [(a_3 - b_4) \mathbf{S}_{xy} + a_4 \mathbf{S}_{yy} - b_3 \mathbf{S}_{xx}]. \tag{9b}$$

From (9a) as the particular case as

$$a_{1y} - b_{1x} = a_{2y} - b_{2x} = b_4 - a_3 = a_4 = b_3 = 0$$

we get the equation

$$(a_1 + b_2) \mathbf{S}_y \wedge \mathbf{S}_x + \mathbf{S} \wedge [a_2 \mathbf{S}_{yy} + (a_1 - b_2) \mathbf{S}_{xy} - b_1 \mathbf{S}_{xx}] + (a_{5y} - b_{5x}) \mathbf{S} +$$

$$(a_{3y} - b_5) \mathbf{S}_x + (a_5 - a_{3x}) \mathbf{S}_y = 0. \tag{10}$$

One of interesting reduction of this equation is the stationary Myrzakulov XIII (M-XIII) equation [28]

$$\mathbf{S} \wedge [a_2 \mathbf{S}_{yy} + (a_1 - b_2) \mathbf{S}_{xy} - b_1 \mathbf{S}_{xx}] + (a_{3y} - b_5) \mathbf{S}_x + (a_5 - a_{3x}) \mathbf{S}_y = 0 \tag{11a}$$

$$a_{5y} - b_{5x} = (a_1 + b_2) \mathbf{S} \cdot (\mathbf{S}_x \wedge \mathbf{S}_y). \tag{11b}$$

It is the stationary case of the following M-XIII equation

$$\mathbf{S}_t = \mathbf{S} \wedge [a_2 \mathbf{S}_{yy} + (a_1 - b_2) \mathbf{S}_{xy} - b_1 \mathbf{S}_{xx}] + (a_{3y} - b_5) \mathbf{S}_x + (a_5 - a_{3x}) \mathbf{S}_y \tag{12a}$$

$$a_{5y} - b_{5x} = (a_1 + b_2) \mathbf{S} \cdot (\mathbf{S}_x \wedge \mathbf{S}_y). \tag{12b}$$

4.1 Heisenberg ferromagnet

Let us consider the following particular case of the system (8)

$$a_1 = a_2 = a_3 = a_4 = b_2 = b_3 = b_4 = b_5 = 0, \quad a_5 = b_1 = 1. \tag{13}$$

So we have (see, e.g., [5-6])

$$\mathbf{r}_x = \mathbf{S}, \quad \mathbf{r}_y = \mathbf{S} \wedge \mathbf{S}_x. \tag{14}$$

Then the system (10) takes the form

$$\mathbf{S}_y = \mathbf{S} \wedge \mathbf{S}_{xx} \tag{15}$$

that is the HF equation.

4.2 Stationary Landau-Lifshitz equation in two dimensions

Let us consider the following particular case of the system (8)

$$\mathbf{r}_x = \mathbf{S} \wedge \mathbf{S}_y, \quad \mathbf{r}_y = -\mathbf{S} \wedge \mathbf{S}_x. \tag{16}$$

Then the system (9) takes the form (see, e.g., the ref. [11])

$$\mathbf{S} \wedge (\mathbf{S}_{yy} + \mathbf{S}_{xx}) = 0 \tag{17}$$

that is the stationary Landau-Lifshitz equation (LLE) which follows from the (2+1)-dimensional LLE

$$\mathbf{S}_t = \mathbf{S} \wedge (\mathbf{S}_{yy} + \mathbf{S}_{xx}). \tag{18}$$

4.3 Stationary M-XIIIA equation

Now we take the following case of the system (8)

$$a_5 = a_{3x} + \phi_x, \quad b_5 = a_{3y} - \phi_y. \tag{19}$$

In this case we have the MF [28]

$$\mathbf{r}_x = a_1 \mathbf{S} \wedge \mathbf{S}_x + a_2 \mathbf{S} \wedge \mathbf{S}_y + a_3 \mathbf{S}_x + (a_{3x} + \phi_x)\mathbf{S} \tag{20a}$$

$$\mathbf{r}_y = b_1 \mathbf{S} \wedge \mathbf{S}_x + b_2 \mathbf{S} \wedge \mathbf{S}_y + a_3 \mathbf{S}_y + (a_{3y} - \phi_y)\mathbf{S}. \tag{20b}$$

