p-wave holographic superconductors with Weyl corrections

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We study the (3+1) dimensional p-wave holographic superconductors with Weyl corrections both numerically and analytically. We describe numerically the behavior of critical temperature T_c with respect to charge density ρ in a limited range of Weyl coupling parameter γ and we find in general the condensation becomes harder with the increase of parameter γ. In strong coupling limit of Yang-Mills theory, we show that the minimum value of T_c obtained from analytical approach is in good agreement with the numerical results, and finally show how we got remarkably a similar result in the critical exponent $\frac{1}{2}$ of the chemical potential μ and the order parameter$\langle J_x^1\rangle$ with the numerical curves of superconductors.

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I. INTRODUCTION

Anti-de Sitter/ Conformal field theory (AdS/CFT) links a d- dimensional strongly coupled conformal field theory on the boundary to a $(d+1)$- dimensional weakly coupled dual gravitational description in the bulk[1–3]. It is necessary to couple a complex scalar field with an Einstein- Maxwell theory to explain the simplest model for holographic superconductors. In a holographic superconductor, below a critical temperature, the gauge symmetry breaks and a black hole is constructed by the unstable developing scalar hair near the horizon. According to the AdS/CFT correspondence, the complex scalar field is dual to a charged operator at the boundary, therefore a superconductor phase transition will be occurred by both the $U(1)$ symmetry-breaking in the gravity and a global $U(1)$ symmetry-breaking in the dual boundary theory[4].

Holographic superconductors have been properly considered in two different models , one with an Abelian-Higgs model, which is the gravity dual of an s-wave superconductor with a scalar order parameter. This model has been studied by many authors see[2, 3, 5–14]. The other type uses a $SU(2)$ gauge theory [15–23].

Hartnoll et al.[3] investigate further this Abelian-Higgs model of superconductivity, and according to AdS/CFT correspondence construct a s-wave holographic superconductor solution with a scalar order parameter displaying the phase transition process at the critical temperature T_c below which the charge condensate form. The Einstein-Yang-Mills (EYM) model of holographic superconductors constructed later by Gubser[24], where spontaneous symmetry-breaking solutions through a condensate of non Abelian gauge fields is presented, and also p-wave and $(p+ip)$-wave backgrounds have been studied [15]. In this case,the CFT have a global $SU(2)$ symmetry and hence three conserved currents . The first effort on analytic methods in this topic was the Herzog's work[25], where critical exponent and the expectation values of the dual operators was attained.

Analytical studies of superconductors have been established in two major methods: *the small parameter perturbation theory* as in[25], and *the variational method* [26, 27]. In the presented paper, we study the Weyl corrected p-wave holographic superconductor composed of a non-Abelian SU(2) gauge field(the matter sector) and a black hole background(the gravity sector) by using the variational method giving only critical temperature T_c^{Min}. As it has been mentioned in[28], for an Abelian gauge field and large range of the Weyl coupling value $\frac{-1}{16} < \gamma < \frac{1}{24}$, the universal relation for the critical temperature $T_c \approx \sqrt[3]{\rho}$ has been found. In this paper we have explored the same validity of the critical temperature relation in a non-Abelian gauge field with Weyl correction numerically and analytically.

The organization of this paper is as follows. In section II we reconstruct the Weyl corrected superconductor's solution of the EYM theory, which is dual to a p-wave superconductor. In section III we present numerical results for

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condensation and critical temperature of the holographic superconductor. Then we investigate the behavior of critical temperature T_c and dual of chemical potential μ with respect to Weyl coupling parameter γ, dual of charge density ρ, and order parameter $\langle J_x^1 \rangle$ analytically in section IV. The conclusion and some discussion are given in section V.

II. WEYL CORRECTED P-WAVE SUPERCONDUCTORS

In this section we study the holographic phase transition for the probe $SU(2)$ Yang-Mills (YM) field A_μ^a in a five dimensional space-time. The bulk action of the Weyl gravity with an $SU(2)$ Yang-Mills in a five dimensional spacetime is:

$$S = \int dtd^4x\sqrt{-g}\{\frac{1}{16\pi G_5}(R+12) - \frac{1}{4g^2}F_{\mu\nu}^a F^{a\mu\nu} + \gamma C^{\mu\nu\rho\sigma}F_{\mu\nu}^a F_{\rho\sigma}^a\} \tag{1}$$

Here G_5 is the five dimensional gravitational constant, g is the Yang-Mills coupling constant, and the negative cosmological constant is satisfied by the factor $\frac{12}{l^2}$, $l = 1$ in the first parenthesis. The field strength component is given as below, where A_μ^a's are the non-Abelian gauge fields.

