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G-essence cosmologies with scalar-fermion interactions

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Abstract. We study the two particular models of g-essence with Yukawa-type interactions between a scalar field ϕ and a classical Dirac field ψ. For the homogeneous, isotropic and flat Friedmann-Robertson-Walker universe filled with the such g-essence; some exact solutions of these models are found. Moreover, we reconstruct the corresponding scalar and fermionic potentials.

1 Introduction

The recent data from type Ia supernovae and cosmic microwave background radiation and so on [1,2] have provided strong evidence for a spatially flat and accelerated expanding universe at the present time. This accelerated expansion of the universe is attributed to the domination of a component with negative pressure, called dark energy. So far, the nature of dark energy remains a mystery. In the literature, plenty of dark-energy candidates has been proposed. Among which are quintessence ($w > -1$), phantom ($w < -1$), k-essence ($w > -1$ or $w < -1$), quintom (w crosses -1), etc. Note that the accelerated expansion of the universe can also be obtained by the modified gravity theories like $F(R)$, $F(G)$, $F(T)$, $F(R,T)$ and so on (see, *e.g.*, [3]). As is well known, the simplest model of dark energy is the cosmological constant with energy density being near the vacuum energy $\rho_\Lambda \approx (10^{-3}\,\mathrm{eV})^4$ withoud varying with time. However, this cosmological-constant model of dark energy suffers from the severe problem of fine-tuning and coincidence.

One of the most interesting models of dark energy is the *k-essence* [4–6] (see also [7,8]). Recently, it has been proposed another model of dark energy —the so-called *f-essence* [9]— which is the fermionic counterpart of the k-essence. Also, it has been proposed the *g-essence*, which is some hybrid construction of k-essence and f-essence (see, *e.g.*, [9–12]).

To our knowledge, in the literature there are relatively few works on the dark-energy models with scalar and fermionic fields and with Yukawa-type interactions (see, *e.g.*, refs. [13–17]). In this paper, we study the g-essence cosmologies with Yukawa-type interactions.

This paper is organized as follows. In sect. 2, we simply review the basic information of g-essence in the Friedmann-Robertson-Walker (FRW) universe. In sect. 3, we present the particular model of g-essence with the Yukawa interactions and construct its exact solution. In sect. 4, we introduce the more general particular model of g-essence and present its two exact solutions. Section 5 is the conclusion of the paper.

2 G-essence

The action of g-essence has the form [9]

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + 2K(X, Y, \phi, \psi, \bar{\psi})\right], \tag{2.1}$$

where K is some function of its arguments, ϕ is a scalar function, $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ is a fermionic function and $\bar{\psi} = \psi^+\gamma^0$ is its adjoint function. Here

$$X = 0.5g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi, \qquad Y = 0.5i\left[\bar{\psi}\Gamma^\mu D_\mu\psi - (D_\mu\bar{\psi})\Gamma^\mu\psi\right] \tag{2.2}$$

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are the canonical kinetic terms for the scalar and fermionic fields, respectively. ∇_μ and D_μ are the covariant derivatives. Note that the fermionic fields are treated here as classically commuting fields. The model (2.1) admits two important reductions: k-essence and f-essence.

We now consider the homogeneous, isotropic and flat FRW universe filled with g-essence. In this case, the metric reads

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2), \tag{2.3}$$

and the vierbein is chosen to be (see, *e.g.*, [18])

$$(e_a^\mu) = \mathrm{diag}(1, 1/a, 1/a, 1/a), \qquad (e_\mu^a) = \mathrm{diag}(1, a, a, a). \tag{2.4}$$

