

GINZBURG–LANDAU EQUATION FROM SU(2) GAUGE FIELD THEORY

VLADIMIR DZHUNUSHALIEV

Freie Universität Berlin, Fachbereich Physik, Arnimallee 14, 14195 Berlin, Germany
and

*Department of Physics and Microel. Engineering, Kyrgyz-Russian Slavic University,
Bishkek, Kievskaya Str. 44, 720000, Kyrgyz Republic*
dzhun@hotmail.kg

DOUGLAS SINGLETON

*Physics Department, CSU Fresno, 2345 East San Ramon Ave.,
M/S 37 Fresno, CA 93740-8031, USA*
dougs@csufresno.edu

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The dual superconductor picture of the QCD vacuum is thought to describe the various aspects of the strong interaction including confinement. Ordinary superconductivity is described by the Ginzburg–Landau (GL) equation. In the present work we show that it is possible to arrive at a GL-like equation from pure SU(2) gauge theory. This is accomplished by using Abelian projection to split the SU(2) gauge fields into an Abelian subgroup and its coset. The two gauge field components of the coset part act as the effective, complex, scalar field of the GL equation. The Abelian part of the SU(2) gauge field is then analogous to the electromagnetic potential in the GL equation. An important feature of the dual superconducting model is for the GL Lagrangian to have a spontaneous symmetry breaking potential, and the existence of Nielsen–Olesen flux tube solutions. Both of these require a tachyonic mass for the effective scalar field. Such a tachyonic mass term is obtained from the condensation of ghost fields.

Keywords: Dual superconductivity; confinement; Abelian projection; strongly interacting Yang–Mills theory.

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1. Introduction

In Ref. 1 it was shown that one could obtain London’s equation from SU(2) gauge theory using the ideas of Abelian projection. This supports the dual superconductor picture of a confining Yang–Mills gauge theory, since London’s equation gives a phenomenological description of the Meissner effect. The vacuum of the SU(3) Yang–Mills theory of the strong interaction is thought to exhibit a dual Meissner effect with respect to chromoelectric fields in analogy to a normal superconductor,

which exhibits the Meissner effect with respect to the magnetic field. In Ref. 1 the quantized SU(2) gauge fields are split into two phases: an ordered and a disordered phase. The ordered phase was the gauge field belonging to the U(1) subgroup (the diagonal, Abelian component of the SU(2) gauge field) and the disordered phase was the coset SU(2)/SU(1) (the off-diagonal components of the SU(2) gauge field). The disordered phase played a role similar to the complex scalar field of the GL equation. In Ref. 1 we simply set this scalar field to a constant vacuum expectation value, i.e. we froze the equations of the SU(2)/U(1) fields making them nondynamical degrees of freedom. In this way we obtained London’s equation. In the present work we want to “unfreeze” the equations connected with the SU(2)/U(1) part, and show that it is possible to obtain a GL-like equation. This helps strengthen the dual superconductor picture of the confining Yang–Mills vacuum. As a reminder the Lagrangian density of GL is

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} + (D_\mu\varphi^*)(D^\mu\varphi) - m^2|\varphi|^2 - \lambda|\varphi|^4, \tag{1}$$

where φ is a complex scalar field, $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, is the field strength tensor of the Abelian field, a_μ , and $D_\mu = \partial_\mu - ie a_\mu$ is the covariant derivative.

For the dual superconducting picture it is important that the potential for the scalar field in (1) be of the spontaneous symmetry breaking form, and have Nielsen–Olesen flux tube solutions.² This means that the mass term in (1) should be tachyonic (i.e. $m^2 < 0$). In this work the tachyonic mass term is generated via the condensation of ghosts as in Ref. 3 (see also Ref. 4).

Strictly the dual superconducting picture of the Yang–Mills vacuum requires a *dual* GL Lagrangian which gives a dual Meissner effect with respect to the “electric” charges of the theory. In the present work we arrive at the ordinary GL Lagrangian which gives the ordinary Meissner effect with respect to the “magnetic” charges of the theory. However, it is simple to perform the construction in this paper starting with the dual gauge fields and arrive at an effective dual GL Lagrangian. A discussion of dual symmetry and dual gauge fields for Yang–Mills theory can be found in Ref. 5. An explicit example of the replacement of the regular gauge fields with the dual gauge fields can be found in Ref. 6.