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \varepsilon^{abc}A_\mu^b A_\nu^c, \{a = 1, 2, 3, \mu = 0, 1, 2, 3\} \tag{2}$$

The Weyl's coupling γ is limited such that its value is in the interval $-\frac{1}{16} < \gamma < \frac{1}{24}$, (for more precise details see [29]). In the probe limit by neglecting the back reactions , the gravity sector is effectively decoupled from the matter field's sector. In this probe limit, the background metric is given by an AdS-Schwarzschild black hole:

$$ds^2 = r^2(-fdt^2 + dx^i dx_i) + \frac{dr^2}{r^2 f} \tag{3}$$

where:

$$f = 1 - (\frac{r_+}{r})^4 \tag{4}$$

The black hole horizon is $r = r_+$. The Smarr-Bekenstein-Hawking temperature of the black hole is determined by the Schwarzschild radius as $T = \frac{r_+}{\pi}$. This is the same as the temperature of the conformal field theory on the boundary of the AdS spacetime. Applying the Euler-Lagrange equation, we can derive the generalized Yang-Mills equation as[28]:

$$\nabla_\mu \left(F^{a\mu\nu} - 4\gamma C^{\mu\nu\rho\sigma}F_{\rho\sigma}^a\right) = -\epsilon_{bc}^a A_\mu^b F^{c\mu\nu} + 4\gamma C^{\mu\nu\rho\sigma}\epsilon_{bc}^a A_\mu^b F_{\rho\sigma}^c \tag{5}$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor and has the following nonzero components in AdS^5:

$$C_{0i0j} = f(r)r_+^4\delta_{ij}, \quad C_{0r0r} = -\frac{3r_+^4}{r^4}, \quad C_{irjr} = -\frac{r_+^4}{r^4 f(r)}\delta_{ij}, \quad C_{ijkl} = r_+^4\delta_{ik}\delta_{jl}. \tag{6}$$

For realization of a holographic p-wave superconductor we take the following anstaz for Yang-Mills gauge field [30]:

$$A = \varphi(r)\sigma^3 dt + \psi(r)\sigma^1 dx \tag{7}$$

(σ^i Pauli's matrixes). The condensation of $\psi(r)$ breaks the $SU(2)$ symmetry and the final state is the superconductor phase transition. The gauge function $\psi(r)$ is dual to the J_x^1 operator on the boundary, choosing x axis as a special direction, the condensation phase of $\psi(r)$ breaks the symmetry and leads to a phase transition, which can be interpreted as a p-wave superconductor phase transition on the boundary. The resulting Yang-Mills equations for metric (3) are given by:

$$\left(1 - \frac{24\gamma r_+^4}{r^4}\right)\varphi'' + \left(\frac{3}{r} + \frac{24\gamma r_+^4}{r^5}\right)\varphi' - \left(1 + \frac{8\gamma r_+^4}{r^4}\right)\frac{\psi^2\varphi}{r^4 f} = 0 \tag{8}$$

$$\left(1 - \frac{8\gamma r_+^4}{r^4}\right)\psi'' + \left[\frac{3}{r} + \frac{f'}{f} - \frac{8\gamma r_+^4}{r^4}\left(-\frac{1}{r} + \frac{f'}{f}\right)\right]\psi' + \left(1 + \frac{8\gamma r_+^4}{r^4}\right)\frac{\varphi^2\psi}{r^4 f^2} = 0 \tag{9}$$

