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Reconstruction of some cosmological models in $f(R,T)$ cosmology

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Abstract In this paper, we reconstruct cosmological models in the framework of $f(R,T)$ gravity, where R is the Ricci scalar and T is the trace of the stress-energy tensor. We show that the dust fluid reproduces ΛCDM, phantom–non-phantom era and phantom cosmology. Further, we reconstruct different cosmological models, including the Chaplygin gas, and scalar field with some specific forms of $f(R,T)$. Our numerical simulation for the Hubble parameter shows good agreement with the BAO observational data for low redshifts, $z<2$.

1 Introduction

From the cosmological observational data [1–4], it is now well established that the present observable Universe is undergoing an accelerating expansion. While the source driving this cosmic acceleration is known as 'dark energy' its origin has not been well understood yet due to absence of a consistent theory of quantum gravity. This acceleration is driven by the negative pressure of the dark energy. The 'cosmological constant' is the most simple and natural candidate for explaining cosmic acceleration but it faces serious problems of fine-tuning and a large mismatch between theory and observations [5–7]. Hence there has been significant development in the construction of dark energy models by modifying the geometrical part of the Einstein–Hilbert action. This phenomenological approach is called Modified Gravity, which can successfully explain the rotation curves of galaxies, the motion of galaxy clusters, the Bullet Cluster, and cosmological observations without the use of dark matter or Einstein's cosmological constant [8–19]. The $f(R)$ theories can produce cosmic inflation, mimic behavior of dark matter and current cosmic acceleration, being compatible with the observational data [20–23] (see also the recent review [24] on $f(R)$ gravity and its cosmological implications).

In a recent paper [25], the authors considered a generalized gravity model $f(R,T)$, with T being the trace of stress-energy tensor, manifesting a coupling between matter and geometry. By choosing different functional forms of f, they solved the dynamical equations of astrophysical and cosmological interest. In the present work, we study the same model by taking different kinds of energy source. Reconstruction of the cosmological models for this theory has been investigated by several authors [28–32].

The paper is organized as follows. In Sect. 2 we present the general action and the equation of the motion for $f(R,T)$. In Sect. 3 we reconstruct the cosmological models from the dust fluid. In Sect. 4 we generalize the dust fluid models to the general fluid with EoS $p=\omega\rho$. Section 5 is devoted to discussion and conclusions.

2 Field equations in $f(R,T)$ gravity

The action of $f(R,T)$ gravity is of the form [25, 26]

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-g}f(R,T)\,d^4x+\int L_m\sqrt{-g}\,d^4x, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the scalar curvature $R=R^\mu_\mu$ and the trace $T=T^\mu_\mu$ of the energy-momentum tensor $T_{\mu\nu}$. We define the Lagrangian density for matter field L_m by

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \tag{2}$$

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The equation of motion (EOM) is obtained by varying the action (1) with respect to $g^{\mu\nu}$ [25]

$$f_R R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) f_R = 8\pi T_{\mu\nu} - f_T T_{\mu\nu} - f_T \Theta_{\mu\nu}, \tag{3}$$

where $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$ and $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$. ∇_μ is the operator for covariant derivative and the box operator (or d'Alembert operator) $\Box$ is defined via

$$\Box \equiv \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu), \qquad \Theta_{\mu\nu} \equiv g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}.$$

Performing a contraction of indices in (3), we obtain

$$R f_R + 3\Box f_R - 2f = 8\pi T - T f_T - \Theta f_T. \tag{4}$$

Here $\Theta \equiv g^{\mu\nu}\Theta_{\mu\nu}$. We will use (4) to reconstruct $f(R,T)$ for different kinds of matter.