2. Ordered and Disordered Phases

In this section we review the Abelian projection decomposition of the SU(2) gauge field⁷ and necessary results from Ref. 1. The SU(2) gauge fields $\mathcal{A}_\mu = \mathcal{A}_\mu^B T^B$ and $\mathcal{F}_{\mu\nu}^B$ can be decomposed as

$$\mathcal{A}_\mu = \mathcal{A}_\mu^B T^B = a_\mu T^a + A_\mu^m T^m, \tag{2}$$

$$a_\mu \in \text{U}(1) \quad \text{and} \quad A_\mu^m \in \text{SU}(2)/\text{U}(1), \tag{3}$$

$$\mathcal{F}_{\mu\nu}^B T^B = \mathcal{F}_{\mu\nu}^3 T^3 + \mathcal{F}_{\mu\nu}^m T^m, \tag{4}$$

where

$$\mathcal{F}_{\mu\nu} = f_{\mu\nu} + \Phi_{\mu\nu} \in U(1), \tag{5}$$

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \in U(1), \tag{6}$$

$$\Phi_{\mu\nu} = g\varepsilon^{3mn} A_\mu^m A_\nu^n \in U(1), \tag{7}$$

$$\mathcal{F}_{\mu\nu}^m = F_{\mu\nu}^m + G_{\mu\nu}^m \in SU(2)/U(1), \tag{8}$$

$$F_{\mu\nu}^m = \partial_\mu A_\nu^m - \partial_\nu A_\mu^m \in SU(2)/U(1), \tag{9}$$

$$G_{\mu\nu}^m = g\varepsilon^{3mn}(A_\mu^n a_\nu - A_\nu^n a_\mu) \in SU(2)/U(1), \tag{10}$$

where ε^{ABC} are the structural constants of $SU(2)$, g is the coupling constant, $a = 3$ is the index of the Abelian subgroup, and $m, n = 1, 2$ are the indices of the coset. After obtaining the classical field equations in terms of this decomposition we applied a quantization technique of Heisenberg⁸ where the classical fields were replaced by operators ($a_\mu \rightarrow \hat{a}_\mu$ and $A_\mu^m \rightarrow \hat{A}_\mu^m$). The differential equations for the field operators are

$$\partial_\nu(\hat{f}^{\mu\nu} + \hat{\Phi}^{\mu\nu}) = -g\varepsilon^{3mn} \hat{A}_\nu^m(\hat{F}^{n\mu\nu} + \hat{G}^{n\mu\nu}), \tag{11}$$

$$D_\nu(\hat{F}^{m\mu\nu} + \hat{G}^{m\mu\nu}) = -g\varepsilon^{3mn}[\hat{A}_\nu^n(\hat{f}^{\mu\nu} + \hat{\Phi}^{\mu\nu}) - \hat{a}_\nu(\hat{F}^{n\mu\nu} + \hat{G}^{n\mu\nu})] \tag{12}$$

(we note that similar approach is known as field correlators in QCD (for review, see Ref. 9) and in Ref. 10 similar ideas were presented to obtain a set of self coupled equations for the field correlators). Equations (11) and (12) can be used to determine the physically measurable expectation values for any field operators such as $\langle Q | \dots \hat{a}_\mu(x) \dots \hat{A}_\nu^m(y) | Q \rangle$, where $|Q\rangle$ is some quantum state. As an example, if we average Eqs. (11) and (12) we obtain equations for $\langle \hat{A}_\nu^m \rangle$ and $\langle \hat{a}_\nu \rangle$. The problem is that the resulting differential equations for these expectation values contain additional terms like $\mathcal{G}_{\mu\nu}^{mn} = \langle A_\mu^m A_\nu^n \rangle$ so the system is not closed. Going back to Eqs. (11), (12) and differentiating, one can obtain an operator equation for the product $\hat{A}_\mu^m \hat{A}_\nu^n$, which then contains new additional terms. Continuing in this way one obtains an infinite set of coupled, differential equations for the various expectation values of field operators of ever increasing powers. This process can be paralleled with the standard Feynman diagram procedure where one has an infinite set of loop diagrams that one must (in principle) calculate. This infinite system of coupled differential equations does not have an exact, analytical solution, so one must use some approximation.

In Ref. 1 we used several assumptions that led to a simplification of this system of differential equations. The two main assumptions were:

1. After quantization the components $\hat{A}_\mu^m(x)$ become stochastic. In mathematical terms this assumption means

$$\langle A_\mu^m(x) \rangle = 0 \quad \text{and} \quad \langle A_\mu^m(x) A_\nu^n(x) \rangle \neq 0. \tag{13}$$

Later we will give a specific form for the nonzero term.