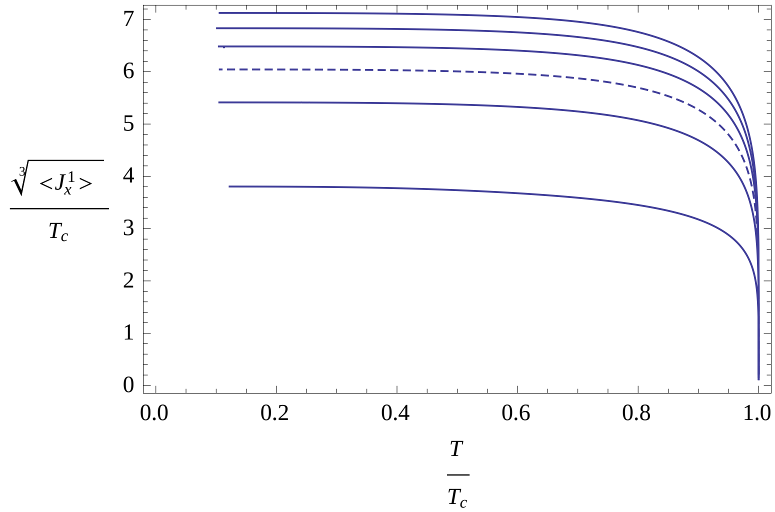

FIG. 1: The condensation as a function of temperature for the operators $< J_x^1 >$. $\gamma = -0.06, -0.04, -0.02, 0, 0.02, 0.04$ from top to bottom and the dotted line is just the case $\gamma = 0$.

where the prime denotes derivative with respect to r. It is more conveint to work in terms of the dimensionless parameter $z = \frac{r_+}{r}$, in which at the horizon $z = 1$, and the boundary at the infinity locates at $z = 0$. Then the the equations of motion (8) and (9) can be reexpressed as:

$$\left(1 - 24\gamma z^4\right)\varphi'' - \frac{1}{z}\left(1 + 72\gamma z^4\right)\varphi' - \left(1 + 8\gamma z^4\right)\frac{\psi^2\varphi}{f} = 0 \tag{10}$$

$$\left(1 - 8\gamma z^4\right)\psi'' + \left[-\frac{1}{z} + \frac{f'}{f} - 8\gamma z^4\left(\frac{3}{z} + \frac{f'}{f}\right)\right]\psi' + \left(1 + 8\gamma z^4\right)\frac{\varphi^2\psi}{f^2} = 0 \tag{11}$$

where the prime now denotes derivative with respect to z. The boundary conditions at infinity, i.e. $z \to 0$, are:

$$\varphi \simeq \mu - \rho z^2 \tag{12}$$

$$\psi \simeq \psi^{(0)} + \psi^{(2)} z^2 \tag{13}$$

μ and ρ are dual to the chemical potential and charge density of the CFT boundary, $\psi^{(0)}$ and $\psi^{(2)}$ are dual to the source term and expectation value of the boundary operator J_x^1 respectively. Further to have a normalizable solution, we always set the source $\psi^{(0)}$ to zero.

γ	-0.06	-0.04	-0.02	0	0.02	0.04
T_c	$0.1701\rho^{1/3}$	$0.1774\rho^{1/3}$	$0.1869\rho^{1/3}$	$0.2005\rho^{1/3}$	$0.2239\rho^{1/3}$	$0.3185\rho^{1/3}$

TABLE I: The critical temperature T_c for different values of Weyl coupling parameter γ(Numerical results).

III. NUMERICAL TREATMENT

In this section we will present numerical results for the condensation and critical temperature due to the shooting method. From EOM's (10) and (11) and the asymptotic behavior of ψ and φ at infinity (12) and (13), we can obtain the regularity condition as:

$$\varphi(1) = \varphi'(0) = \psi'(0) = \psi'(1) = 0 \tag{14}$$

combining boundary condition (12) and (13) with regularity condition (14), we can solve EOM's (10) and (11) numerically by using a shooting method, and plot the FIG.1 to demonstrate the condensation as a function of temperature for the operator$< J_x^1 >$. The curve in FIG.1 is qualitatively similar to that obtained in the holographic superconductors[28, 32], where the condensation of $< J_x^1 >$ goes to a constant at zero temperature. As we can see

from the FIG.1, it is easy to find that the critical temperatures of Weyl corrected superconductors is increasing as the parameter γ varies in the range of -0.06 to 0.04, therefore we conclude that when $\gamma < 0$ the critical temperature is smaller and the formation of scalar hair is harder and vice versa when $\gamma > 0$.
We have also presented the critical temperature T_c with different values of the parameter γ in the TABLE I. According to the results in the TABLE I, we can conclude that by minimizing the coupling parameter γ, the critical temperature decrease smoothly .

IV. ANALYTICAL TREATMENT

In this section we compute the critical temperature and critical exponent via an analytical method, which has been proposed recently [26]. In this method by defining appropriate equation matching with field's boundary conditions, the field's EOM will be transformed to Sturm-Liouville self adjoint form. Therefore according to the general variational method to solve the Sturm-Liouville problem ([33] or appendix of [26]), the eigenvalue λ^2 minimizing the Sturm-Liouwille equation can be found. Using this minimum value of λ, one can obtain the minimum critical temperature T_c^{Min}. In this section we calculate this T_c^{Min} and discuss the critical exponent.

