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Some thick brane solutions in $f(R)$-gravity

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Abstract: The thick brane model is considered in $f(R) \sim R^n$ gravity. It is shown that regular asymptotically anti-de Sitter solutions exist in some range of values of the parameter n. A peculiar feature of this model is the existence of a fixed point in the phase plane where all solutions start, and the brane can be placed at this point. The presence of the fixed point allows to avoid fine tuning of the model parameters to obtain thick brane solutions.

Keywords: Integrable Equations in Physics, Field Theories in Higher Dimensions

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1 Introduction

At the present time, the study of the structure and evolution of the Universe is at an interesting stage. The necessity of a consistent description of the present Universe demands the creation of a unified theory of elementary particles and cosmology. A very promising way is the consideration of models of the Universe in higher-dimensional theories. Such investigations were initiated in the works of Kaluza and Klein in the 1920s for the unification of the two fundamental interactions known at that time — gravitation and electromagnetism — within the framework of a unified five-dimensional theory. Later on, similar ideas were used for the unified description of the four currently known fundamental interactions within the framework of superstring theories with several extra space dimensions. As in the case of the Kaluza-Klein theories, in superstring theories, it is supposed that our four-dimensional space-time results from the spontaneous compactification of a higher-dimensional space.

At the same time, models of the Universe with non-compact (and even infinite) extra dimensions are under consideration [1, 2] (for a review, see also [3, 4]). In such a theory, it is supposed that we live on a thin leaf (brane) embedded into some higher-dimensional space (bulk), and matter is somehow confined (trapped) on the brane. The existence of extra dimensions then allows to resolve a number of old problems in high-energy physics (such as the problem of mass hierarchy, stability of the proton, etc.).

All branes can be divided into two classes: thin branes and thick branes. In the first case, one has a delta-like localization of matter on the brane [1, 2]. From a realistic point of view, however, the brane should have some thickness. The inclusion of a brane thickness then yields new possibilities and new problems (for a review, see, e.g., [5]). Such a brane must satisfy two major requirements: 1) the solutions should be regular and asymptotically flat, or de Sitter ones (or anti- de Sitter ones); 2) ordinary matter should be confined to the brane.

Most thick brane models employ scalar fields within the framework of Einstein's theory of gravity (see e.g. the review [5] and references therein). However, one might expect the existence of brane-like solutions also for some kinds of modified gravity theories, the so-called higher-order gravity theories (HOGT). In such theories the action of the Einstein-Hilbert gravitational Lagrangian is supplemented by further terms, which are curvature invariants [6]. (Such a modification is based on the effect of the interaction of quantum matter fields with the classical gravitational field.) This allowed to avoid an initial cosmological singularity and to construct regular cosmological models of the early Universe [7–10]. Later it was shown that in such type of models a stage of inflation can exist [11].

Currently, this last possibility is widely used for the description of the present accelerated expansion of the Universe. This acceleration can be explained by the presence of some antigravitating substance - the so-called dark energy. The description of dark energy can also be realized within the framework of $f(R)$ theory, where $f(R)$ is some arbitrary function of the scalar curvature R. By choosing $f(R) \sim R^n$, it was shown that such models are in good agreement with several different sets of observations [12–14, 21–24]. On the other hand, such theories can be successfully employed for the description of dark matter [25] as well.

HOGT with more complicated combinations of curvature invariants are also under consideration. In particular, in the low-energy limit of M-theory the Gauss-Bonnet invariant appears

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}.$$

It was shown that such models, on the one hand, do not contradict observations within the Solar System and, on the other hand, successfully describe the present accelerated expansion of the Universe [26, 27]. These models can be used in the description of an effective equation of state both for an effective cosmological constant and for the dynamic case (quintessence, phantom dark energy), and also for the description of the transition of one type of dark energy (quintessence) into another one (phantom energy). Also there are theories which employ both $f(R)$ and Gauss-Bonnet terms to describe dark energy [28].