2. The gauge potentials a_μ and A_μ^m are not correlated, and a_μ behaves in a classical manner. Mathematically this means that

$$\langle f(a_\mu)g(A_\nu^m) \rangle = \langle f(a_\mu) \rangle \langle g(A_\mu^m) \rangle = f(a_\mu) \langle g(A_\mu^m) \rangle, \tag{14}$$

where f, g are any functions of a_μ and A_μ^m respectively. The classical behavior of a_μ results in $\langle f(a_\mu) \rangle \rightarrow f(a_\mu)$.

One way in which the present work will deviate from the procedure of Ref. 1 is that rather than working in terms of the equations of motion of the SU(2) gauge theory, we will focus on the Lagrange density.

3. Ginzburg–Landau Lagrangian

We now want to show that an effective, complex, Higgs-like, scalar field can be obtained from the SU(2)/U(1) coset part of the SU(2) gauge theory. The self-interaction of this effective scalar field is a consequence of nonlinear terms in the original Yang–Mills Lagrangian. The mass term for the scalar field is generated via the condensation of ghosts fields as discussed in the following section.

Making a connection between scalar and gauge fields is not a new idea. In Ref. 11 it was shown that by setting a non-Abelian gauge field to some combination of a scalar field and its derivatives it was possible to obtain massless $\lambda\varphi^4$ theory. One could obtain a massive $\lambda\varphi^4$ theory by starting with Yang–Mills theory with a mass term inserted by hand.¹² The final scalar field Lagrangian that we will arrive at is also a massless $\lambda\varphi^4$ theory with the addition of a coupling to a U(1) gauge field. This is (except for the U(1) gauge field coupling) similar to the result of Ref. 11. In the present paper we exchange the two gauge fields of the SU(2)/U(1) coset with a complex scalar field, whereas in Refs. 11 and 12 one gauge field is exchanged for a real scalar field. We begin by taking the expectation of the SU(2) Lagrangian (the gauge fixing and Faddeev–Popov terms are dealt with in the next section)

$$-\frac{1}{4} \langle Q | \mathcal{F}_{\mu\nu}^A \mathcal{F}^{A\mu\nu} | Q \rangle = -\frac{1}{4} \langle \mathcal{F}_{\mu\nu}^A \mathcal{F}^{A\mu\nu} \rangle = -\frac{1}{4} \langle \mathcal{F}_{\mu\nu}^3 \mathcal{F}_{\mu\nu}^3 + \mathcal{F}_{\mu\nu}^m \mathcal{F}_{\mu\nu}^m \rangle. \tag{15}$$

This contains many terms which we will now consider in order. First we consider the term

$$\langle \mathcal{F}_{\mu\nu}^3 \mathcal{F}_{\mu\nu}^3 \rangle = \langle (f_{\mu\nu} + \Phi_{\mu\nu})(f^{\mu\nu} + \Phi^{\mu\nu}) \rangle, \tag{16}$$

where the field $f_{\mu\nu}$ is in the ordered phase and $\Phi_{\mu\nu}$ is in the disordered phase. From the second assumption of the previous section a_μ and $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, behave as classical fields so that

$$\langle f_{\mu\nu} \Phi^{\mu\nu} \rangle = f_{\mu\nu} \langle \Phi^{\mu\nu} \rangle, \tag{17}$$

$$\langle f_{\mu\nu} f^{\mu\nu} \rangle = f_{\mu\nu} f^{\mu\nu}, \tag{18}$$

$$\langle \Phi_{\mu\nu} \rangle = g \varepsilon^{3mn} \langle A_\mu^m A_\nu^n \rangle. \tag{19}$$

For the expectation of $A_\mu^m(y)A_\nu^n(x)$ we take the form

$$\langle A_\mu^m(y)A_\nu^n(x) \rangle = -\delta^{mn}\eta_{\mu\nu}\mathcal{G}(y, x) \tag{20}$$

$\mathcal{G}(y, x)$ is an arbitrary function. We note that strictly speaking this form of the expectation is an approximation since it reduces the degrees of freedom of the initial set of coset gauge fields, A_μ^m (this is our purpose — to find some approximate description of nonperturbative quantization). Nevertheless these degrees of freedom should be quantized at the next level of approximation. We hope that they can be quantized by perturbative Feynman diagram techniques.