3 Reconstruction of $f(R,T)$ using dust

We consider the metric of a flat Friedmann–Robertson–Walker (FRW) spacetime,

$$g_{\mu\nu} = \mathrm{diag}\big(1, -a^2(t), -a^2(t), -a^2(t)\big). \tag{5}$$

Following the conservation of the energy-momentum tensor $T^{\mu\nu}_{;\mu} = 0$ for metric (5) we obtain

$$\dot{T} = -3HT. \tag{6}$$

The 00-component of (3) reads

$$f_R\left(\frac{2}{3}R - \ddot{R}\right) + \frac{1}{6}f = \frac{2}{3}T(8\pi + f_T) + \dot{R}^2 f_{RR} + 9H^2T^2 f_{TT} - 6HT\dot{R} f_{RT} - 3T\left(\frac{R}{6} - 5H^2\right) f_T. \tag{7}$$

In (7), if we know the functional form of the Hubble parameter H as a function of the two independent variables R and T, then reconstruction of the model $f(R,T)$ is straightforward. But always we have a physical intuition about the form of $H(R,T)$. We clarify it here more. In a FRW model, the form of the Ricci scalar is $R = 6(\dot{H} + 2H^2)$. Any cosmological era has specific matter fields, radiation, phantom field or a mixture of them. For example in ΛCDM model, we have $H^2 = H_0^2 + \frac{\kappa^2 \rho_0 a^{-3}}{3}$. It is easy to see that we can write R in terms of H as $H^2 = \frac{R}{3} - 3H_0^2$. Thus, usually, we can write $H^2 = H^2(R)$. Thus (7) can be integrated easily to give the functional form $f(R,T)$. Also, since $\dot{T} = -3H(R)T$, $\ddot{T} = -3T(\frac{R}{6} - 5H^2)$ all terms of (6) are functions of R, T. The term $\ddot{R}$ we can write as $\ddot{R} = \frac{\dot{H}}{\frac{dH}{dR}} = \frac{\frac{R}{6} - 2H(R)^2}{\frac{dH}{dR}}$. Mathematically, (7) is a second order partial differential equation for $f(R,T)$.

3.1 Cosmological implications

We now discuss the solutions of (7) relevant in cosmological context. In the coming sections we will show that any cosmological epoch (radiation, matter, dark energy dominated eras) can be constructed in a model of $f(R,T)$ only with dust fluid as the source. We mention here that the form of the action $f(R,T)$ is not unique. For this reason we can continue our investigation of the cosmological reconstruction for another model with $f(R,T) = f_1(R) + f_2(T)$ [25]. When we set $f_2(T) = 0$, we recover the $f(R)$ theory, which has been discussed in [27]. Adopting the technique of the reconstruction of $f(R,T)$ models, we have the following different models reconstructed from the dust fluid.

3.2 ΛCDM model

In Einstein gravity, the Hubble parameter for a flat FRW model with real matter field describes the ΛCDM model by $H^2 = H_0^2 + \frac{\kappa^2}{3}\rho_0 a^{-3}$, in units $\kappa^2 = 8\pi G\ c = 1$. Since

$$H^2 = \frac{R}{3} - 3H_0^2, \tag{8}$$

$$\dot{R} = -9\left(\frac{R}{3} - 3H_0^2\right)^{3/2}, \tag{9}$$

$$\ddot{R} = \frac{9}{2}\big(R - 12H_0^2\big)\big(R - 9H_0^2\big), \tag{10}$$

then the field equation (7) converts to the following form:

$$\left[\frac{2}{3}R - \frac{9}{2}\big(R - 12H_0^2\big)\big(R - 9H_0^2\big)\right] f_R + \frac{1}{6}f = \frac{2}{3}T(8\pi + f_T) + 81\left(\frac{R}{3} - 3H_0^2\right)^3 f_{RR} + 9\left(\frac{R}{3} - 3H_0^2\right)T^2 f_{TT} + 54\left(\frac{R}{3} - 3H_0^2\right)^2 T f_{RT} - 3T\left(\frac{R}{6} - 5H^2\right) f_T. \tag{11}$$

The solutions of (11) can be obtained where we have two special cases:

- $f(R,T) = F(R)$ and $F(R) = AR + B$ where A, B and T are constants. Equation (15) gives $B = 32\pi T$ and $A = 0$.
- $f(R,T) = G(T)$ and $G(T) = CT + D$ where C, D and R are constants. Equation (15) gives $C = \frac{-16/3\pi}{\frac{1}{2} + \frac{9}{2}R + 9H_0^2}$ and $D = 0$.

