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F -essence

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In the present work, we study the cosmological model with fermionic field and with the non-canonical kinetic term (*fermionic k-essence* or *f-essence*). We also present some important reductions of the model as well as its some generalizations. We also found the exact solution of the model and examine the influence of such gravity-fermion interaction on the observed accelerated expansion of our universe.

1. The observational evidence from different sources for the present stage of accelerated expansion of our universe has driven the quest for theoretical explanations of such feature. Assuming the validity of the theory of gravity, one attempt of explanation is the existence of an unregarded, but dominated at present time, ingredient of the energy content of the universe, known as dark energy, with unusual physical properties. The other possibility is modifying the general theory of relativity at large scales. In cosmology, the investigation for the constituents responsible for the accelerated periods in the evolution of the universe is of great interest. The mysterious dark energy has been proposed as a cause for the late time dynamics of the current accelerated phase of the universe.

During last years theories described by the action with non-standard kinetic terms, k-essence, attracted a considerable interest. Such theories were first studied in the context of k-inflation [1], and then the k-essence models were suggested as dynamical dark energy for solving the cosmic coincidence problem [2]-[4]. The action of the k-essence scalar field ϕ minimally coupled to the gravitational field $g_{\mu\nu}$ we write in the form (see e.g. [1]-[4])

$$S = \int d^4x\sqrt{-g}[R + K_1(X, \phi)], \tag{1}$$

where

$$X = 0.5g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi, \tag{2}$$

is the canonical kinetic term, ∇_μ is the covariant derivative associated with metric $g_{\mu\nu}$. The important particular reductions of the scalar k-essence (1.1) are: i) $K_1 = A_1(X)$ (purely kinetic case); ii) $K_1 = A_1(X)B_1(\phi)$; iii) $K_1 = A_1(X) + B_1(\phi)$.

In the recent years several approaches were made to explain the accelerated expansion by choosing fermionic fields as the gravitational sources of energy (see e.g. refs. [5]-[18]). In particular, it was shown that the fermionic field plays very important role in: i) isotropization of initially anisotropic spacetime; ii) formation of singularity free cosmological solutions; iii) explaining late-time acceleration. In the present work, we study the cosmological model with fermionic field, the M_{33} - model, which has the non-canonical kinetic term (*f-essence*). We examine the influence of such gravity-fermionic interaction on the accelerated expansion of the Universe.

The formulation of the gravity-fermionic theory has been discussed in detail elsewhere [19]-[22]., so we will only present the result here.

In order to have this work self-consistent, in this section we present briefly the techniques that are used to include fermionic sources in the Einstein theory of gravitation and for a more detailed analysis the reader is referred to[19]-[22]. The Einstein-Dirac action reads as

$$S = \int d^4x\sqrt{-g}[R + \epsilon Y - V], \tag{3}$$

where $\epsilon = \pm 1$ ($\epsilon = 1$ is the usual case and $\epsilon = -1$ is the phantom case) and

$$Y = 0.5i[\bar{\psi}\Gamma^\mu D_\mu\psi - (D_\mu\bar{\psi})\Gamma^\mu\psi], \quad V = V(\bar{\psi}, \psi). \tag{4}$$

The closed system of the equations for this model looks like (see e.i. [8])

$$R_{\mu\nu} - 0.5Rg_{\mu\nu} + T_{\mu\nu} = 0, \quad (5)$$

$$i\Gamma^{\mu}D_{\mu}\psi - \frac{dV}{d\bar{\psi}} = 0, \quad (6)$$

$$iD_{\mu}\bar{\psi}\Gamma^{\mu} + \frac{dV}{d\psi} = 0, \quad (7)$$

$$\dot{\rho}_f + 3H(\rho_f + p_f) = 0, \quad (8)$$

where the density of energy and pressure are given by

$$\rho_f = -V, \quad p_f = -Y + V. \quad (9)$$

2. Let us now we consider the M_{33} - model, which has the action

$$S = \int d^4x\sqrt{-g}[R + K_2(Y, \psi, \bar{\psi})], \quad (10)$$

where

$$Y = 0.5i[\bar{\psi}\Gamma^{\mu}D_{\mu}\psi - (D_{\mu}\bar{\psi})\Gamma^{\mu}\psi]. \quad (11)$$

We work with a space-time metric of the form

$$ds^2 = -dt^2 + a^2(dx^2 + dy^2 + dz^2), \quad (12)$$

that is the FRW metric. In this case, the equations of motion look like

$$3H^2 + 0.5[K_2 + 0.5(K_{2\psi}\psi + K_{2\bar{\psi}}\bar{\psi})] = 0, \quad (13)$$

$$2\dot{H} + 3H^2 + 0.5K_2 = 0, \quad (14)$$

$$K_{2Y}\dot{\psi} + 0.5(3HK_{2Y} + \dot{K}_{2Y})\psi - i\gamma^0 K_{2\bar{\psi}} = 0, \quad (15)$$

$$K_{2Y}\dot{\bar{\psi}} + 0.5(3HK_{2Y} + \dot{K}_{2Y})\bar{\psi} + iK_{2\psi}\gamma^0 = 0, \quad (16)$$

$$\dot{\rho}_f + 3H(\rho_f + p_f) = 0, \quad (17)$$

where $Y = 0.5i(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi)$ and

$$\rho_f = -[K_2 + 0.5(K_{2\psi}\psi + K_{2\bar{\psi}}\bar{\psi})], \quad p_f = K_2 \quad (18)$$

are the energy density and pressure of the fermionic field. If $K_2 = Y - V$, then from the system (13)-(17) we get the equations corresponding to the Einstein-Dirac model that is the equations (5)-(8). We note that the model (10) contents some important submodels. For example: a) the Langrangian of the M_{33A} - model has the form $K_2 = A_2(Y)$; b) the M_{33B} - model with the Langrangian $K_2 = A_2(Y)B_2(\psi, \bar{\psi})$; c) the M_{33C} - model with the Langrangian $K_2 = A_2(Y) + B_2(\psi, \bar{\psi})$.