The form given in (20) is consistent with the color and Lorentz indices of the left-hand side. One might think of adding a term with an index structure like $\eta_{\mu\nu}\varepsilon^{3mn}$. However, this is antisymmetric under exchange of $A_\mu^m(y)A_\nu^n(x)$ (i.e. exchanging both Lorentz and group indices) which is not consistent with the bosonic statistics of the gauge fields. The quantity in (20) is a mass dimension 2 condensate. The role of such gauge non-invariant quantities in the Yang–Mills vacuum has been discussed by several authors.^{5,13}

From (20) we find

$$\langle \Phi_{\mu\nu} \rangle = 0, \tag{21}$$

$$\begin{aligned} \langle \Phi_{\mu\nu}\Phi^{\mu\nu} \rangle &= g^2(\langle A_\mu^1 A_\nu^2 A^{1\mu} A^{2\nu} \rangle + \langle A_\mu^2 A_\nu^1 A^{2\mu} A^{1\nu} \rangle \\ &\quad - \langle A_\mu^2 A_\nu^1 A^{1\mu} A^{2\nu} \rangle - \langle A_\mu^1 A_\nu^2 A^{2\mu} A^{1\nu} \rangle). \end{aligned} \tag{22}$$

We now assume that these quartic gauge field terms can be approximated as products of quadratic gauge expectations from (20)

$$\begin{aligned} \langle A_\alpha^m(x_1)A_\beta^n(x_2)A_\mu^p(x_3)A_\nu^q(x_4) \rangle &= \delta^{mp}\delta^{nq}\eta_{\alpha\mu}\eta_{\beta\nu}\mathcal{G}(x_1, x_3)\mathcal{G}(x_2, x_4) \\ &\quad + \delta^{mq}\delta^{np}\eta_{\alpha\nu}\eta_{\beta\mu}\mathcal{G}(x_1, x_4)\mathcal{G}(x_2, x_3) \\ &\quad + \delta^{mn}\delta^{pq}\eta_{\alpha\beta}\eta_{\mu\nu}\mathcal{G}(x_1, x_2)\mathcal{G}(x_3, x_4) \end{aligned} \tag{23}$$

which gives

$$\langle \Phi_{\mu\nu}(x)\Phi^{\mu\nu}(x) \rangle \approx 24g^2\mathcal{G}^2(x, x). \tag{24}$$

The next term is

$$\begin{aligned} &\langle \mathcal{F}_{\mu\nu}^m(y)\mathcal{F}^{m\mu\nu}(x) \rangle \Big|_{y=x} \\ &= ([\partial_{\mu y}A_\nu^m(y) - ga_\mu(y)\varepsilon^{3mn}A_\nu^n(y)] - [\partial_{\nu y}A_\mu^m(y) - ga_\nu(y)\varepsilon^{3mn}A_\mu^n(y)]) \\ &\quad \times ([\partial_x^\mu A^{m\nu}(x) - ga^\mu(x)\varepsilon^{3mp}A^{p\nu}(x)] - [\partial_x^\nu A^{m\mu}(x) - ga^\nu(x)\varepsilon^{3mp}A^{p\mu}(x)]) \Big|_{y=x}. \end{aligned} \tag{25}$$

We begin by expanding the first term of $\mathcal{F}_{\mu\nu}^m(y)$ in square brackets against the first term of $\mathcal{F}^{m\mu\nu}(x)$. This has four terms, the first of which is

$$\begin{aligned}
 \langle \partial_{\mu y} A_{\nu}^m(y) \partial_x^{\mu} A^{m\nu}(x) \rangle \Big|_{y=x} &= \partial_{\mu y} \partial_x^{\mu} \langle A_{\nu}^m(y) A^{m\nu}(x) \rangle \Big|_{y=x} \\
 &= -\partial_{\mu y} \partial_x^{\mu} \delta^{mn} \eta_{\nu}^{\nu} \mathcal{G}(y, x) \Big|_{y=x} \\
 &= -8 \partial_{\mu y} \partial_x^{\mu} \mathcal{G}(y, x) \Big|_{y=x}.
 \end{aligned} \tag{26}$$

The next two terms are

$$g^2 \varepsilon^{3mn} \varepsilon^{3mp} a_{\mu}(x) a^{\mu}(x) \langle A_{\nu}^n(x) A^{p\nu}(x) \rangle = -8g^2 a_{\mu} a^{\mu}(x) \mathcal{G}(x, x), \tag{27}$$

$$\langle [\partial_{\mu y} A_{\nu}^m(y)] [g \varepsilon^{3mn} a^{\mu}(x) A^{n\nu}(x)] \rangle \Big|_{y=x} = g \varepsilon^{3mn} a^{\mu}(x) \langle [\partial_{\mu y} A_{\nu}^m(y)] A^{n\nu}(x) \rangle \Big|_{y=x}. \tag{28}$$

