Ann. Phys. (Leipzig) **15**, No. 9, 671 – 677 (2006) / **DOI** 10.1002/andp.200610208

Strings and branes under microscope

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Received 10 September 2004, accepted 20 February 2006 by K. Fredenhagen
Published online 5 April 2006

Key words Kaluza-Klein gravity, wormhole-like solution, string.
PACS 04.50.+h

It is shown that in the standard vacuum 5D Kaluza-Klein gravity there exist wormhole-like solutions which look like strings attached to two D-branes. The asymptotic behaviour of the corresponding metric is investigated.

1 Introduction

In string theory the strings are the matter and consequently the inner structure of a string is not defined. It is like to the situation in classical and quantum field theories: the point-like electron does not have any inner structure. It is well known that the structureless electron leads to such difficulties as the infiniteness electron mass, infinities in loops of Feynman diagrams and so on. In string theory the pointlike electron is stretched out in one dimension and the inner structure remains yet unknown.

Einstein has offered an idea that pointlike electron has an inner structure and it is a bridge (wormhole in the modern language) between remote parts of a Universe. Up to now this program is not realized as there are many difficulties connected with this idea. For instance the geometrical interpretation of the spin is not clear in this electron model. In [1] it is shown that this idea may be applicable for strings and not for pointlike particles. In these papers it is shown that in the standard 5D Kaluza-Klein gravity there exist flux tube solutions filled with electric and magnetic fields and a certain part of this solution can be superlong and superthin. The cross section of this flux tube can be choosen in the Planck region and consequently the tube can be considered as 1-dimensional object, namely a $\Delta-$string. Δ means that the ends of the tube (where the string is attached to an external space) are similar to a river delta because a spacetime foam spreads these attached points.

In [1] the properties of the central part of the solution are investigated. In this paper the peripheral parts of the $\Delta-$string solution will be studied. We will see that they have asymptotically flat spaces (tails) and intermediate regions. By neglecting the intermediate regions the whole construction is similar to D-branes with strings stretched between them.

2 The metric and $\Delta-$string part of solution

We investigate the 5D metric $G_{AB}, A, B = 0, 1, 2, 3, 5$ in the following form

$$ds^2 = \frac{a(r)}{\Delta(r)}dt^2 - dr^2 - a(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - \frac{\Delta(r)}{a(r)}e^{2\psi(r)}\left(d\chi + \omega(r)dt + Q\cos\theta d\varphi\right)^2 \quad (1)$$

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the functions $a(r)$, $\Delta(r)$ and $\psi(r)$ are even functions and consequently the metric has a wormhole-like form; Q is the magnetic charge. The form of $G_{5\mu} = (\omega, 0, 0, Q\cos\theta)$ $(\mu = 0, 1, 2, 3)$ leads to the appearance of radial electric and magnetic F fields. The 5D Einstein's equations are

$$R_{AB} - \frac{1}{2}\eta_{AB}R = 0 \; . \tag{2}$$

Here A, B are 5-bein indices; R_{AB} and R are 5D Ricci tensor and the scalar curvature respectively; $\eta_{AB} = diag\{1, -1, -1, -1, -1\}$. The 5D Einstein's equations for the metric (1) are

$$R_{15} = \omega'' + \omega'\left(-\frac{a'}{a} + 2\frac{\Delta'}{\Delta} + 3\psi'\right) = 0, \tag{3}$$

$$-R_{33} = \frac{a''}{a} + \frac{a'\psi'}{a} - \frac{2}{a} + \frac{Q^2\Delta e^{2\psi}}{a^3} = 0, \tag{4}$$

$$R_{11} - R_{55} = \psi'' + \psi'^2 + \frac{a'\psi'}{a} - \frac{Q^2\Delta e^{2\psi}}{2a^3} = 0, \tag{5}$$

$$2R_{11} + R_{22} - R_{33} - R_{55} = \frac{\Delta''}{\Delta} - \frac{\Delta' a'}{\Delta a} + 3\frac{\Delta'\psi'}{\Delta} + \frac{2}{a} - 6\frac{a'\psi'}{a} = 0, \tag{6}$$

$$\begin{aligned} -R_{55} + R_{22} - 2R_{33} + 2R_{11} = {} & \frac{\Delta'^2}{\Delta^2} + \frac{4}{a} - \frac{q^2 e^{-4\psi}}{\Delta^2} - \frac{Q^2\Delta e^{2\psi}}{a^3} - 6\frac{a'\psi'}{a} \\ & -2\frac{\Delta' a'}{\Delta a} + 2\frac{\Delta'\psi'}{\Delta} = 0. \end{aligned} \tag{7}$$