Another usage of HOGT consists in the consideration of higher-dimensional cosmological and astrophysical models. In particular, in ref. [29] brane world and black hole models with higher-dimensional action

$$S = \int d^d x\sqrt{-g}\left[aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \frac{1}{k^2}R - \Lambda + L_m\right],$$

(where L_m is the matter Lagrangian; a, b, c are arbitrary constants) were considered. The obtained results allow to estimate general properties of models within the framework of HOGT.

Furthermore there are some results obtained by the application of HOGT for the creation of brane-world models [30–33]. In particular, Parry et al. [34] considered a brane model in $f(R) = R + \alpha R^2$ theory. Making use of the conformal equivalence of such gravity models and Einstein-Hilbert gravity with a scalar field, the authors rewrote the $f(R)$-equations in the form of the Einstein equations with some scalar field source. They showed that in such models brane-like solutions exist.

Note here that using the conformal transformation, i.e., the transition from the original Jordan frame to the Einstein frame, is not always possible. For example, if there exist any other matter fields then the conformal transformation to a theory with scalar fields may lead to ambiguities (see, e.g., ref. [35] about this question). That is why we prefer to study here the thick brane model without using this transformation.

Whereas Parry et al. [34] considered thin branes, in this paper we investigate thick branes in such $f(R)$ theories to see, whether this leads to new and physically more acceptable results.

2 Equations and solutions in $f(R) \sim R^n$ theory

We will work in a five-dimensional spacetime. The corresponding gravitational action can be taken in the form

$$S = \int d^5x\sqrt{-{}^5g}\left[-\frac{R}{2} + f(R)\right], \tag{2.1}$$

where $f(R)$ is an arbitrary function of the scalar curvature R. (Here we employ units such that $8\pi G = c = 1$.) Variation of the action (2.1) with respect to the 5-dimensional metric tensor g_{AB} led to gravitational equations:

$$R_A^B - \frac{1}{2}\delta_A^B R = \hat{T}_A^B, \tag{2.2}$$

where capital Latin indices run over $A, B, \ldots = 0, 1, 2, 3, 5$, and

$$\hat{T}_A^B = -\left\{\left(\frac{\partial f}{\partial R}\right) R_A^B - \frac{1}{2}\delta_A^B f + \left(\delta_A^B g^{LM} - \delta_A^L g^{BM}\right)\left(\frac{\partial f}{\partial R}\right)_{;L;M}\right\} \tag{2.3}$$

specifies the effective geometric matter source with the nontrivial dependence on curvature; the semicolon denotes the covariant derivative. One can see that the gravitational equations in $f(R)$-gravity rewritten in the form (2.2) have a structure that coincides with the standard general relativity equations when the source of the gravitational field is the effective energy-momentum tensor (2.3). One can check that the energy-momentum conservation law is satisfied as well (see, e.g., [7–10]).

Here we will focus on a special choice of $f(R)$ in the form

$$f(R) = -\alpha R^n, \tag{2.4}$$

where $\alpha > 0$ and n are constants. As shown in refs. [12–14], where the present accelerated expansion of the Universe was considered, there are some ranges of n which do not contradict the observational cosmological data. Therefore it seems natural to consider these values of n for brane models as well.

We will adopt the flat brane model with the metric

$$ds^2 = e^{2y(z)}\eta_{\alpha\beta}dx^\alpha dx^\beta - dz^2, \tag{2.5}$$

where the warp factor function depends on the fifth coordinate z only, and $\eta_{\alpha\beta} = \{1, -1, -1, -1\}$ is the Minkowski metric. Inserting this metric into eqs. (2.2) and (2.3), one obtains from the $\binom{z}{z}$ component of the Einstein equations

$$p\frac{d^2p}{dy^2} + \left(\frac{dp}{dy}\right)^2 + 5p\frac{dp}{dy} = \frac{1}{32p^2 f_{RR}}\left[4p\left(\frac{dp}{dy} + p\right) f_R - \frac{1}{2}f - 6p^2\right], \tag{2.6}$$

where the new variable $p = dy/dz$ was introduced, and the index R denotes the derivative with respect to the scalar curvature R. Eq. (2.6) is a third-order differential equation with respect to the metric function y, and the remaining components of the Einstein equations are fourth-order ones. Using the expression for $f(R)$ from (2.4) one can obtain the equation for y in the form