The compatibility condition of the equations (20) or the system (11) gives the stationary M-XIIIA equation

$$\mathbf{S} \wedge [a_2 \mathbf{S}_{yy} + (a_1 - b_2)\mathbf{S}_{xy} - b_1 \mathbf{S}_{xx}] + \phi_y \mathbf{S}_x + \phi_x \mathbf{S}_y = 0 \tag{21a}$$

$$\phi_{xy} = \frac{(a_1 + b_2)}{2}\mathbf{S}\cdot(\mathbf{S}_x \wedge \mathbf{S}_y). \tag{21b}$$

It follows from the M-XIIIA equation [28]

$$\mathbf{S}_t = \mathbf{S}\wedge[a_2\mathbf{S}_{yy} + (a_1 - b_2)\mathbf{S}_{xy} - b_1\mathbf{S}_{xx}] + \phi_y\mathbf{S}_x + \phi_x\mathbf{S}_y \tag{22a}$$

$$\phi_{xy} = \frac{(a_1 + b_2)}{2}\mathbf{S}\cdot(\mathbf{S}_x \wedge \mathbf{S}_y) \tag{22b}$$

in the stationary limit.

4.4 Stationary M-XIIIB equation

Let

$$a_5 = a_{3x} + \phi_y, \quad b_5 = a_{3y} - \phi_x \tag{23}$$

so that instead of (8) we get the following MF [28]

$$\mathbf{r}_x = a_1\mathbf{S}\wedge\mathbf{S}_x + a_2\mathbf{S}\wedge\mathbf{S}_y + a_3\mathbf{S}_x + (a_{3x} + \phi_y)\mathbf{S} \tag{24a}$$

$$\mathbf{r}_y = b_1\mathbf{S}\wedge\mathbf{S}_x + b_2\mathbf{S}\wedge\mathbf{S}_y + a_3\mathbf{S}_y + (a_{3y} - \phi_x)\mathbf{S}. \tag{24b}$$

In this case the system (11) takes the form

$$\mathbf{S}\wedge[a_2\mathbf{S}_{yy} + (a_1 - b_2)\mathbf{S}_{xy} - b_1\mathbf{S}_{xx}] + \phi_x\mathbf{S}_x + \phi_y\mathbf{S}_y = 0 \tag{25a}$$

$$\phi_{xx} + \phi_{yy} = (a_1 + b_2)\mathbf{S}\cdot(\mathbf{S}_x \wedge \mathbf{S}_y) \tag{25b}$$

that is the stationary M-XIIIB equation. Itself the M-XIIIB equation can be written as [28]

$$\mathbf{S}_t = \mathbf{S}\wedge[a_2\mathbf{S}_{yy} + (a_1 - b_2)\mathbf{S}_{xy} - b_1\mathbf{S}_{xx}] + \phi_x\mathbf{S}_x + \phi_y\mathbf{S}_y \tag{26a}$$

$$\phi_{xx} + \phi_{yy} = (a_1 + b_2)\mathbf{S}\cdot(\mathbf{S}_x \wedge \mathbf{S}_y). \tag{26b}$$

4.5 Stationary Ishimori equation

Similarly can be shown that the stationary Ishimori equation

$$\mathbf{S}\wedge(\mathbf{S}_{xx} + \alpha^2\mathbf{S}_{yy}) + \phi_x\mathbf{S}_y + \phi_y\mathbf{S}_x = 0 \tag{27a}$$

$$\alpha^2\phi_{yy} - \phi_{xx} = \alpha^2\mathbf{S}\cdot(\mathbf{S}_x \wedge \mathbf{S}_y) \tag{27b}$$

also can be obtained from the compatibility condition of the particular case of the MF (8) [28].

5 Surfaces induced by the generalized Myrzakulov formula

Now we consider the following generalized MF [28]

$$\mathbf{r}_x = a_1\mathbf{N}\wedge\mathbf{N}_x + a_2\mathbf{N}\wedge\mathbf{N}_y + a_3\mathbf{N}_x + a_4\mathbf{N}_y + a_5\mathbf{N} + \mathbf{A} + C\mathbf{r} + \mathbf{R}_1\wedge\mathbf{r} \quad (28a)$$

$$\mathbf{r}_y = b_1\mathbf{N}\wedge\mathbf{N}_x + b_2\mathbf{N}\wedge\mathbf{N}_y + b_3\mathbf{N}_x + b_4\mathbf{N}_y + b_5\mathbf{N} + \mathbf{B} + D\mathbf{r} + \mathbf{R}_2\wedge\mathbf{r}. \quad (28b)$$

We cite a few particular examples of this generalized MF (for details see, e.g., the ref. [28]).