In the case of the FRW metric (2.3), the equations corresponding to action (2.1) look like [9]

$$3H^2 - \rho = 0, \tag{2.5}$$
$$2\dot{H} + 3H^2 + p = 0, \tag{2.6}$$
$$K_X\ddot{\phi} + (\dot{K}_X + 3HK_X)\dot{\phi} - K_\phi = 0, \tag{2.7}$$
$$K_Y\dot{\psi} + 0.5(3HK_Y + \dot{K}_Y)\psi - i\gamma^0 K_{\bar{\psi}} = 0, \tag{2.8}$$
$$K_Y\dot{\bar{\psi}} + 0.5(3HK_Y + \dot{K}_Y)\bar{\psi} + iK_\psi\gamma^0 = 0, \tag{2.9}$$
$$\dot{\rho} + 3H(\rho + p) = 0, \tag{2.10}$$

where the kinetic terms, the energy density and the pressure take the form

$$X = 0.5\dot{\phi}^2, \qquad Y = 0.5i(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi), \tag{2.11}$$

and

$$\rho = 2XK_X + YK_Y - K, \qquad p = K. \tag{2.12}$$

3 Model with Yukawa interactions

In this paper, we consider the g-essence action (2.1) with

$$K = X + Y - V_1(\phi) - V_2(u) - \eta\phi u, \tag{3.1}$$

where $\eta = \mathrm{const}$, $u = \bar{\psi}\psi$. Then the system (2.5)–(2.10) takes the form

$$3H^2 - \rho = 0, \tag{3.2}$$
$$2\dot{H} + 3H^2 + p = 0, \tag{3.3}$$
$$\ddot{\phi} + 3H\dot{\phi} + \eta u - V_{1\phi} = 0, \tag{3.4}$$
$$\dot{\psi} + \frac{3}{2}H\psi + iV_2'\gamma^0\psi + i\eta\gamma^0\psi\phi = 0, \tag{3.5}$$
$$\dot{\bar{\psi}} + \frac{3}{2}H\bar{\psi} - iV_2'\bar{\psi}\gamma^0 - i\eta\phi\bar{\psi}\gamma^0 = 0, \tag{3.6}$$
$$\dot{\rho} + 3H(\rho + p) = 0, \tag{3.7}$$

where

$$\rho = 0.5\dot{\phi}^2 + V_1 + \eta\phi u + V_2, \tag{3.8}$$
$$p = 0.5\dot{\phi}^2 - V_1 - V_2 + V_2'u. \tag{3.9}$$

Now we show that the system (3.2)–(3.7) admits exact analytical solutions. To do so, we will use the methods, *e.g.*, of [19–21]. In fact, for example, we find the following particular solution:

$$a = a_0t^\lambda, \tag{3.10}$$
$$\phi = \phi_0 t^{2-3\lambda}, \tag{3.11}$$
$$\psi_l = \frac{c_l}{a_0^{1.5}t^{1.5\lambda}}e^{-iD} \qquad (l = 1, 2), \tag{3.12}$$
$$\psi_k = \frac{c_k}{a_0^{1.5}t^{1.5\lambda}}e^{iD} \qquad (k = 3, 4), \tag{3.13}$$

where c_j obey the following condition: $c = |c_1|^2 + |c_2|^2| - |c_3|^2 - |c_4|^2$, $\phi_0 = -\frac{\eta c}{a_0^3(2-3\lambda)}$ and

$$D = -\frac{2\lambda a_0^3}{c(3\lambda-1)}t^{3\lambda-1} + \frac{\eta^2 c}{3a_0^3(1-\lambda)}t^{3(1-\lambda)} + D_0, \qquad D_0 = \text{const}. \tag{3.14}$$

The corresponding potentials take the form

$$V_1(\phi) = \frac{\phi_0^2\delta^2(\delta-1+3\lambda)}{2(\delta-1)}\left(\frac{\phi}{\phi_0}\right)^{\frac{2(\delta-1)}{\delta}} + \frac{\phi_0\delta\eta c}{a_0^3(\delta-3\lambda)}\left(\frac{\phi}{\phi_0}\right)^{\frac{\delta-3\lambda}{\delta}} + V_{10}, \tag{3.15}$$

$$V_2(u) = 3\lambda^2\left(\frac{ua_0^3}{c}\right)^{\frac{2}{3\lambda}} + \frac{3\phi_0^2\delta^2\lambda}{2(\delta-1)}\left(\frac{ua_0^3}{c}\right)^{-\frac{2(\delta-1)}{3\lambda}} + \frac{3\lambda\eta\phi_0 c}{a_0^3(\delta-3\lambda)}\left(\frac{ua_0^3}{c}\right)^{-\frac{\delta-3\lambda}{3\lambda}} - V_{10}, \tag{3.16}$$

where $V_{10} = \text{const}$, $\delta = 2-3\lambda$, $u = \frac{c}{a^3} = \frac{c}{a_0^3 t^{3\lambda}}$. For this solution, the equation of state and the deceleration parameters take the form

$$w = -1 + \frac{2}{3\lambda}, \qquad q = \frac{1-\lambda}{\lambda}. \tag{3.17}$$