$$y''' - \frac{1}{n}\frac{y''^2}{y'} + \left[5 - \frac{\frac{7n}{2} - 5}{n(n-1)}\right] y'y'' - \frac{5}{2}\frac{n - \frac{5}{2}}{n(n-1)} y'^3 = \frac{12y'}{\alpha 8^n n(n-1)} \left(y'' + \frac{5}{2}y'^2\right)^{2-n}, \quad (2.7)$$

where the prime denotes the derivative with respect to z. We note, that by introducing the scaled variable $\bar{z} = \bar{\alpha} z$ with $\bar{\alpha} = \alpha^{\frac{1}{2(1-n)}}$, eq. (2.7) becomes independent of α. Thus it is sufficient to solve the equation for $\alpha = 1$. All other solutions are obtained from this solution by scaling.

One can also see from eq. (2.7) that the first derivative y' cannot take the value zero unless $y''(z) = 0$ as well. As will be shown below there is only one point in the phase plane where both y' and y'' are equal to zero - the fixed point.

Due to its complexity, we will seek numerical solutions of eq. (2.7). But before, let us investigate the qualitative behavior of solutions of eq. (2.7). For this purpose, we rewrite it as a system of three first-order differential equations

$$\begin{aligned} y' &= p, \\ p' &= v, \\ v' &= \frac{1}{n}\frac{v^2}{p} - \left[5 - \frac{\frac{7n}{2} - 5}{n(n-1)}\right] p\, v + \frac{5}{2}\frac{n - \frac{5}{2}}{n(n-1)} p^3 + \frac{12p}{\alpha 8^n n(n-1)} \left(v + \frac{5}{2}p^2\right)^{2-n}. \end{aligned} \quad (2.8)$$

The fixed point of the system is

$$\mathcal{A} = \{p \to 0, v \to 0,\ w := v' \to 0\}. \quad (2.9)$$

In order to analyze the behavior of solutions at the fixed point, let us seek a solution in the neighborhood of the fixed point in the form

$$y = y_{fp} + \gamma (z - z_{fp})^{\beta}, \quad (2.10)$$

where β, γ are some constants and the position of the fixed point is denoted by $z = z_{fp}$. Then we have:

$$p = y' = \gamma\beta(z - z_{fp})^{\beta-1}, \quad v = y'' = \gamma\beta(\beta-1)(z - z_{fp})^{\beta-2}, \quad w = y''' = \gamma\beta(\beta-1)(\beta-2)(z - z_{fp})^{\beta-3}.$$

To provide the finiteness of all these expressions we have to put $\beta > 3$. Substituting these expressions in the third equation from (2.8), we find

$$\begin{aligned} \gamma\beta(\beta-1)(\beta-2) = {}& \frac{1}{n}\gamma\beta(\beta-1)^2 - \left[5 - \frac{\frac{7n}{2} - 5}{n(n-1)}\right] \gamma^2\beta^2(\beta-1)(z - z_{fp})^{\beta} \\ &+ \frac{5}{2}\frac{n - \frac{5}{2}}{n(n-1)} \gamma^3\beta^3(z - z_{fp})^{2\beta} + \frac{12}{\alpha 8^n n(n-1)} (\gamma\beta)^{3-n}(\beta-1)^{2-n}(z - z_{fp})^{\beta(2-n)+2(n-1)}. \end{aligned} \quad (2.11)$$

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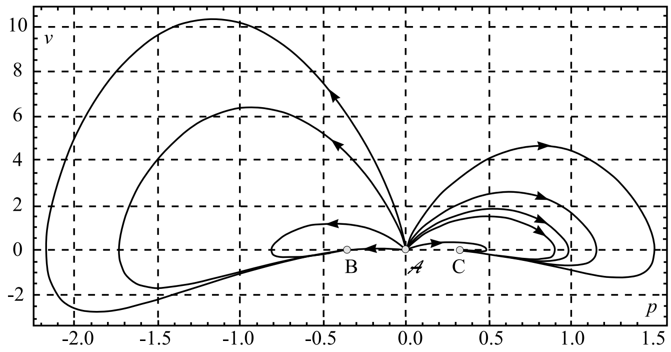

Figure 1. The phase portrait for the case $n = 4/3$, $\alpha = 1$. $\mathcal{A}$ is a repulsive node, B, C denote the asymptotic points (2.15) at $z \to \mp\infty$, respectively.

