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Review of solution methods and applications in real and complex analysis for Functional Equations

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Review of solution methods and applications in real and complex analysis for Functional Equations. At this time, how can we solve functional problems and find solutionsactual question in math world. It was popular in 16 century and many scientiests had tried to find an answer in minimum steps. And now a lot of people are getting interested in this area of mathematics. Our aim was to search how can we solve functional equations. We stopped in Cauchy equations like:

$$f : Q \to Qf(x+y) = f(x) + f(y), then f(x) = f(1)x$$

Also it was shown in this project class of continuous functions. It is easy to show continuous function in figure but we used only mathematical definition: function at point is not breaking-off point then the next rule true : for every ϵ >0 exist δ >0 if |x-a|< δ then |f(x)-f(a)|< ϵ . This rule defined Cauchy and also we defined Geine: fiction at point a is not breaking-off point then next rule true :if x(n) $\to$ a for n $\to \infty$ then f(x(n)) $\to$ f(a).

In the next parts we studied continious function and limited function. In this paragraph it was shown 4 types of continious function:
1) increasing function, 2) not decreasing function , 3) not increasing function, 4) decreasing function. All this types are similar, and when in one problem asking of continious function we can take any one of them. Also differentiated functions were expected. It means that

$$f'(x) = lim \Delta f/\Delta x for \Delta x \to 0, where \Delta f = f(x+\Delta x) - f(x)$$

And in the other to π cs we tried to explain in detail but it was so big profect, so wanted to take the main idea of every paragraph such as exponential function. The main idea of this part is Cauchy equation f(x+y)=f(x)f(y). In this equation solution will be f (x) = ax as we show in that to pic. At this time, how can we solve functional problems and find solutionsactual question in math world. It was popular in 16 century and many scientiests had tried to find an answer in minimum steps. And now a lot of people are getting interested in this area of mathematics. Our aim was to search how can we solve functional equations. We stopped in Cauchy equations like:

$$f : Q \to Qf(x+y) = f(x) + f(y), then f(x) = f(1)x.[1], [5].$$

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A functional equation is the equation in which one or more functions are unknown. For example,

$$f(x) + xf(x+1) = 1$$

$$(x) + g(1-x) = f(g(\frac{2}{x+1}))$$

Some functional equations f (x) = f (-x), f (-x) = - f (x), f (x+T) = f (x) set such properties of functions, as parity, oddness, periodicity. The problem of the solving of the functional equations is one of the oldest in the mathematical analysis. They appeared almost simultaneously with the origins of the theory of functions. The first blossoming of this subject is connected with the problem of the parallelogramme of forces. In 1769 Dalamber has reduced the law of addition of forces to the solution of the functional equation

$$f(x+y)+f(x-y)=2\dot{f}(x)\dot{f}(y) \tag{1}$$

The same equation and with the same purpose was considered by Poisson in 1804 with an assumption of analyticity, whereas in 1821 Cauchy (1789 - 1857) found the common solutions

$$f(x)=\cos ax$$

$$f(x)=chax=\frac{e^{ax}+e^{-ax}}{2}$$

$$f(x)=0$$

of this equation, assuming only the continuity f (x) [3].

Even the known formula of Non-Euclidean Geometry for a parallelism corner

$$f(x)=\tan\frac{1}{2}\prod(x)=e^{-\frac{x}{k}}$$

was obtained by N.I.Lobachevsky (1792 - 1856) from the functional equation

$$f^2(x)=f(x-y)\cdot(x+y) \tag{2}$$

which he solved using a method similar to the method of Cauchy. It is possible to lead it to the equation

$$f(\frac{x+y}{2})=\frac{f(x)+f(y)}{2}$$

A number of the geometrical problems leading to the functional equations were considered by English mathematician C.Babbedzh (1792-1871). He studied, for instance, periodic curves of the second order defined by the following property for any pair of points of a curve: if the abscissa of the second point is equal to the ordinate of the first then the ordinate of the second point is equal to the abscissa of the first. Let such curve be the graph of the function y = f(x); (x, f (x)) - its ordinary point. Then, according to the condition, the point with the abscissa f(x) has the ordinate x. Thus,

f(f(x))=x (3)

In particular the following functions satisfy the functional equation (3) :

$$f(x)=\sqrt{a^2-x^2}, x\in[0;|a|], f(x)=\frac{a}{x}, a\neq 0[4]$$

One of the elementary functional equations are the equations of Cauchy

$$f(x+y)=f(x)+f(y), \tag{4}$$

$$f(x+y)=f(x)\cdot f(y), \tag{5}$$

$$f(xy) = f(x) + f(y), \tag{6}$$

$$f(xy) = f(x) \cdot f(y), \tag{7}$$

Cauchy studied in detail in his book (the Analysis Course) published in 1821. Continuous solutions of these four basic equations are

$$f(x) = ax, a^x, \log_a x, x^a(x.0), respectively.$$

In the class of discontinuous functions there can be other solutions. The equation (4) was considered earlier by Legendre and Gauss while obtaining the basic theorem of projective geometry and researching of Gauss' law of distribution of probabilities.