We want to consider this last term more closely. From Eqs. (20), (26) and (27) it can be seen that there are quantum correlations only between fields with the same color (δ^{mn}) and coordinate ($\eta_{\mu\nu}$) indices. In the absence of some principle which forbids it, one would in general expect that there should physically be some interaction between gauge fields of different colors. In Eq. (20) we excluded terms proportional to $\eta_{\mu\nu} \varepsilon^{3mn}$ because of the Bose symmetry of the gauge fields. In the present expression the gauge fields do not appear symmetrically (one is acted on by a derivative operator) so such a term can be included. Thus we take the expectation of (28) to have the general form

$$\langle [\partial_{\mu y} A_{\alpha}^m(y)] A_{\beta}^n \rangle = -\delta^{mn} \eta_{\alpha\beta} \partial_{\mu y} \mathcal{G}(y, x) - i \varepsilon^{3mn} \eta_{\alpha\beta} \partial_{\mu y} \mathcal{P}(y, x), \tag{29}$$

where $\mathcal{P}(y, x)$ is some general function. This new term will mix gauge bosons of different colors. Using this form Eq. (28) becomes

$$\langle [\partial_{\mu y} A_{\nu}^m(y)] [g \varepsilon^{3mn} a^{\mu}(x) A^{n\nu}(x)] \rangle \Big|_{y=x} = -8iga^{\mu}(x) \partial_{\mu y} \mathcal{P}(y, x) \Big|_{y=x}. \tag{30}$$

Making the same assumption for the other cross term

$$\langle A_{\alpha}^n(y) [\partial_{\mu x} A_{\beta}^m(x)] \rangle = -\delta^{nm} \eta_{\alpha\beta} \partial_{\mu x} \mathcal{G}(y, x) - i \varepsilon^{3nm} \eta_{\alpha\beta} \partial_{\mu x} \mathcal{P}^*(y, x) \tag{31}$$

yields

$$\langle A_{\nu}^n(y) [g \varepsilon^{3mn} a_{\mu}(x) \partial_x^{\mu} A^{m\nu}(x)] \rangle = 8iga^{\mu}(x) \partial_{\mu x} \mathcal{P}^*(y, x) \Big|_{y=x}. \tag{32}$$

Next we expand the first term of $\mathcal{F}_{\mu\nu}^m(y)$ in square brackets against the second term of $\mathcal{F}^{m\nu}(x)$. This also has four terms which are

$$\langle [\partial_{\mu y} A_{\nu}^m(y)] [\partial_x^{\nu} A^{m\mu}(x)] \rangle \Big|_{y=x} = -2 \partial_{\mu y} \partial_x^{\mu} \mathcal{G}(y, x) \Big|_{y=x}, \tag{33}$$

$$\langle [\partial_{\mu y} A_{\nu}^m(y)] [ga^{\nu}(x) \varepsilon^{3mp} A^{p\mu}(x)] \rangle \Big|_{y=x} = -2iga^{\nu}(x) \partial_{\nu y} \mathcal{P}(y, x) \Big|_{y=x}, \tag{34}$$

$$\langle [ga_{\mu}(y) \varepsilon^{3mn} A_{\nu}^n(y)] [\partial_x^{\nu} A^{m\mu}(x)] \rangle = 2iga^{\mu}(x) \partial_{\mu x} \mathcal{P}^*(y, x) \Big|_{y=x}, \tag{35}$$

$$\langle [ga^{\mu}(y) \varepsilon^{3mn} A_{\nu}^n(y)] [ga^{\nu}(x) \varepsilon^{3mp} A^{p\mu}(x)] \rangle = -2g^2 a_{\mu}(x) a^{\mu}(x) \mathcal{G}(x, x). \tag{36}$$