Since β is positive then the second and third terms on the right hand side tend to 0 at $z \to z_{fp}$. The fourth term tends to zero if

$$\beta(2-n) + 2(n-1) > 0. \tag{2.12}$$

If this condition is fulfilled then, omitting the terms proportional to powers of $(z - z_{fp})$, we have from (2.11)

$$\beta - 2 = \frac{1}{n}(\beta - 1),$$

whence

$$\beta = \frac{2n-1}{n-1}. \tag{2.13}$$

Using this expression, we have from (2.12) that n must be positive. Next, due to the above condition $\beta > 3$, we have from the last expression that

$$1 < n < 2. \tag{2.14}$$

Thus we see that the solutions of the system (2.8) allowing the existence of the fixed point $\mathcal{A}$ of the type $\frac{0}{0}$ do exist only in the case of the fulfilment of the condition (2.14) for n. In this case all integral curves of equation (2.7) must pass through the point $\mathcal{A}$.

Specifying the boundary conditions in the neighborhood of the fixed point $\mathcal{A}$, one can then draw a phase portrait of the system (2.8) for the model with $n = 4/3$ (figure 1).

The asymptotical form of the solution for arbitrary n is given by

$$y_\infty = k_n|z|, \quad k_n = \left[\frac{12}{\alpha\left(1 - \frac{2n}{5}\right) 20^n}\right]^{\frac{1}{2(n-1)}}. \tag{2.15}$$

One can see that there is an upper bound for the parameter n, $n < 5/2$, when this asymptotical solution is valid.

The presence of the fixed point allows to place the brane at this point directly. Indeed, when considering thick brane models the first derivative of the metric function y is usually chosen to be zero on the brane. Such choice of the derivative allows to find Z_2-symmetric solutions (see e.g. [37]). In our case the first derivative is equal to zero just at the fixed

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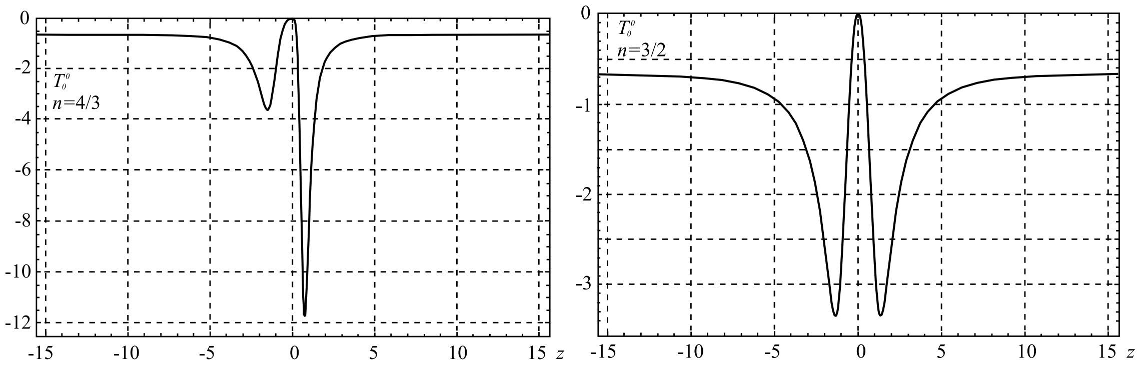

Figure 2. The effective energy density $\hat{T}_0^0$ for the case $n = 4/3$ (left panel) and $n = 3/2$ (right panel), $\alpha = 1$.

point in the range of the parameter n mentioned above (see eq. (2.14)). However, in our case, generally speaking, solutions will not be Z_2-symmetric. This can be seen from the behavior of the solution in the neighborhood of the fixed point which is given by the expression (2.10). Since eq. (2.7) does not depend on the coordinate z then it is always possible to shift the position of the fixed point (and correspondingly the brane) at the point $z_{fp} = 0$ by the corresponding transformation of coordinates.