3. In this section we want to construct a solution of the M_{33}-model. Let $K_2 = K_2(Y, u)$, where $u = \bar{\psi}\psi$. Then the system (13)-(17) becomes

$$3H^2 + 0.5[K_2 + K_2^{'}u] = 0, \quad (19)$$

$$2\dot{H} + 3H^2 + 0.5K_2 = 0, \quad (20)$$

$$K_{2Y}\dot{\psi} + 0.5(3HK_{2Y} + \dot{K}_{2Y})\psi - i\gamma^0 K_2^{'}\psi = 0, \quad (21)$$

$$K_{2Y}\dot{\bar{\psi}} + 0.5(3HK_{2Y} + \dot{K}_{2Y})\bar{\psi} + iK_2^{'}\bar{\psi}\gamma^0 = 0, \quad (22)$$

$$\dot{\rho}_f + 3H(\rho_f + p_f) = 0, \quad (23)$$

where $K_2^{'} = dK_2/du$ and

$$\rho_f = -[K_2 + K_2^{'}u]. \quad p_f = K_2. \quad (24)$$

We now consider the M_{33C}-model, where we assume that $A_2 = \alpha Y^n, \quad B_2 = \beta u^m$ that is the case $K_2 = \alpha Y^n + \beta u^m$. Let $a = a_0 t^\lambda$. Then we have the following solution

$$Y = \left\{ \left[-\frac{6m\lambda^2 - 4(m+1)\lambda}{\alpha m} \right] t^{-2} \right\}^{\frac{1}{n}}, \quad u = \left\{ \left[\frac{-4\lambda}{\beta m} \right] t^{-2} \right\}^{\frac{1}{m}}, \tag{25}$$

where

$$m = \frac{1}{3\lambda - 1}, \quad n = \frac{1}{3\lambda}, \quad c = \alpha n a_0^3 \left[\frac{-4\lambda}{\beta m} \right]^{\frac{1}{m}} \left[-\frac{6m\lambda^2 - 4(m+1)\lambda}{\alpha m} \right]^{\frac{1-n}{n}}. \tag{26}$$

Finally we present the following formulas

$$u = ca^{-3} K_{2Y}^{-1}, \quad \psi_j = c_j a^{-1.5} K_{2Y}^{-0.5} e^{i\gamma^0 \int K_2' K_{2Y}^{-1} dt}, \tag{27}$$

where $c = |c|_1^2 + |c|_2^2 - |c|_3^2 - |c|_4^2, \quad c_j = consts.$

4. We briefly summarize the present work. We first derived the equations of the M_{33} - model for the FRW space-time. Then we found their exact solution for the *f-essence* $K_2 = \alpha Y^n + \beta u^m$. Finally, let us we present the expression for the equation of state parameter w. For the our particular solution (25) it takes the form

$$w_f = \frac{p_f}{\rho_f} = -1 - \frac{2}{3\lambda}. \tag{28}$$

This formula tells us that the M_{33} - model can describes the observed accelerated expansion of our universe. Finally we would like to present the M_{34} - model which has the following action

$$S = \int d^4x \sqrt{-g}[R + K(X, Y, \phi, \psi, \bar{\psi})]. \tag{29}$$

It contents some important particular submodels. For example: i) the scalar k-essence (1) as $K = K_1(X, \phi)$; ii) the M_{33} - model (10) as $K = K_2(Y, \psi, \bar{\psi})$; iii) the M_{34A} - model as $K = K_1(X, \phi) K_2(Y, \psi, \bar{\psi})$; vi) the M_{34B} - model as $K = K_1(X, \phi) + K_2(Y, \psi, \bar{\psi})$. We believe that the M_{34} - model can desctibes the late-time acceleration of the universe.

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Цыба П.Ю. Нугманова Г.Н. *F* **-эссенция**

Осы жұмыста фермиондың өрісі және канондық емес кинетикалық мүшесі (фермиондық к-эссенция немесе *F* -эссенция) бар космологиялық моделі зерттелген. Модельдің кейбір маңызды жеңілдетері, сонымен бірге оның кейбір жалпылануы келтірілген. Модельдің дәл шешімі табылған және бақаланытын Әлемнің үделемі ұлғаюына фермиондардың гравитациялық әсерлесуінің ықпалы зерттелген.

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В настоящей работе изучена космологическая модель с фермионным полем и с неканоническим кинетическим членом (фермионных K-эссенции или F-эссенции). Приведены также некоторые важные упрощения модели, а также некоторые ее обобщения. Найдено точное решение модели и изученно влияние гравитационного взаимодействия фермионов на наблюдаемое ускоренное расширение Вселенной.

Поступила в редакцию 11.10.10

Рекомендована к печати 30.10.10