Finally we need to expand the second term of $\mathcal{F}_{\mu\nu}^m(y)$ against the second term of $\mathcal{F}^{m\mu\nu}(x)$, and also the second term of $\mathcal{F}_{\mu\nu}^m(y)$ against the first term of $\mathcal{F}^{m\mu\nu}(x)$. These yield, respectively, the same results as Eqs. (26), (27), (30) and (32) and Eqs. (33)–(36). Collecting all these terms together gives

$$\begin{aligned} \langle \mathcal{F}_{\mu\nu}^m \mathcal{F}^{m\mu\nu} \rangle &= -20[\partial_{\mu y} \partial_x^\mu \mathcal{G}(y, x) + g^2 a_\mu(x) a^\mu(x) \mathcal{G}(x, x) \\ &\quad - i g a^\mu(x) \partial_{\mu x} \mathcal{P}^*(y, x) + i g a^\mu(x) \partial_{\mu x} \mathcal{P}(y, x)] \Big|_{y=x}. \end{aligned} \tag{37}$$

We now make the approximation that both functions, $\mathcal{G}(y, x)$ and $\mathcal{P}(y, x)$, can be rewritten in terms of a single complex scalar function as

$$\mathcal{G}(y, x) = \mathcal{P}(y, x) = \frac{1}{5} \varphi^*(y) \varphi(x). \tag{38}$$

Setting both $\mathcal{G}(y, x)$ and $\mathcal{P}(y, x)$ equal to the same product of a complex scalar field is called the *one function approximation or ansatz*. The factor of $1/5$ is to ensure that the kinetic term of this scalar field will have a factor of 1 in front of it. Taking Eqs. (20), (29), (38) together, we want to note the similarity between this ansatz for the gauge fields, and the ansatz used in Ref. 11. Both have similar forms for the indexed terms, and both involve some form of scalar field and its derivatives. This complex function, φ , will act as the effective scalar field. In Ref. 1 we set $\varphi^* \varphi = \text{const.}$, making the degrees of freedom connected with the $SU(2)/U(1)$ coset space nondynamical. (Note that in Ref. 1 we used a single scalar field. The scalar field of Ref. 1 is proportional to $\varphi^*(x) \varphi(x)$ here.) By letting φ be a function of the coordinates we make these dynamical degrees of freedom. Using (38) in (37) we find

$$\begin{aligned} \langle \mathcal{F}_{\mu\nu}^m \mathcal{F}^{m\mu\nu} \rangle &= -4[(\partial^\mu \varphi^*)(\partial_\mu \varphi) + g^2 a_\mu^2 \varphi^* \varphi - i g a^\mu (\partial_\mu \varphi^*) \varphi + i g a^\mu \varphi^* (\partial^\mu \varphi)] \\ &= -4|\partial_\mu \varphi - i g a_\mu \varphi|^2. \end{aligned} \tag{39}$$

Thus the total Lagrangian density is

$$\langle \mathcal{L} \rangle = -\frac{1}{4} \langle \mathcal{F}_{\mu\nu}^A \mathcal{F}^{A\mu\nu} \rangle = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + (D_\mu \varphi^*)(D^\mu \varphi) - \frac{6g^2}{25} |\varphi|^4, \tag{40}$$

where $D_\mu = \partial_\mu - i g a_\mu$. This is the GL Lagrangian, with a *massless*, effective scalar field. This scalar field is connected with the off-diagonal gauge fields by (20) and (38). This lack of a mass term is a shortcoming for the effective Lagrangian of (40). Without it there is no spontaneous symmetry breaking and no Nielsen–Olesen flux tube solutions, both are critical to make the connection to the dual superconducting picture of the QCD vacuum. In the next section we show how a condensation of ghosts fields can lead to a mass term for the effective scalar field φ . This mass term is of the correct form (i.e. tachyonic) to give rise to spontaneous symmetry breaking and Nielsen–Olesen flux tube solutions. Also, as remarked previously, to get the dual superconducting QCD vacuum we should construct a dual GL Lagrangian. This is easily accomplished by starting with the dual gauge fields instead of the ordinary gauge fields.

4. Tachyonic Mass Term via Ghost Condensation

In Ref. 3 (see also Ref. 4) it was shown that a tachyonic mass term is generated for the off-diagonal gauge fields of a pure SU(2) Yang–Mills via a condensation of ghost and anti-ghost fields. This result was taken to be somewhat of a problem since this would apparently give the off-diagonal gluons a tachyonic mass. In the present work having a tachyonic mass for our effective scalar field (which is related to the off-diagonal gauge fields via (20) and (38)) is a desired result. We sketch the parts of Ref. 3 relevant to the present work. To take care of gauge fixing and integration over gauge equivalent field configurations one must add a gauge fixing (\mathcal{L}_{GF}) and Faddeev–Popov part (\mathcal{L}_{FP}) to the Yang–Mills Lagrangian in (40)