Then, as it can be seen from (2.10), the values of both the function y and its derivatives will depend, generally speaking, on where we are situated – to the right or the left side of the fixed point (of the brane). The arbitrary constant γ in (2.10) defines the behavior of the solution near the fixed point or, in fact, specifies the boundary conditions for eq. (2.7) in the neighborhood of the fixed point. Since we seek regular solutions then it is necessary to make a corresponding choice of γ for given β, defined in eq. (2.14). If β is an even number then the solution will be symmetric relative to the axis $z = 0$ (Z_2-symmetric). In this case γ must be positive on either side of the brane that gives the regular solutions. On the other hand, if β is an odd number then γ must have different signs on the left and on the right sides of the brane (plus at $z > 0$ and minus at $z < 0$). Only by making such a choice of the signs, the solutions will be regular on either side of the brane. In this case it is obvious that the solutions will not already be Z_2-symmetric. Thus, generally speaking, there will be two different spaces on either side of the brane glued on the brane at the fixed point. But, as opposed to a thin brane, where a derivative discontinuity of the metric function takes place, in the case of the thick brane considered here, the metric and its derivatives remain smooth functions.

Such a behavior of the solutions can be illustrated by the distribution of the effective energy density $\hat{T}_0^0 = -3\left(y'' + 2y'^2\right)$ for the case $n = 4/3$ ($\beta = 5$) and the case $n = 3/2$ ($\beta = 4$), shown in figure 2. As one can see from the figure, in the case of odd $\beta = 5$ the solution is nonsymmetric relative to $z = 0$, and in the case of even $\beta = 4$ — it is symmetric. Taking into account the expression (2.15), one can see that asymptotically $\hat{T}_0^0$ goes to a constant negative value: $\hat{T}^0_{0(\pm\infty)} \to -6y'^2_\infty \approx -0.64$ (for $n = 4/3$) and $= -0.675$ (for $n = 3/2$), and the 5-dimensional scalar curvature $R^{(5)} = 8y'' + 20y'^2$ goes to a positive constant, $R^{(5)}_{(\pm\infty)} \approx 2.12$ (for $n = 4/3$) and $= 2.25$ (for $n = 3/2$). Thus this corresponds to asymptotical anti-de Sitter solutions.

3 Trapping of matter

At the beginning of this section let us note that the usual statement of a problem within the framework of the brane theory is the following: 1) a search of localized multidimensional gravitating solutions; 2) one also needs to show that such solutions trap test matter fields of various spins.

The first task has been considered in the previous section. In this section we consider the trapping of a test scalar field on the brane considered above. For this purpose, let us use the approach suggested in the paper [37]. We consider the test complex scalar field χ with the Lagrangian

$$L_\chi = \frac{1}{2}\partial_A\chi^*\partial^A\chi - \frac{1}{2}m_0^2\chi^*\chi,$$

where m_0 is the mass of the test field. Using this Lagrangian, we find the equation for the scalar field

$$\frac{1}{\sqrt{-^5g}}\frac{\partial}{\partial x^A}\left(\sqrt{-^5g}g^{AB}\frac{\partial\chi}{\partial x^B}\right) = -m_0^2\chi. \tag{3.1}$$

Here χ is a function of all coordinates, $\chi = \chi(x^A)$. Taking into account that the canonically conjugate momenta $p_\mu = (E, \vec{p})$ are integrals of motion, we will seek a solution in the form

$$\chi(x^A) = X(z)\exp(-ip_\mu x^\mu).$$

Inserting this ansatz into eq. (3.1), leads to the equation for $X(z)$

$$X'' + 4y'X' + (p^\mu p_\mu - m_0^2)X = 0,$$

or, taking into account that $p^\mu p_\mu = e^{-2y}\left(E^2 - \vec{p}^{\,2}\right)$, we obtain

$$X'' + 4y'X' + \left[\left(E^2 - \vec{p}^{\,2}\right)e^{-2y} - m_0^2\right]X = 0,$$

where the prime denotes differentiation with respect to z. According to eq. (2.15), asymptotically $y_\infty = k_n|z|,\ k_n > 0$. That is why, in the above equation, we can neglect the term with e^{-2y} in comparison with m_0^2. Thus we have

$$X'' + 4k_nX' - m_0^2X = 0$$

with the asymptotically decaying solution

$$X_\infty \approx C\exp\left[-2\left(k_n + \sqrt{k_n^2 + m_0^2/4}\right)|z|\right], \tag{3.2}$$

where C is an integration constant.