A. Critical temperature T_C^{Min}

Considering the non linear system (10,11). If there is a second order continuous phase transition at the critical temperature, the solution of the EOMs at the T_C should be:

$$\psi(z) = 0, \varphi(z) = \lambda h_c(1 - z^2) \tag{15}$$

Here $\lambda = \frac{\rho}{h_c^3}$, h_c is the radius of the horizon corresponding to $T = T_c$. At a temperature slightly below T_c, the EOM for ψ becomes:

$$z^2 \frac{d}{dz}((\frac{(1-z^4)}{2g^2 z} + 4\gamma h_c^3 z^3)\frac{d\psi}{dz}) + \lambda^2[h_c^3(\frac{z}{2g^2} + 4\gamma z^5)\frac{1-z^4}{1+z^4}]\psi = 0 \tag{16}$$

It is appropriate to define:

$$\psi(z) = \frac{\langle J_x^1 \rangle}{h} z^2 F(z) \tag{17}$$

Matching the boundary condition at the boundary $z = 0$, we normalize the function as $F(0) = 1, F'(0) = 0$. The equation for F(z) is:

$$\frac{d}{dz}(k(z)\frac{dF(z)}{dz}) - p(z)F(z) + \lambda^2 q(z)F(z) = 0 \tag{18}$$

where:

$$k(z) = \frac{z^3(1-z^4)}{2g^2} + 4\gamma h_c^3 z^7 \tag{19}$$

$$p(z) = -2z^5(-\frac{2}{g^2} + 16\gamma h_c^3) \tag{20}$$

$$q(z) = h_c^3 z^2(\frac{h_c z}{2g^2} + 4\gamma z^5)\frac{1-z^4}{1+z^4} \tag{21}$$

The eigenvalue λ minimizes the expression (18) is obtained from the following functional:

$$\lambda^2 = \frac{\int_0^1 (k(z)F'(z)^2 + p(z)F(z)^2)dz}{\int_0^1 q(z)F(z)^2 dz} \tag{22}$$

To estimate it, we use the trial function $F(z) = 1 - \alpha z^2$. We then obtain:

$$\lambda_\alpha^2 = \frac{2g^2\left(-1.6h_c^3\alpha^2\gamma + 8.h_c^3\alpha\gamma - 5.33333h_c^3\gamma + \frac{0.533333\alpha^2}{g^2} - \frac{\alpha}{g^2} + \frac{0.666667}{g^2}\right)}{h_c^3\left(\alpha^2\left(-0.21929\gamma g^2 - 0.0151862\right) + \alpha\left(0.628086\gamma g^2 + 0.052961\right) - 0.485957\gamma g^2 - 0.0568528\right)} \tag{23}$$

Which attains a minimum at $\alpha = 0.304936$, and from the $\lambda = \frac{\rho}{h_c^3}$ and $T_c = \frac{h_c}{\pi}$ the minimum value of the critical temperature can be read as:

$$T_c^{Min(\pm)} = 0.256926\sqrt[3]{\frac{-0.128539 \pm 0.3125g^2\sqrt{\frac{1.90164\gamma g^2\rho^2(1.00743\gamma g^2+0.134769)+0.169187}{g^4}}}{\gamma g^2}} \tag{24}$$

We know that in strong-coupling regime of the YM theory, the quantities can be expanded in series of $\frac{1}{g}$ [34]. Since for some values of g, we may be have $T_c^{Min(-)} < 0$, therefore only the $T_c^{Min(+)}$ is acceptable and can be read in strong limit of order $\frac{1}{g^2}$ as:

$$T_c^{Min(+)} \approx 0.1943040830\rho^{\frac{1}{3}} + \frac{0.1497404(-0.128539\gamma + 0.028931219\gamma\rho)}{\gamma^2\rho^{\frac{2}{3}}g^2} \tag{25}$$

In prob limit by neglecting the back reaction, the large values of the YM coupling is accessible. Comparing equation (25) with the TABLE I we observe that the analytic value of the leading order $T_c^{Min} \approx 0.1943040830\rho^{\frac{1}{3}}$ obeys the well known role $T_c \propto \rho^{\frac{1}{3}}$ and it is the lower bound for tabulated values of T_c, given in TABLE I. According to the numerical results of TABLE I, the minimum value of T_c reads as:

$$T_{c(numeric)}^{Min(+)} \approx 0.1701\rho^{\frac{1}{3}} \tag{26}$$

which shows that the analytic values in relation (25) and the numerical estimate in equation (26) are in good agreement with each other. It seems that there is a deep relation between the strong limit of YM part of the action and the analytical results of the p-wave superconductors .