3.3 $f(R,T)$ reproducing the system with phantom and non-phantom matter

In Einstein gravity, a Universe filled with a mixture of phantom and non-phantom components obey the following expression for H: $H^2 = \frac{\kappa^2}{3}(\rho_q a^{-c_1} + \rho_p a^{c_1})$. Here the set of the variables ρ_q (energy density of non-phantom matter) ρ_p, (energy density of phantom matter) c_1 are the parameters of the model. In the early Universe when the scale factor was small, the first term in $\rho_q a^{-c_1}$ dominates and *it behaves as the Universe described by Einstein gravity with a matter whose EoS parameter is* $w=-1+c_1/3>-1$, which means that the matter field is non-phantom like. But for present era we have $w=-1-c_1/3<-1$, which means we live in a phantom epoch of the Universe.

As a special case, $c_1=4$, which is a special form discussed in [27], we have

$$H^2=\frac{R}{3}-3H_0^2, \tag{12}$$

$$\dot{R}=2\frac{\sqrt{\frac{a}{R}+bR(\frac{R}{6}-2H^2)}}{b-\frac{a}{R^2}}, \tag{13}$$

$$\ddot{R}=\left\{2\frac{\sqrt{\frac{a}{R}+bR(\frac{R}{6}-2H^2)}}{b-\frac{a}{R^2}}\right\}\frac{d}{dR}\times\left[2\frac{\sqrt{\frac{a}{R}+bR(\frac{R}{6}-2H^2)}}{b-\frac{a}{R^2}}\right]. \tag{14}$$

Thus (7) converts to the following form:

$$\begin{aligned}&f_R\left(\frac{2}{3}R-\left\{2\frac{\sqrt{\frac{a}{R}+bR(\frac{R}{6}-2H^2)}}{b-\frac{a}{R^2}}\right\}\frac{d}{dR}\times\left[2\frac{\sqrt{\frac{a}{R}+bR(\frac{R}{6}-2H^2)}}{b-\frac{a}{R^2}}\right]\right)+\frac{1}{6}f\\&=\frac{2}{3}T(8\pi+f_T)+\left(2\frac{\sqrt{\frac{a}{R}+bR(\frac{R}{6}-2H^2)}}{b-\frac{a}{R^2}}\right)^2f_{RR}\\&+9\left(\frac{R}{3}-3H_0^2\right)T^2f_{TT}\\&-6\sqrt{\frac{R}{3}-3H_0^2}T\left\{2\frac{\sqrt{\frac{a}{R}+bR(\frac{R}{6}-2H^2)}}{b-\frac{a}{R^2}}\right\}f_{RT}\\&-3T\left(\frac{R}{6}-5H^2\right)f_T.\end{aligned} \tag{15}$$

We solve equation (15) using the ansatz $f(R,T)=F(T)$. The solution of (15) is

$$F(T)=C_1T^{k_1}+C_2T^{k_2}+T-\frac{32\pi T}{3+27R-270H_0^2}, \tag{16}$$

where C_1 and C_2 are two constants of integration and

$$k_1=\frac{1}{36}\frac{-4-9R+108H_0^2+\sqrt{16+144R-1512H_0^2+81R^2-1944RH_0^2+11664H_0^4}}{R-9H_0^2}, \tag{17}$$

$$k_2=\frac{-1}{36}\frac{4+9R-108H_0^2+\sqrt{16+144R-1512H_0^2+81R^2-1944RH_0^2+11664H_0^4}}{R-9H_0^2}. \tag{18}$$

If we choose $f(R,T)=C_3$ a pure constant, then (15) gives $C_3=32\pi T$ where T is also a constant. Furthermore if $f(R,T)=F(R)$ then (15) cannot be solved analytically or numerically.

3.4 de Sitter Universe in $f(R,T)$ gravity

If we live in a Universe filled by dust and the scale factor increases exponentially with time as $a(t)=a_0e^{H_0t}$, then the Hubble parameter is constant. Such a model, which has been proposed firstly by de Sitter, is called a de Sitter Universe. In this solution, it is assumed that the only source of matter filling the Universe is dust. Thus it is contained in our treatment on dust formation. For a de Sitter Universe in its static patch we have

$$H=H_0, \tag{19}$$

$$R=12H_0^2, \tag{20}$$

$$\dot{R}=\ddot{R}=0. \tag{21}$$

Equation (7) reduces to

$$\frac{2}{3}Rf_R+\frac{1}{6}f=\frac{2}{3}T(8\pi+f_T)+9H_0^2T^2f_{TT}-3T\left(\frac{R}{6}-5H_0^2\right)f_T. \tag{22}$$