As a necessary condition for the trapping of matter on the brane, one should require finiteness of the field energy per unit 3-volume of the brane, i.e.,

$$E_{\rm tot}[\chi] = \int\limits_{-\infty}^{\infty} T_0^0 \sqrt{-{}^5 g}\, dz = \int\limits_{-\infty}^{\infty} e^{4k_n|z|} \left[e^{-2k_n|z|}(E^2 + \vec{p}^{\,2})X^2 + m_0^2 X^2 + X'^2 \right] dz < \infty, \tag{3.3}$$

and also finiteness of the norm of the field χ

$$||\chi||^2 = \int\limits_{-\infty}^{\infty} \sqrt{-{}^5 g}\, \chi^* \chi\, dz = \int_{-\infty}^{\infty} e^{4k_n|z|}\, X^2\, dz.$$

From the solution (3.2) it is evident that both $E_{\rm tot}$ and $||\chi||$ converge asymptotically.

Thus it is obvious from the above analysis that the localized solutions found in section 2 trap the test scalar field. It indicates that such solutions may be interpreted as brane solutions (see the beginning of this section).

4 Conclusion

We have considered the 5-dimensional thick brane model in $f(R) \sim R^n$ theory. Special attention was called to the regular solutions starting from the fixed point $\mathcal{A}$, see eq. (2.9). It follows from the analytical analysis of the solutions in the neighborhood of this point that such a point exists only at definite values of the parameter n from (2.14), $1 < n < 2$. In this case all the derivatives of the metric function y are equal to zero, that allows to place the brane at the fixed point $z = z_{fp}$ directly. Since eq. (2.7) is invariant under the shift of the independent variable $z \to z + z_0$, the position of the brane is arbitrary and it can be placed at any point on the axis z, including $z = z_{fp} = 0$. An interesting feature of the model under consideration is that the presence of the fixed point provides existence of both Z_2-symmetric and nonsymmetric solutions. It depends on a value of the parameter n (see figure 2). In the case under consideration all solutions (on either side of the brane) start from the neighborhood of the repulsive fixed point $\mathcal{A}$ (node) and tend to the asymptotic value (2.15). It allows to not provide any special conditions for the model parameters (fine-tuning conditions) that is typically necessary for other brane models (see, for example, ref. [37]).

Consideration of the behavior of a test scalar field in the bulk has shown that such a field is trapped by the $f(R)$-brane. Note that the trapping is purely gravitational.

Equation (2.7) allows regular solutions for a case when n lies outside the range $1 < n < 2$ as well. In particular, such regular solutions *without* a fixed point do exist for an important case with $n = 2$. This case within the framework of the $f(R)$ theory has also been employed for the thin brane model [34]. In the vicinity of the brane (located at $z = 0$) a similar behavior of the metric function is found, $y(z) \sim z^3$ (compare with (2.10) at $\beta = 3$). However, at some finite value z_s, is encountered. Thus the thin brane model appears to be plagued by bulk singularities, very much in contrast to the thick brane model considered in this paper.

An interesting further avenue would be to consider the existence of thick brane solutions in a bulk anti-de Sitter black hole spacetime. In refs. [38, 39] the thin brane model on the background of AdS black hole within the framework of 5D higher derivative gravity with the Gauss-Bonnet term was considered. The thin brane model considered in ref. [40] is being described by the higher derivative Lagrangian containing the terms $L \sim aR^2 + bR_{\mu\nu}R^{\mu\nu} - \Lambda$. The corresponding equations have an exact AdS black hole solution. In the case of $b = 0$ this can correspond to the AdS black hole spacetime without a brane in R^2 gravity plus Λ-term. Inserting a thick brane into this spacetime and changing R^2 by R^n should lead to a set of partial differential equations in the case of cosmology on the brane. The resulting thick brane solution might provide an interesting model for cosmological considerations.

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