B. Relation of $< J_x^1 > -(\mu - \mu_c)$

Now we want to know the behavior of the order parameter at T_c, by solving the equation for the scalar potential close to T_c, therefore by substituting (13) in (10) we have:

$$\frac{d}{dz}((-\frac{1}{2zg^2} + 12\gamma z^3)\frac{d\varphi}{dz}) + (\frac{z}{h_c})^2(\frac{z}{2(1-z^4)g^2} + \frac{4\gamma z^5}{1-z^4})(\frac{< J_x^1 >}{h_c})^2F(z)^2\varphi = 0 \tag{27}$$

Since near the critical point, the order parameter $< J_x^1 >$ is small, we can expand φas a series form of the small parameter as below:

$$\varphi = \mu_c + < J_x^1 > \chi(z) + ... \tag{28}$$

The boundary condition imposes that $\chi(z)$ to be $\chi(1) = 0$ .The EOM for $\chi(z)$ can be obtained as below:

$$\frac{d}{dz}((-\frac{1}{2zg^2} + 12\gamma z^3)\frac{d\chi(z)}{dz}) + (\frac{z}{h_c})^2(\frac{z}{2(1-z^4)g^2} + \frac{4\gamma z^5}{1-z^4})\frac{J_x^1}{h_c^2}F(z)^2\mu_c = 0 \tag{29}$$

By integration from both sides of the (29), the EOM for $\chi(z)$ can be reduced to:

$$(-\frac{1}{2z\zeta^2} + 12\gamma z^3)\frac{d\chi(z)}{dz} = -\mu_c\frac{< J_x^1 >}{h_c^4}\int z^2(\frac{z}{2(1-z^4)g^2} + \frac{4\gamma z^5}{1-z^4})F(z)^2dz \tag{30}$$

By regularity condition, we have $\chi'(0) = 0$, so we take $F(z) = (1 - z^2)(1 - \alpha z^2)$. Now $\varphi(z)$ can be expanded as:

$$\varphi(z) \sim \mu - \rho z^2 \approx \mu_c + < J_x^1 > (\chi(0) + \chi'(0)z + ...) \tag{31}$$

Now from (31), by comparing the coefficients of z^0 term in both sides of the above formula, we obtain:

$$\mu - \mu_c \approx \frac{(< J_x^1 >)^2 \mu_c}{34560 h^4 \gamma^2 g^4} (3003.22 \gamma^{3/2} g^3 \left(8 \gamma g^2 + 1\right) \mathrm{Li}_2 \left(\frac{12.\sqrt{\gamma} g}{12.\sqrt{\gamma} g - 2.44949}\right) + (9047.79 + 6359.31) \gamma^2 g^4) \tag{32}$$

Where $\alpha = 0.304936$ is a parameter minimizing the equation (23), and $\mathrm{Li}_2(z)$ gives the polylogarithm function. This critical exponent $\frac{1}{2}$ for the condensation value and $(\mu - \mu_c)$ qualitatively match the numerical curves for superconductors with Weyl corrections[28].

V. CONCLUSIONS

In this letter we have investigated the Weyl corrected p-wave Holographic superconductors at the probe limit using numerical and analytical solutions. We obtained the behavior of the critical temperature T_c as a function of the dual charge density ρ in different values of Weyl coupling parameter $\frac{-1}{16} < \gamma < \frac{1}{24}$. We have found that the critical temperature increases by growing the Weyl coupling parameter, therefore the condensation becomes harder when $\gamma < 0$, and vice versa when $\gamma > 0$. As a final point, obtaining the critical exponent $\frac{1}{2}$ for the chemical potential μ and order parameter$\langle J_x^1 \rangle$ show a good agreement with the numerical curves of Weyl corrected s-wave holographic superconductors. Furthermore, we have shown that in the strong limit of YM theory, the analytical and numerical values of the minimum value of the critical temperature are in good agreement.

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