Solution of (22) is

$$f(R,T)=F_1T^{k_3}+F_2T^{k_4}-\frac{32\pi T}{3+54H_0^2}, \tag{23}$$

where F_1 and F_2 are two arbitrary constants and

$$k_3=\frac{1}{54}\left(\frac{-2+\sqrt{4+54H_0^2}}{H_0^2}\right), \tag{24}$$

$$k_4=\frac{-1}{54}\left(\frac{2+\sqrt{4+54H_0^2}}{H_0^2}\right). \tag{25}$$

3.5 Einstein static Universe in $f(R,T)$ model

In this case since $H=0$, we have $\dot{H}=R=\dot{R}=\ddot{R}=0$. Thus we have

$$\frac{1}{6}f=\frac{2}{3}T(8\pi+f_T). \tag{26}$$

The solution for this purely T dependent equation is

$$f(T)=-\frac{32\pi T}{3}+C_4T^{1/4}. \tag{27}$$

4 Models for $p=\omega\rho$

Starting from (4) and assuming EoS $p=\omega\rho$, we have

$$\Theta_{\mu\nu}=-2(1+\omega)\rho u_\mu u_\nu+p.g_{\mu\nu}. \tag{28}$$

The trace of (28) is given by

$$\Theta=2\rho(\omega-1). \tag{29}$$

Since $T=\rho(1-3\omega)$, we have

$$\Theta=\frac{2\omega-2}{1-3\omega}T. \tag{30}$$

Now we rewrite (4) in the following form:

$$Rf_R+3\Box f_R-2f=8\pi T-Tf_T-\frac{2\omega-2}{1-3\omega}Tf_T, \tag{31}$$

which can be written in suitable form as

$$Rf_R+3\Box f_R-2f=8\pi T-\omega' Tf_T, \tag{32}$$

where $\omega'=-\frac{\omega+1}{1-3\omega}$. We will search for exact solutions of $f(R,T)$ in the following cases.

4.1 Solutions in the form $f(R,T)=R+2f(T)$

Substituting this form $f(R,T)=R+2f(T)$ in (32) we obtain

$$-2R-4f(T)=8\pi T-2\omega' Tf_T. \tag{33}$$

There is only one possibility to get an exact solution: when R is constant. In this case, the solution for (33) reads

$$f(T)=c_1T^{2/\omega'}+\frac{2(2-\omega')R+16\pi T}{4(\omega'-2)}. \tag{34}$$

Thus we have

$$f(R,T)=R+2\left(c_1T^{2/\omega'}+\frac{2(2-\omega')R+16\pi T}{4(\omega'-2)}\right). \tag{35}$$

4.2 Solutions in the form $f(R,T)=f_1(R)+f_2(T)$

Substituting this form of $f(R,T)=f_1(R)+f_2(T)$ in (32) we obtain

$$\begin{aligned}&Rf_1'(R)+3\Box f_1'(R)-2f_1(R)\\&\quad=8\pi T-\omega' Tf_2'(T)+2f_2(T).\end{aligned} \tag{36}$$

Note that the left and right hand sides of (36) are functions of R and T, respectively. Thus solving the T dependent part of (36), we obtain

$$f_2(T)=CT^{\frac{2}{\omega'}}+\frac{16\pi T+(\omega'-2)c_1}{2(\omega'-2)}. \tag{37}$$

The solution for the R dependent part of the (36) is complicated. Indeed we must solve the following equation:

$$Rf_1'(R)+3\big(\partial_{tt}f_1'(R)+3H\partial_t f_1'(R)\big)-2f_1(R)=c_1, \tag{38}$$

or the following equivalent form:

$$\begin{aligned}&Rf_1'(R)+3\big(\ddot{R}f_1''(R)+\dot{R}^2f_1'''(R)\\&\quad+3H\dot{R}f_1''(R)\big)-2f_1(R)=\frac{c_1}{6}.\end{aligned} \tag{39}$$

One simple but interesting solution is obtained by taking the Ricci curvature to be constant. From (39) we have

$$f_1(R)=-\frac{c_1}{2}. \tag{40}$$

Thus one of the interesting models is

$$f(R,T)=-\frac{1}{2}c_1+CT^{\frac{2}{\omega'}}+\frac{16\pi T+(\omega'-2)c_1}{2(\omega'-2)}. \tag{41}$$