5.1 Example 1.

Let us assume that the tangent vectors are given by

$$\mathbf{r}_x = C\mathbf{r}, \quad \mathbf{r}_y = D\mathbf{r} \quad (29)$$

where C, D are 3×3 matrices. Hence we get

$$C_y - D_x + [C, D] = 0. \quad (30)$$

Let

$$C = \begin{pmatrix} 0 & k & 0 \\ -k & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & \omega_1 & 0 \end{pmatrix}. \quad (31)$$

If we introduce the complex function $\psi = \frac{k}{2}e^{-i\partial_x^{-1}\tau}$. Then from (30) we get [1]

$$i\psi_t + \psi_{xx} + 2|\psi|^2\psi = 0 \quad (32)$$

which is the NLSE.

5.2 Example 2.

Now we consider the case when the tangent vectors are given by

$$\mathbf{r}_x = \mathbf{R}_1\wedge\mathbf{r}, \quad \mathbf{r}_y = \mathbf{R}_2\wedge\mathbf{r} \quad (33)$$

where $\mathbf{R}_i$ are some vectors. Hence we get

$$\mathbf{R}_{1y} - \mathbf{R}_{2x} + 2\mathbf{R}_1\wedge\mathbf{R}_2 = 0. \quad (34)$$

6 Conclusion

In this note, we have considered some spin surfaces induced by SS in two dimensions. The corresponding SS are presented. These SS include, the stationary Ishimori equation, Heisenberg ferromagnet, the LLE and so on. At last, in Appendix, we presented some generalized SS, so-called Myrzakulov equations, which describe spin-phonon or magnetoelastic systems.

7 *Appendix. Some magnetoelastic systems in 1+1 dimensions*

In this Appendix we present some spin-phonon or magnetoelastic systems in 1+1 dimensions, so-called Myrzakulov equations, which were obtained in [28]. Some of these equations are integrable, for instance, the Myrzakulov V (M-V), Myrzakulov XXXIV (M-XXXIV) and Myrzakulov LXIX (LXIX) equations.

Table 1. *Nonlinear magnetoelastic systems of the 0-type (The Landau-Lifshitz equations with potentials)*

Name of the equation	Equation of motion
the M-LVII equation	$2iS_t = [S, S_{xx}] + u[S, \sigma_3]$
the M-LVI equation	$2iS_t = [S, S_{xx}] + uS_3[S, \sigma_3]$
the M-LV equation	$2iS_t = \{(\mu\vec{S}_x^2 - u + m)[S, S_x]\}_x$
the M-LIV equation	$2iS_t = n[S, S_{xxxx}] + 2\{(\mu\vec{S}_x^2 - u + m)[S, S_x]\}_x$
the M-LIII equation	$2iS_t = [S, S_{xx}] + 2iuS_x$

Table 2. *Nonlinear magnetoelastic systems of the 1-type*

Name of the equation	Equation of motion
the M-LII equation	$2iS_t = [S, S_{xx}] + u[S, \sigma_3]$ $\rho u_{tt} = \nu_0^2 u_{xx} + \lambda(S_3)_{xx}$
the M-LI equation	$2iS_t = [S, S_{xx}] + u[S, \sigma_3]$ $\rho u_{tt} = \nu_0^2 u_{xx} + \alpha(u^2)_{xx} + \beta u_{xxxx} + \lambda(S_3)_{xx}$
the M-L equation	$2iS_t = [S, S_{xx}] + u[S, \sigma_3]$ $u_t + u_x + \lambda(S_3)_x = 0$
the M-XLIX equation	$2iS_t = [S, S_{xx}] + u[S, \sigma_3]$ $u_t + u_x + \alpha(u^2)_x + \beta u_{xxx} + \lambda(S_3)_x = 0$

Table 3. *Nonlinear magnetoelastic systems of the 2-type*

Name of the equation	Equation of motion
the M-XLVIII equation	$2iS_t = [S, S_{xx}] + uS_3[S, \sigma_3]$ $\rho u_{tt} = \nu_0^2 u_{xx} + \lambda(S_3^2)_{xx}$
the M-XLVII equation	$2iS_t = [S, S_{xx}] + uS_3[S, \sigma_3]$ $\rho u_{tt} = \nu_0^2 u_{xx} + \alpha(u^2)_{xx} + \beta u_{xxxx} + \lambda(S_3^2)_{xx}$
the M-XLVI equation	$2iS_t = [S, S_{xx}] + uS_3[S, \sigma_3]$ $u_t + u_x + \lambda(S_3^2)_x = 0$
the M-XLV equation	$2iS_t = [S, S_{xx}] + uS_3[S, \sigma_3]$ $u_t + u_x + \alpha(u^2)_x + \beta u_{xxx} + \lambda(S_3^2)_x = 0$