The solution of Maxwell equation (3) is

$$\omega' = \frac{qae^{-3\psi}}{\Delta^2} \tag{8}$$

here q is the electric charge. The solutions are parametrized by electric q and magnetic Q charges [3]: as the relative strengths of the electric and magnetic fields are varied it is found that the solutions evolve in a following way:

1. $0 < Q < q$. The solution is a wormhole-like object. The throat between the surfaces at $\pm r_H$ (r_H is defined as follows: $\Delta(\pm r_H) = 0$) is filled with both electric and magnetic fields. The longitudinal distance between the $\pm r_H$ surfaces increases by $q \to Q$.
2. $q = Q$. In this case the solution is an infinite flux tube filled with constant electrical and magnetic fields. The cross-sectional size of this solution is constant ($a = const.$).
3. $0 < q < Q$. In this case we have a singular finite flux tube located between two (+) and (-) electrical and magnetic charges located at $\pm r_0$. Thus the longitudinal size of this object is finite, but now the cross sectional size decreases as $r \to r_0$.

By $q = Q$ the solution is the infinite flux tube

$$a(r) = a(0) = \frac{Q_0^2}{2} = const, \tag{9}$$

$$e^{\psi(r)} = \frac{a(0)}{\Delta(r)} = \cosh\frac{r}{\sqrt{a(0)}}, \tag{10}$$

$$\omega(r) = \sqrt{2}\sinh\frac{r}{\sqrt{a(0)}}, \tag{11}$$

$$E = \frac{q_0}{a(0)}, \quad H = \frac{Q_0}{a(0)} \tag{12}$$

here we have parallel electric E and magnetic H fields with equal $q_0 = Q_0 = \sqrt{2a(0)}$ electric and magnetic charges, $a_0 = a(0)$. This solution is the 5D analog of the 4D Levi-Civita-Robertson-Bertotti solution [4]-[5]. The $\Delta-$string solution is the solution with $\delta = 1 - q/q_0 \ll 1$, $q > Q$. In [2] this solution in the region $|r| < r_H$ is investigated. The topology of this spacetime is $R \times S^1 \times S^2 \times [-r_H, +r_H]$, where R is the time dimension; S^1 is the 5-th dimension; S^2 is the 2-sphere spanned on θ and φ angles; $r \in [-r_H, +r_H]$ is the radial coordinate. The linear sizes of S^1 and S^2 are in the Planck region.

In [2] the next relation using numerical and approximate analytical calculations is derived

$$a(r) + \Delta(r)e^{2\psi(r)} \approx 2a(0). \tag{13}$$

For $q = 0$ and $q = Q$ this relation is exact but for the $q < Q$ and in the region $r \in [-r_H, +r_h]$ it was verified using numerical and approximate analytical calculations only. This equation shows us that in the region $|r| < r_H$ the cross section of the flux tube has the same order: $a(0) < a(r) < 2a(0)$. If $a(0) \approx l_{Pl}^2$ then the tube can be considered as one dimensional object because l_{Pl} is the least length in nature. Also the investigations show us that the length L of the tube can be arbitrary long depending on δ: $L \xrightarrow{\delta \to 0} \infty$. This allows us to call this flux tube as a string-like object: $\Delta-$string.