For this model, we can study the evolution of the Hubble parameter and the scale factor. For this purpose, we assume $f_1(R)=R^2$ and rewrite (39) in terms of the Hubble parameter $H(z)$ (and using $dt=-\frac{1}{1+z}\frac{dz}{H(z)}$) and the scale factor $a(t)$:

$$\begin{aligned}&18H\frac{dH}{dz}-6(1+z)\left(\frac{dH}{dz}\right)^2-6(1+z)H\frac{d^2H}{dz^2}\\&\quad+3H\left(12H^2-6H(1+z)\frac{dH}{dz}\right)=\frac{c_1}{6},\end{aligned} \tag{42}$$

$$a^2\frac{d^4a}{dt^4}-5\left(\frac{da}{dt}\right)^2\frac{d^2a}{dt^2}+a\left(\frac{d^2a}{dt^2}\right)^2+3a\frac{da}{dt}\frac{d^3}{dt^3}=\frac{c_1}{6}a^3. \tag{43}$$

The two equations (42) and (43) are numerically solved and the result is shown in Figs. 1 and 2. The initial conditions are determined using the definitions of Hubble parameter and the jerk parameter. Specifically $a(0)=1$, $H_0=74.2$, $\dot{a}(0)=H_0a_0=74.2$, $q_0=-0.67$, $j_0=-0.98$ [33], $\ddot{a}(0)=-H_0^2q_0$, $\dddot{a}(0)=-j_0H_0^3=4\times10^5$ [33]. Note that our model is compatible with the BAO data [34–36] for the Hubble parameter.

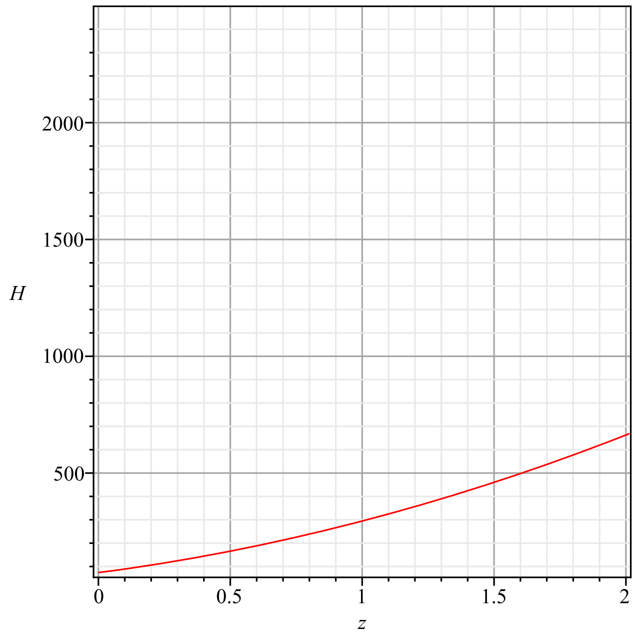

Fig. 1 Evolution of Hubble parameter (42) against redshift z. We choose $c_1 = 6$, $H(0) = H_0 = 74.2$, $\frac{dH}{dz}(z = 0) = 22.26$