So we conclude that, in this case, the *g*-essence model (2.1) with (3.1) can describe the accelerated expansion of the universe.

4 General model with Yukawa-type interactions

In this section, we consider the *g*-essence action (2.1) with

$$K = \epsilon X + \sigma Y - V_1(\phi) - V_2(u) - \eta U_1(\phi)U_2(u), \tag{4.1}$$

where ϵ and σ are some constants. Here we can note that $\epsilon = 1$ ($\epsilon = -1$) corresponds to the usual (phantom) case. Then the system (2.5)–(2.10) takes the form

$$3H^2 - \rho = 0, \tag{4.2}$$

$$2\dot{H} + 3H^2 + p = 0, \tag{4.3}$$

$$\epsilon\ddot{\phi} + 3\epsilon H\dot{\phi} + \eta U_2 U_{1\phi} - V_{1\phi} = 0, \tag{4.4}$$

$$\sigma\dot{\psi} + \frac{3}{2}\sigma H\psi + iV_2'\gamma^0\psi + i\eta U_1 U_2'\gamma^0\psi = 0, \tag{4.5}$$

$$\sigma\dot{\overline{\psi}} + \frac{3}{2}\sigma H\overline{\psi} - iV_2'\overline{\psi}\gamma^0 - i\eta U_1 U_2'\overline{\psi}\gamma^0 = 0, \tag{4.6}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{4.7}$$

where

$$\rho = 0.5\epsilon\dot{\phi}^2 + V_1 + \eta U_1 U_2 + V_2, \tag{4.8}$$

$$p = 0.5\epsilon\dot{\phi}^2 - V_1 - V_2 + V_2' u. \tag{4.9}$$

As in the previous case, we can construct the exact analytical solutions of the system (4.2)–(4.7).

i) As an example, here we present the following particular solution:

$$a = a_0 t^\lambda, \tag{4.10}$$

$$\phi = \phi_0 t^\delta, \tag{4.11}$$

$$\psi_l = \frac{c_l}{a_0^{1.5}t^{1.5\lambda}}e^{-iD} \qquad (l = 1, 2), \tag{4.12}$$

$$\psi_k = \frac{c_k}{a_0^{1.5}t^{1.5\lambda}}e^{iD} \qquad (k = 3, 4), \tag{4.13}$$

where c_j obey the following condition: $c = |c_1|^2 + |c_2|^2| - |c_3|^2 - |c_4|^2$, $\phi_0 = -\frac{\eta c}{a_0^3(2-3\lambda)}$ and

$$D = \frac{i}{\sigma}\left(-\frac{2\lambda a_0^3}{c(3\lambda-1)}t^{3\lambda-1} + \frac{\epsilon\phi_0^2\delta^2 a_0^3}{c(2\delta-1+3\lambda)}t^{2\delta-1+3\lambda} + \frac{\eta a_0^3 U_{10}U_{20}}{c(n+l+3\lambda+1)}\left(1+\frac{n}{3\lambda}\right)t^{n+l+3\lambda+1} + D_0\right). \tag{4.14}$$