Now we would like to show that the $\Delta-$string solution is nonsingular at the points $\pm r_H$ where $\Delta(\pm r_H) = 0$. For this we investigate the solution near to the point $|r| \approx r_H$ where

$$a(r) = a_0 + a_1 (r - r_H) + a_2 (r - r_H)^2 + \cdots, \tag{14}$$

$$\psi(r) = \psi_H + \psi_1 (r - r_H) + \psi_2 (r - r_H)^2 + \cdots, \tag{15}$$

$$\Delta(r) = \Delta_1 (r - r_H) + \Delta_1 \Delta_2 (r - r_H)^2 + \cdots. \tag{16}$$

The substitution in Eqs. (3)–(7) yields the following solution

$$\Delta_1 = \pm q e^{-2\psi_H}, \quad (+) \quad \text{for} \quad r \to -r_H \quad \text{and} \quad (-) \quad \text{for} \quad r \to +r_H, \tag{17}$$

$$\psi_2 = -\psi_1 \frac{a_1 + a_0 \psi_1}{2a_0}, \tag{18}$$

$$\Delta_2 = \frac{-3a_0\psi_1 + a_1}{2a_0}, \tag{19}$$

$$a_2 = \frac{2 - a_1\psi_1}{2}. \tag{20}$$

In this case Eq. (8) has the following behaviour near to the points $r = \pm r_H$

$$\omega'(r) = \frac{a_0 e^{\psi_H}}{q} \frac{1}{(r - r_H)^2} + \omega_1 + \mathcal{O}(r - r_H) \tag{21}$$

where ω_1 is some constant depending on $a_{0,1}, \psi_{1,2}, \Delta_{1,2}$. It leads to the following $\omega(r)$ behaviour

$$\omega(r) = -\frac{a_0 e^{\psi_H}}{q} \frac{1}{(r - r_H)} + \omega_0 + \mathcal{O}(r - r_H) \tag{22}$$

where ω_0 is some integration constant. The G_{tt} metric component is

$$G_{tt} = \frac{a(r)}{\Delta(r)} - \frac{\Delta(r)e^{2\psi(r)}}{a(r)}\omega^2(r) = -e^{2\psi_H} \frac{2qe^{-\psi_H}\omega_0 - a_1 - a_0\psi_1}{q} + \mathcal{O}(r - r_H). \tag{23}$$

Then the metric (1) has the following approximate behaviour near to $r = \pm r_H$:

$$\begin{aligned} ds^2 = & \left[g_H + \mathcal{O}(r - r_H)\right] dt^2 - \mathcal{O}(r - r_H)(d\chi + Q\cos\theta d\varphi)^2 - \\ & \left[e^{\psi_H} + \mathcal{O}(r - r_H)\right] dt\,(d\chi + Q\cos\theta d\varphi) - dr^2 - \\ & \left[a(r_H) + \mathcal{O}(r - r_H)\right]\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \approx g_H dt^2 - e^{\psi_H}(d\chi + Q\cos\theta d\varphi)\,dt - \\ & dr^2 - a(r_H)\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \end{aligned} \tag{24}$$

where $g_H = -e^{2\psi_H}\left(2qe^{-\psi_H}\omega_0 - a_1 - a_0\psi_1\right)/q$. It means that at the points $r = \pm r_H$ the metric (1) is nonsingular.

3 The tails of the Δ−string

Now we would like to consider the $|r| > r_H$ parts of the solution. The numerical investigation of Eqs. (3)–(7) are presented on Figs. 1–4,

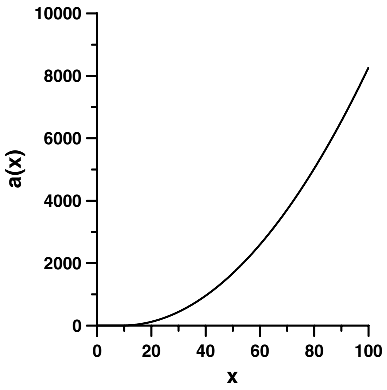

Fig. 1 The function $a(x), \delta \approx 10^{-9}. x = r/\sqrt{a(0)}$ is the dimensionless radius.

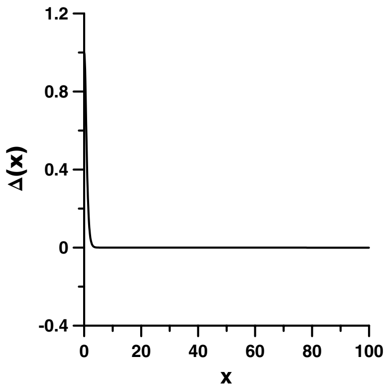

Fig. 2 The function $\Delta(x)$.