The functional equation (4) was one more time applied by Jean Gaston Darboux to the problem of the parallelogram of forces and to the basic theorem of projective geometry; his main achievement - considerable easing of assumptions. We know that the functional equation of Cauchy (4) characterises the linear homogeneous function f (x) = ax. In continuous Darboux showed that any solution is continuous at least in one point or limited from above (or from below) in any small interval, also should look like f (x) = ax. The further results on easing of assumptions followed quickly one after another (integration, measurability on set of a positive measure of the function). There is a question: Is there any additive function (i.e. satisfying (4)), different from the linear homogeneous. Finding such function is really hard! During work we will show that at rational x values of any additive function should coincide with values of some linear homogeneous function, i.e. f (x) = ax for x $\in$ Q [6] .

It would seem that then f (x) = ax for all valid x. If f (x) is continuous, it is valid; if it is not then the given assumption is wrong. The first example different from f (x) = ax.

The invalid solution of the functional equation (4) was constructed in 1905 by the German mathematician G.Gamel by means of the basis of real numbers previously introduced by him.

Many functional equations do not define the given function, and set a wide class of functions, i.e. express the property characterising this or that class of functions. For example, the functional equation f (x+1) = f (x) characterises a class of the functions having the period 1, and the equation f (1+x) = f (1-x) – a class of the functions symmetric to the straight line x = 1, and etc.

In general, we know few common methods of the solutions of the functional equations which are not reduced to differential or integrated. Some approaches will be considered,which allow us to solve the functional equations.

In the given work the functional equations and some ways of their solutions have been considered. During the work we made sure that the functional equations is a general class of the equations in which some functions are required. The differential equations, the integrated equations and the equations in essence concern the functional equations in final differences. As the functional equation in the narrow sense we understand the equations in which required functions are connected with known functions of one or several variables by means of operation of formation of difficult function. The functional equation can be considered also as expression of the property characterising this or that class of functions.

The functional equations have the large application. So, for example, in the theory of analytical functions are often applied to the introduction of new classes of functions. For example, double periodic functions are characterised by the functional equations f (z +a) = f (z) and f (z + b) = f (z). If the function is known in some area then knowledge for the functional equation allows to expand range of definition of this function. For example, the functional equation f (x + 1) = f (x) for periodic functions allows to define their value in any point on values on a [0, 1]. This is often useful for analytical continuation of functions of the complex variable. For example, using functional equation Γ (z + 1) = zΓ (z) and knowing values of function Γ (z) (Γ (z) - Gamma function) in the $0 \leq_R ez \leq 1$, it is possible to continue it on all plane z.[7]

The conditions of symmetry which are available in any physical problem, cause certain laws of transformation of solutions of this problem at those or other transformations of co-ordinates. It defines the functional equations with which should satisfy the solution of the given problem. Value of the corresponding functional equations in many cases facilitates a finding of solutions.

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Гюль М.

В это время, есть фактический вопрос в математическом мире, как мы можем решает функциональные задачи и находит фактические решения. Это было популярно в 16-ом столетии, и многие ученые попытались найти ответ в минимальных шагах. И теперь многие люди интересуются этой областью математики. Наша цель состояла в поиске методов решении функциональных уравнений. Мы остановились на уравнениях Коши :

$$f : Q \to Q f(x+y) = f(x) + f(y), then f(x) = f(1)x$$

Также в этойстатье показалинепрерывные функций. Легко показать непрерывную функцию на графике, но мы использовали только математическое определение: функция в точке, не точка разрыва, тогда следующее верное правило: для каждого $\epsilon > 0$ существуют $\Delta > 0$ если |x-a | $< \Delta$ тогда |f (x)-f (a) | $< \epsilon$. Это правило определил Коши,а также мы определили Geine: функция в точке не точка разрыва тогда следующее правило верно: если x (n) $\to$ a для n $\to \infty$ тогда f (x (n)) $\to$ f (a).

В следующих частях мы изучили продолжительнкю и ограниченную функцию. В этом параграфе показали 4 типа продолжительной функции: 1) возрастающая функция, 2) не убывающая функция, 3) не возрастающая функция, 4) убывающая функция. Все эти виды подобны, и когда в задаче спрашивается продолжительная функция, мы можем взять любой из этих видов. Также билирассмотрены дифференцированные функции. Это означает это

$$f'(x) = lim \Delta f/\Delta x for \Delta x \to 0, where \Delta f = f(x+\Delta x) - f(x)$$

В других главах мы попытались объяснить подробно, но это был очень комплекснаяработа, чтобы взять главную идею каждого параграфа, такого как показательная функция. Главная идея этой части - уравнение Коши f (x+y) =f (x) f (y).

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