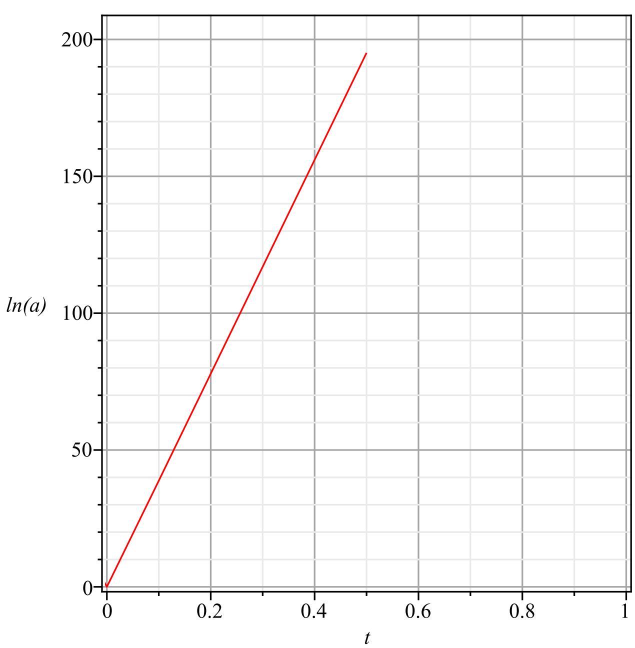

Fig. 2 Evolution of $\log(a)$ (43) against time t. We choose $c_1 = 6$, $a(0) = 1$, $\dot{a}(0) = 74.2$, $\ddot{a}(0) = 2.73 \times 10^5$, $\dddot{a}(0) = 4 \times 10^5$

4.3 Reconstruction using Chaplygin gas

The Chaplygin gas (CG) has the equation of state $p = \frac{-A}{\rho}$. There are some extensions of this model, like the generalized Chaplygin gas, but we restrict ourselves only to the CG case. Thus we have the quantities

$$\Theta = -2T + 4\frac{A}{\rho}, \tag{44}$$

$$T = \rho + 3\frac{A}{\rho}. \tag{45}$$

Solving (45) for ρ and inserting the solution in (44) and by defining a new parameter as $12A = T_0^2$ we have

$$\Theta \equiv \Theta(T) = \frac{2}{3}\frac{T_0^2 - 3T^2 - 3T\sqrt{T^2 - T_0^2}}{T + \sqrt{T^2 - T_0^2}}. \tag{46}$$

Using (46) in (4), we have

$$Rf_R + 3\Box f_R - 2f = 8\pi T - Tf_T - \frac{2}{3}\frac{T_0^2 - 3T^2 - 3T\sqrt{T^2 - T_0^2}}{T + \sqrt{T^2 - T_0^2}} f_T. \tag{47}$$

We are searching for constant curvature solutions in which $R = R_0 \Rightarrow \Box f_R = 0$. Thus solution of (47) is

$$f(T) = R_0 + e^{2\int \frac{dT}{T+\Theta(T)}}\left[C + 8\pi \int \frac{T\,dT}{g(T)(T + \Theta(T))}\right]. \tag{48}$$

4.4 $f(R,T)$ models for scalar field

We know that there is an important duality between $f(R)$ models and scalar fields [24]. The Lagrangian for a scalar field which is minimally coupled to the background reads

$$L = -\frac{1}{2}\omega\phi_{,\mu}\phi^{,\mu}. \tag{49}$$

Here ω is a free parameter. The corresponding expression for stress-energy tensor is

$$T_{\mu\nu} = -\frac{1}{2}\omega\left(\phi_{,\mu}\phi_{,\mu} - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}\right). \tag{50}$$

Now, the expressions for Θ and T read

$$\Theta = -3\omega\phi_{;\alpha}\phi^{;\alpha}, \tag{51}$$

$$T = \frac{1}{2}\omega\phi_{;\alpha}\phi^{;\alpha}. \tag{52}$$

Eliminating the term $\phi_{;\alpha}\phi^{;\alpha}$ from (51) and (52) we have

$$\Theta = -6T. \tag{53}$$

We can rewrite (4) using (53)

$$Rf_R + 3\Box f_R - 2f = 8\pi T + 5Tf_T. \tag{54}$$

Again, we limit ourselves to constant curvature solutions in which $R = R_0$ (choosing $f(R,T) \longrightarrow R + f(T)$). Then the general solution for (54) is

$$f(R,T) = R - \frac{4\pi}{7}T + \frac{C}{T^{\frac{2}{5}}}. \tag{55}$$

5 Conclusion

In summary, we reconstruct cosmological models in the framework of a newly proposed model of $f(R,T)$ gravity, where R is the Ricci scalar and T is the trace of the stress-energy tensor. We show that the dust fluid reproduces ΛCDM, Einstein static Universe, de Sitter Universe, phantom–non-phantom era and phantom cosmology. Further, we reconstruct different cosmological models, including Chaplygin gas, minimally coupled scalar field, with some specific forms of $f(R,T)$. We demonstrated that for one case, $f(R,T)=R^2+f(T)$, we found that the behavior of Hubble parameter $H(z)$ and scale factor is compatible with the observational data of BAO for small redshift $z<2$.

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