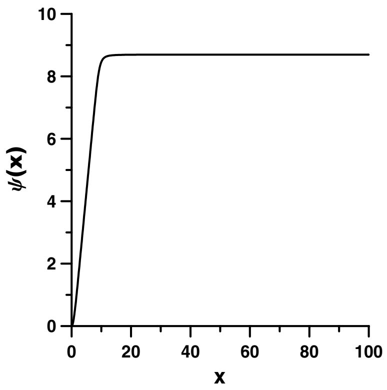

Fig. 3 The function $\psi(x)$.

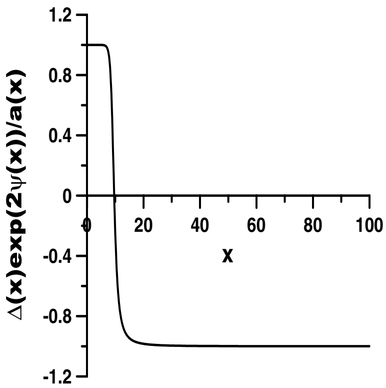

Fig. 4 The function $\frac{\Delta(x)e^{2\psi(x)}}{a(x)}$.

We search the asymptotical behavior of the metric in the form

$$a(r) = r^2 + m_1 r + q_1 + \cdots, \tag{25}$$

$$\Delta(r) = -\Delta_\infty r^2 + \Delta_\infty m_2 r + \Delta_\infty q_2 + \cdots , \tag{26}$$

$$\psi(r) = \psi_\infty + \frac{\psi_1}{r^2} + \cdots . \tag{27}$$

The solution is

$$\psi_1 = -\frac{Q^2 \Delta_\infty e^{2\psi_\infty}}{4}, \tag{28}$$

$$q_1 = \frac{q^2 e^{-4\psi_\infty} + 3Q^2\Delta_\infty^3 e^{2\psi_\infty} - \Delta_\infty^2 m_2^2 - 2\Delta_\infty^2 m_1 m_2}{4\Delta_\infty^2}, \tag{29}$$

$$q_2 = \frac{q^2 e^{-4\psi_\infty} - 3Q^2\Delta_\infty^3 e^{2\psi_\infty} - \Delta_\infty^2 m_2^2}{4\Delta_\infty^2}. \tag{30}$$

The numerical investigation (see, Fig. 4) shows that at the infinity

$$\frac{\Delta(r)e^{2\psi(r)}}{a(r)} \approx -\Delta_\infty e^{2\psi_\infty}\left(1 - \frac{m_1+m_2}{r} - \frac{m_1^2 - q_1 - q_2 + 2\psi_1}{r^2}\right) \to -1 \tag{31}$$

and consequently

$$\Delta_\infty = e^{-2\psi_\infty}. \tag{32}$$

After substitution in Eqs. (28)–(30) we have

$$\psi_1 = -\frac{Q^2}{4}, \tag{33}$$

$$q_1 = \frac{q^2 + 3Q^2 - m_2^2 - 2m_1 m_2}{4}, \tag{34}$$

$$q_2 = \frac{q^2 - 3Q^2 - m_2^2}{4}. \tag{35}$$

As $q \approx l_{Pl}$ and $Q \approx l_{Pl}$ then at the tails of the $\Delta-$string solution ($|r| \gg r_H$)

$$\psi_1 \approx 0, \tag{36}$$

$$q_1 \approx -\frac{2m_1 m_2 + m_2^2}{4}, \tag{37}$$

$$q_2 \approx -\frac{m_2^2}{4}. \tag{38}$$

This fact allows us to say that the size of the intermediate region of the $\Delta-$string solution is $l_{int} \approx m_{1,2}$.

Thus the $\Delta-$string solution has the superthin and superlong flux tube, two intermediate regions and two almost flat spaces, see Fig. 5. If the cross section of the throat is in the Planck region then whole construction looks like a string attached to two D-branes (if we neglect the intermediate regions). Such comparison has a deep physical meaning as we can isometrically insert the considered spacetime in a sufficiently large Minkowski spacetime and then the throat of the superthin and superlong flux tube is a 1D object as the Planck cell is the minimally accessible volume in the nature.