$$\mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} = i\delta_B \bar{\delta}_B \left(\frac{1}{2} A_\mu^m A^{\mu m} - \frac{\alpha}{2} i C^m \bar{C}^m \right), \tag{41}$$

where α is a gauge parameter, C^m and \bar{C}^m are the off-diagonal ghost and anti-ghost fields, and δ_B and $\bar{\delta}_B$ are the BRST and anti-BRST transformations. These gauge fixing and Faddeev–Popov Lagrangians can be transformed into

$$\begin{aligned} \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} = & -\frac{1}{2\alpha} (D_\mu^{mn} A^{\mu n})^2 + i\bar{C}^m D_\mu^{mp} D^{\mu pn} C^n - ig^2 \varepsilon^{mq} \varepsilon^{pn} \bar{C}^m C^n A^{\mu p} A_\mu^q \\ & + \frac{\alpha}{4} g^2 \varepsilon^{mn} \varepsilon^{pq} \bar{C}^m \bar{C}^n C^p C^q, \end{aligned} \tag{42}$$

where ε^{mn} is the antisymmetric symbol for the off-diagonal indices ($\varepsilon^{12} = -\varepsilon^{21} = 1$ and $\varepsilon^{11} = \varepsilon^{22} = 0$), and $D_\mu^{mn} = \partial_\mu \delta^{mn} - g\varepsilon^{mn} a_\mu$.

Next, the last term in (42) is a four-ghost interaction term which can be replaced by^{3,4}

$$\frac{\alpha}{4} g^2 \varepsilon^{mn} \varepsilon^{pq} \bar{C}^m \bar{C}^n C^p C^q \rightarrow -\frac{1}{2\alpha g^2} \psi^2 - i\psi \varepsilon^{mn} \bar{C}^m C^n \tag{43}$$

with ψ being an auxiliary field. Extracting the ghost kinetic energy term from the second term in (42) and adding it to (43) one arrives at a subset of the Lagrangian involving only the ghost and auxiliary field

$$\mathcal{L}_{\text{ghost}} = i\bar{C}^m \partial_\mu \partial^\mu C^m - \frac{1}{2\alpha g^2} \psi^2 - i\psi \varepsilon^{mn} \bar{C}^m C^n. \tag{44}$$

The Coleman–Weinberg mechanism¹⁴ can be applied to $\mathcal{L}_{\text{ghost}}$ leading to a spontaneous symmetry breaking potential for ψ of the form

$$V_{\text{eff}}(\psi) = \frac{\psi^2}{2\alpha \bar{g}^2} + \frac{\psi^2}{32\pi^2} \left(\ln \frac{\psi^2}{\bar{\mu}^4} - 3 \right). \tag{45}$$

This potential has a nonzero minimum at $\psi = \pm v = \pm \bar{\mu}^2 \exp\left(1 - \frac{8\pi}{\alpha \bar{g}^2 \bar{\mu}^2}\right)$ (the bars over the quantities indicate that the $\overline{\text{MS}}$ dimensional regularization scheme is being employed). Investigating the ghost propagator in this nonzero vacuum, it is found³ that a ghost condensation occurs

$$\langle i\bar{C}^m C^m \rangle = -\frac{v}{16\pi} < 0 \tag{46}$$

since $v > 0$. The third term in the Lagrangian in (42) now becomes

$$\begin{aligned}
 ig^2 \varepsilon^{mq} \varepsilon^{pn} \bar{C}^m C^n A^{\mu p} A_\mu^q &\rightarrow \frac{1}{2} g^2 \langle i \bar{C}^m C^m \rangle \langle A_\mu^n A^{\mu n} \rangle = \frac{1}{2} g^2 \left(-\frac{v}{16\pi} \right) (-8\mathcal{G}) \\
 &= \frac{vg^2}{20\pi} \varphi^* \varphi, \tag{47}
 \end{aligned}$$

where (20) and (38) have been used. Putting this term together with the quartic term from the effective GL Lagrangian in (40) we find that φ has developed an effective potential of the form

$$V_\varphi = -\frac{vg^2}{20\pi} |\varphi|^2 + \frac{6g^2}{25} |\varphi|^4. \tag{48}$$

A tachyonic mass term ($m^2 = -(vg^2)/(20\pi) < 0$) has been generated for the effective scalar field, φ , with the consequence that spontaneous symmetry breaking occurs. We have arrived at an effective *massive* GL-Lagrangian with the tachyonic mass term for the φ field being generated by ghost condensation. This effective GL-Lagrangian will also have Nielsen–Olesen flux tube solutions.