Table 4. *Nonlinear magnetoelastic systems of the 3-type*

Name of the equation	Equation of motion
the M-XLIV equation	$2iS_t = \{(\mu\vec{S}_x^2 - u + m)[S, S_x]\}_x$ $\rho u_{tt} = \nu_0^2 u_{xx} + \lambda(\vec{S}_x^2)_{xx}$
the M-XLIII equation	$2iS_t = \{(\mu\vec{S}_x^2 - u + m)[S, S_x]\}_x$ $\rho u_{tt} = \nu_0^2 u_{xx} + \alpha(u^2)_{xx} + \beta u_{xxxx} + \lambda(\vec{S}_x^2)_{xx}$
the M-XLII equation	$2iS_t = \{(\mu\vec{S}_x^2 - u + m)[S, S_x]\}_x$ $u_t + u_x + \lambda(\vec{S}_x^2)_x = 0$
the M-XLI equation	$2iS_t = \{(\mu\vec{S}_x^2 - u + m)[S, S_x]\}_x$ $u_t + u_x + \alpha(u^2)_x + \beta u_{xxx} + \lambda(\vec{S}_x^2)_x = 0$

Table 5. *Nonlinear magnetoelastic systems of the 4-type*

Name of the equation	Equation of motion
the M-XL equation	$2iS_t = [S, S_{xxxx}] + 2\{(\mu\vec{S}_x^2 - u + m)[S, S_x]\}_x$ $\rho u_{tt} = \nu_0^2 u_{xx} + \lambda(\vec{S}_x^2)_{xx}$
the M-XXXIX equation	$2iS_t = [S, S_{xxxx}] + 2\{(\mu\vec{S}_x^2 - u + m)[S, S_x]\}_x$ $\rho u_{tt} = \nu_0^2 u_{xx} + \alpha(u^2)_{xx} + \beta u_{xxxx} + \lambda(\vec{S}_x^2)_{xx}$
the M-XXXVIII equation	$2iS_t = [S, S_{xxxx}] + 2\{(\mu\vec{S}_x^2 - u + m)[S, S_x]\}_x$ $u_t + u_x + \lambda(\vec{S}_x^2)_x = 0$
the M-XXXVII equation	$2iS_t = [S, S_{xxxx}] + 2\{(\mu\vec{S}_x^2 - u + m)[S, S_x]\}_x$ $u_t + u_x + \alpha(u^2)_x + \beta u_{xxx} + \lambda(\vec{S}_x^2)_x = 0$

Table 6. *Nonlinear magnetoelastic systems of the 5-type*

Name of the equation	Equation of motion
the M-XXXVI equation	$2iS_t = [S, S_{xx}] + 2iuS_x$ $\rho u_{tt} = \nu_0^2 u_{xx} + \frac{\lambda}{4}(tr(S_x^2))_{xx}$
the M-XXXV equation	$2iS_t = [S, S_{xx}] + 2iuS_x$ $\rho u_{tt} = \nu_0^2 u_{xx} + \alpha(u^2)_{xx} + \beta u_{xxxx} + \frac{\lambda}{4}(tr(S_x^2))_{xx}$
the M-XXXIV equation	$2iS_t = [S, S_{xx}] + 2iuS_x$ $u_t + u_x + \frac{\lambda}{4}(tr(S_x^2))_x = 0$
the M-XXXIII equation	$2iS_t = [S, S_{xx}] + 2iuS_x$ $u_t + u_x + \alpha(u^2)_x + \beta u_{xxx} + \frac{\lambda}{4}(tr(S_x^2))_x = 0$

Table 7. *Nonlinear magnetoelastic systems of the 6-type*

Name of the equation	Equation of motion
the M-LXIX equation	$\mathbf{S}_t = \frac{1}{\sqrt{\mathbf{S}_x^2}}(-\sqrt{\mathbf{S}_x^2 - u^2}\mathbf{S}_x + u\mathbf{S} \wedge \mathbf{S}_x)$ $u_x = v\sqrt{\mathbf{S}_t^2 - u^2}$ $v_t = -\mathbf{S} \cdot (\mathbf{S}_t \wedge \mathbf{S}_x)$

Table 8. *Nonlinear magnetoelastic systems of the 7-type*

Name of the equation	Equation of motion
the M-V equation	$iS_t = \frac{1}{2}[S, S_{xx}] + \frac{3}{2}[S^2, (S^2)_{xx}], \quad S^3 = S \in osp(2\|1)$

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