Finally we would like to show that the solution considered here is not an ordinary black metrics presented in review [7]. At first we consider the asymptotical form of the metric (1)

$$\begin{aligned} ds^2 \approx &\left(1 - \frac{m_1+m_2}{r} - \frac{m_1^2 - q_1 - q_2 + 2\psi_1}{r^2}\right)(d\chi + \omega dt + Q\cos\theta d\varphi)^2 - \\ &\frac{1}{\Delta_\infty}\left(1 + \frac{m_1 - m_2}{r} + \frac{q_1 - q_2 + m_2^2}{r^2}\right)dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\big) = \\ &G_{55}\left(d\chi + \omega dt + Q\cos\theta d\varphi\right)^2 + g_{\mu\nu}dx^\mu dx^\nu \end{aligned} \tag{39}$$

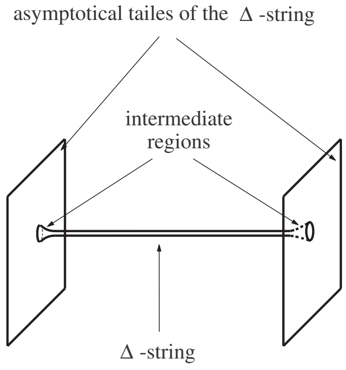

Fig. 5 The spacetime of the $\Delta-$string.

where the asymptotical form of the metric is

$$G_{55} \approx 1 - \frac{m_1 + m_2}{r} - \frac{m_1^2 - q_1 - q_2 + 2\psi_1}{r^2}, \tag{40}$$

$$g_{tt} \approx -\frac{1}{\Delta_\infty}\left(1 + \frac{m_1 - m_2}{r} + \frac{q_1 - q_2 + m_2^2}{r^2}\right), \tag{41}$$

$$g_{\theta\theta} \approx -r^2, \tag{42}$$

$$g_{\theta\theta} \approx -r^2 \sin^2\theta. \tag{43}$$

Immediately we see that the 5^{th} coordinate χ in this region becomes timelike and the time coordinate t spacelike.

For the clarification of the physical meaning of the asymptotical metric we must transform it to the standard Kaluza-Klein form with new 4D metric $\widetilde{g}_{\mu\nu}$ and new electromagnetic potential $\widetilde{A}_\mu$

$$\begin{aligned} ds^2 \approx & \widetilde{G}_{55}\left(dt + \widetilde{A}_\mu dx^\mu\right)^2 + \widetilde{g}_{\mu\nu}dx^\mu dx^\nu = \\ & \left(G_{55}\omega^2 - g_{tt}\right)\left[dt + \frac{G_{55}\omega}{G_{55}\omega^2 - g_{tt}}\left(d\chi + Q\cos\theta d\varphi\right)\right]^2 - \\ & \frac{g_{tt}G_{55}}{G_{55}\omega^2 - g_{tt}}\left(d\chi + Q\cos\theta d\varphi\right)^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\big) \end{aligned} \tag{44}$$

where $\widetilde{G}_{AB}$ $(A, B = 0, 1, 2, 3, 5)$ is the new form of the metric on the tails. Here the t coordinate is the new 5^{th} coordinate and χ is a new time coordinate. This form of the metric shows that there is a rotation connected with the term $d\chi d\varphi$. The φ-component of electromagnetic potential depends on the r and θ coordinates and consequently we have a radial and θ-components of the magnetic field in addition to the radial electric field. All of this leads to the appearance of the angular momentum density for the electromagnetic field which is the origin for the rotation term $d\chi d\varphi$. It allows us to say that the considered metric is none of the black hole solutions presented in the review [7].

4 Conclusions

This investigation shows that in pure multidimensional vacuum gravity there exist wormhole-like solutions which can be approximately considered as two D-branes connected by a string with some intermediate regions. One interesting characteristic property of the presented regular solution is that the metric signature is changed at the points $\pm r_H$: $(+,-,-,-,-)$ for $|r| < r_H$ is changed to $(-,-,-,-,+)$ for $|r| > r_H$. It means that on the $\Delta-$string the time coordinate is t but on the tails (D-branes) the time coordinate is the 5-th coordinate χ.

Acknowledgements I am very grateful to the ISTC grant KR-677 for the financial support.

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