With this tachyonic mass term for φ , spontaneous symmetry breaking occurs and the $U(1)$ field, a^μ , will develop a mass of $\sqrt{(5g^2v)/(24\pi)}$. It might be thought that the ghost condensate would also contribute some to the mass for a^μ , since the second term in the gauge fixing and Faddeev–Popov Lagrangian of (42) contains a term proportional to $i\bar{C}^a C^a a_\mu a^\mu$. However, in Ref. 3 it was argued that this term is canceled by one-loop contributions coming from other terms in (40) which are proportional to $\varepsilon^{mn}(\partial_\mu \bar{C}^m) C^n a^\mu$ and $\varepsilon^{mn} \bar{C}^m (\partial_\mu C^n) C^n a^\mu$. Thus the mass of a^μ is generated purely from the spontaneous symmetry breaking of the effective GL Lagrangian (40).

5. Conclusions

In this paper we have combined several ideas (Abelian projection, quantization methods originally proposed by Heisenberg, and some assumptions about the forms of various expectation values of the gauge fields) to show that one can construct an effective scalar field within a pure $SU(2)$ gauge field theory. The system of effective scalar field plus the remaining Abelian field is essentially scalar electrodynamics or the relativistic version of the GL-Lagrangian. There is also a connection between the present work and Cho’s¹⁵ “magnetic symmetry” study of the dual Meissner effect within Yang–Mills theory. In Ref. 15 a Lagrangian similar to our Eq. (40) is obtained, with the complex scalar field being associated with a monopole coupled to a $U(1)$ dual magnetic gauge boson. This may offer one possible physical interpretation of our scalar field of Eq. (38): it may represent some effective monopole-like field which results from the $SU(2)/U(1)$ coset fields. This is in accord with lattice QCD simulations which indicate that monopole condensation plays a role in color confinement.¹⁶

The effective mass term for the Lagrangian in (38) comes from a condensation of ghost fields. This condensation is of the correct character (i.e. tachyonic) to give spontaneous symmetry breaking and the existence of Nielsen–Olesen flux tube solutions.² Both of these are thought to be important features in explaining confinement via the dual superconducting model of the QCD vacuum. The apparently problematic result (i.e. the tachyonic ghost condensate and resulting tachyonic nature for the off-diagonal gauge field masses) of Refs. 3 and 4 is actually a desired result in the present work. In our model a tachyonic mass term is necessary for the effective field, φ in order to have a GL-like Lagrangian that exhibits both spontaneous symmetry breaking and Nielsen–Olesen flux tube solutions. Note that φ is related to the condensate of the off-diagonal gauge fields via (20) and (38).

The essential physics here is that one has disordered fields (the gauge fields of the $SU(2)/U(1)$ coset space or equivalently the effective, complex, scalar field) which “pushes out” (i.e. exhibits the Meissner effect) the ordered field (the Abelian, $U(1)$ field) except in the interior of the flux tubes. This is a continuation of Ref. 1 which supports the dual superconducting picture of the Yang–Mills vacuum for a *pure* gauge field. The scalar field comes from some subset of the gauge fields rather than being put in by hand. An interesting and open question is if the procedure in this paper can be applied to the $SU(3)$ gauge theory of the strong interaction. In Ref. 6 the $SU(3)$ Lagrangian *with quarks* was studied, and using a procedure similar to the one in the present paper, it was found that the dual Meissner effect did occur.

The main assumptions used to arrive at the effective GL-like Lagrangian from a pure non-Abelian gauge theory are enumerated as follows:

- $SU(2)$ quantized gauge fields can be separated on two components: ordered (Abelian) and disordered (coset) phases;
- ordered and disordered fields are correlated according to Eq. (14);
- Green’s functions for the disordered phase can be expressed via one function approximation as in Eq. (38).

In conclusion we would like to make the following remark: in this paper we have shown that in nonlinear quantum systems one can have well-ordered objects (Nielsen–Olesen flux tube in our case). This can be compared with classical self-organizing systems like Benard cells,¹⁷ or Belousov–Zhabotinsky¹⁸ reactions (and references therein). The most important difference is that in the classical case the self-organization arises from some external flux of energy, but in our quantum case the self-organization arises from inside the nonlinear, quantum system without the need for any external sources.

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