In this case, the corresponding potentials look like

$$V_1(\phi) = \frac{\epsilon\phi_0^2\delta^2(\delta-1+3\lambda)(2\delta+1+3\lambda)}{2(\delta-1)(4(\delta-1)-n+3\lambda)}\left(\frac{\phi}{\phi_0}\right)^{\frac{2(\delta-1)}{\delta}} + V_{10}, \tag{4.15}$$

$$V_2(u) = 3\lambda^2\left(\frac{ua_0^3}{c}\right)^{\frac{2}{3\lambda}} + \frac{3\epsilon\phi_0^2\delta^2\lambda(2(\delta-1)-n-3\lambda)}{2(\delta-1)(4(\delta-1)-n+3\lambda)}\left(\frac{ua_0^3}{c}\right)^{-\frac{2(\delta-1)}{3\lambda}} - V_{10}, \tag{4.16}$$

$$U_1(\phi) = -\frac{2\epsilon\phi_0^2\delta^2(\delta-1+3\lambda)}{\eta U_{20}(4(\delta-1)-n+3\lambda)}\left(\frac{\phi}{\phi_0}\right)^{\frac{2(\delta-1)-n}{\delta}}, \tag{4.17}$$

$$U_2(u) = U_{20}\left(\frac{ua_0^3}{c}\right)^{-\frac{n}{3\lambda}}, \tag{4.18}$$

where $V_{10} = \text{const}$, $\delta = 2-3\lambda$, $u = \frac{c}{a^3} = \frac{c}{a_0^3t^{3\lambda}}$. Also, we can present the equation of state and the deceleration parameters for this solution. We have

$$w = -1 + \frac{2}{3\lambda}, \qquad q = \frac{1-\lambda}{\lambda}. \tag{4.19}$$

So, also for this solution, the g-essence model (2.1) with (4.1) can describe the accelerated expansion of the universe for some values of λ.

ii) As a second example, let us consider the de Sitter-like solution. It reads

$$a = a_0e^{\beta t}, \tag{4.20}$$

$$\phi = \phi_0e^{\kappa t}, \tag{4.21}$$

$$\psi_l = \frac{c_l}{a_0^{1.5}e^{1.5\beta t}}e^{-iD} \qquad (l=1,2), \tag{4.22}$$

$$\psi_k = \frac{c_k}{a_0^{1.5}e^{1.5\beta t}}e^{iD} \qquad (k=3,4), \tag{4.23}$$

where c_j obey the following condition: $c = |c_1|^2 + |c_2|^2 - |c_3|^2 - |c_4|^2$ and

$$D = \frac{i}{\sigma}\left(\frac{\epsilon a_0^3\phi_0^2\kappa^2}{c(2\kappa+3\beta)}e^{(2\kappa+3\beta)t} + \frac{\eta a_0^3U_{10}U_{20}}{c(l+n+3\beta)}\left(1+\frac{n}{3\beta}\right)e^{(l+n+3\beta)t} + D_0\right). \tag{4.24}$$

For the potentials we obtain the following expressions:

$$V_1(\phi) = \frac{\epsilon\phi_0^2\kappa(\kappa+3\beta)(n+3\beta)}{2(4\kappa-n+3\beta)}\left(\frac{\phi}{\phi_0}\right)^2 + V_{10}, \tag{4.25}$$

$$V_2(u) = \frac{3\epsilon\phi_0^2\kappa\beta(2\kappa-n-3\beta)}{2(4\kappa-n+3\beta)}\left(\frac{ua_0^3}{c}\right)^{-\frac{2\kappa}{3\beta}} - V_{10} + 3\beta^2, \tag{4.26}$$

$$U_1(\phi) = -\frac{2\epsilon\phi_0^2\kappa^2(\kappa+3\beta)}{\eta U_{20}(4\kappa-n+3\beta)}\left(\frac{\phi}{\phi_0}\right)^{\frac{2\kappa-n}{\kappa}}, \tag{4.27}$$

$$U_2(u) = U_{20}\left(\frac{ua_0^3}{c}\right)^{-\frac{n}{3\lambda}}, \tag{4.28}$$

where V_{10}, $n = \text{const}$. In this case, the equation of state and the deceleration parameters take the form $w = q = -1$, that is we have the de Sitter spacetime.

5 Conclusion

In this paper, we have studied two particular cases of g-essence with the Yukawa-type interactions between the scalar and the fermion fields. We constructed some examples of exact analytical solutions of these models. The corresponding scalar and fermionic potentials are presented. These results show that the g-essence with the Yukawa interactions can describe the accelerated expansions of the universe. Finally, we would like to note that, quite recently, the models with the classical fermionic fields were studied also in [22,23].

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