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RESEARCH ARTICLE

Creation/annihilation of wormholes supported by the Sine-Gordon phantom (ghost) field

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Abstract The possible process of creation/annihilation of traversable wormholes in the model with phantom (ghost) scalar field is described. It is shown that such process can be realized only for some special choice of a potential energy, in particular, for the Sine-Gordon potential.

Keywords Scalar fields · Wormholes

1 Introduction

In the paper [1], the *conditions* of existence of traversable wormholes have been considered. In summary, the following conclusion had been done: *"...However, any hope that they (wormholes) might be constructable must rely on the future discovery of an exotic field or quantum state of known fields with tension that exceeds energy density on macroscopic length scales ..."*. The last condition means violation of the weak energy condition (WEC) which states that $\rho + p \geq 0$, where ρ and p are

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effective energy density and pressure of matter. It can be done in a few ways (for a review, see [2]). One of possible ways of violation of the WEC can be realized by using some phantom (or ghost) scalar fields as a source of matter. There are some works in this direction (see, e.g., Refs. [3,4]). In these papers some effective hydrodynamical energy-momentum tensor with the violated WEC was chosen as a source of matter. But a distribution of this matter was added by hand and accordingly the nonself-consistent models of the traversable wormholes were considered. In Ref. [5] the self-consistent model of the traversable wormhole supported by two interacting phantom and ghost scalar fields was considered.

But these investigations just show that there is a possibility of existence of static wormhole-like solutions, leaving aside the question of dynamical evolution of the models. Of course, investigation of such dynamical problem demands a consideration of a self-consistent problem of evolution of a traversable wormhole with time. Apparently, the construction of such models is quite complicated problem from the technical point of view. That is why at the initial stage of researches, it is possible either a consideration of some general properties of such models, or investigations of the simplest models allowing to estimate parameters of the dynamical process.

In particular, in Ref. [6] an example of possible enlarging of a submicroscopic wormhole to macroscopic size on the background of inflationary solution is considered. In [7] the cosmological model with the generalized Morris–Thorne traversable wormhole was investigated. Note that in the first paper violation of the WEC takes place, and in the second one not. In Ref. [8] the discussion of some general properties of time-dependent (dynamical) wormholes is presented. In the work [9] some general consideration of a possible process of dynamic formation of a wormhole united with a black hole is also presented. In the paper [10] a specific analytic model of wormhole formation with the massless ghost scalar field is suggested. In this model, there are two special features: (1) the spacetime is constructed by gluing the Minkowski and Roberts spacetimes at null hypersurfaces in a regular manner; (2) there appears a weak curvature singularity at the moment of the wormhole formation.

In this paper we consider more modest problem. We try to show that there exists a set of static *regular* wormhole-like solutions with smoothly varying throat radius up to the zero value. It might be interpreted as a *possible* process of creation/annihilation of wormholes by adding/subtraction of phantom (ghost) matter to some spherically symmetric object created by the phantom (or ghost) matter with the Sine-Gordon potential energy. Our main purpose is to show that for such phantom field there exists a sequence of wormhole solutions with a vanishing throat radius. It might be interpreted as follows: increasing or decreasing the phantom matter, one can decrease or increase the throat radius. If to change a phantom mass of the throat in such a way to get a zero throat radius, then, as a result, one will obtain two spherically symmetric solutions connected (strictly speaking) in one point. If there is a possibility to break up these two spaces in this point, then one might obtain two separate spaces, i.e., the wormhole will be broken up. One can suppose that the inverse process of creation of a wormhole also exists.

2 Wormholes supported by a phantom Sine-Gordon scalar field

Let us consider a gravitating system with one phantom scalar field φ with the Lagrangian

$$L = -\frac{R}{16\pi G} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi), \tag{1}$$

where R is the scalar curvature, G is the Newton's gravitational constant, and V is the Sine-Gordon potential with the reversed sign

$$V = \frac{m^4}{\lambda}\left[\cos\left(\frac{\sqrt{\lambda}}{m}\varphi\right) - 1\right]. \tag{2}$$

Here m is a mass of the field, and λ is a coupling constant. The corresponding energy-momentum tensor will then be:

$$T_i^k = -\partial_i\varphi\partial^k\varphi - \delta_i^k\left[-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right], \tag{3}$$

and variation of the Lagrangian (1) gives the gravitational and field equations in the form

$$G_i^k = 8\pi G T_i^k, \tag{4}$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\, g^{\mu\nu}\frac{\partial\varphi}{\partial x^\nu}\right) = \frac{\partial V}{\partial\varphi}. \tag{5}$$

Let us take the spherically symmetric metric in the form

$$ds^2 = e^{2F(r)}dt^2 - \frac{dr^2}{A(r)} - (r^2 + r_0^2)(d\theta^2 + \sin^2\theta d\phi^2), \tag{6}$$

where the metric functions $F(r)$ and $A(r)$ depend only on the radial coordinate r.

Introducing new dimensionless variables $\phi = (\sqrt{\lambda}/m)\varphi,\ x = mr$, one can obtain from (3), (4) and (5) the following equations

$$-\frac{A'}{A}x + \frac{1}{A} + \frac{x^2}{x^2 + x_0^2} - 2 = \frac{x^2 + x_0^2}{A}\beta\left(-\frac{A}{2}\phi'^2 + \cos\phi - 1\right), \tag{7}$$

$$\frac{2x^2}{x^2 + x_0^2} - 2 - \frac{A'}{A}x + 2xF' = -\beta(x^2 + x_0^2)\phi'^2, \tag{8}$$

where $\beta = m^2/\lambda$, and Eqs. (7) and (8) are $\binom{t}{t}$ and $\left[\binom{t}{t} - \binom{x}{x}\right]$ components of the Einstein equations (4), respectively, and the scalar field equation is

$$\phi'' + \left(F' + \frac{2x}{x^2 + x_0^2} + \frac{1}{2}\frac{A'}{A}\right)\phi' = \frac{1}{A}\sin\phi, \tag{9}$$

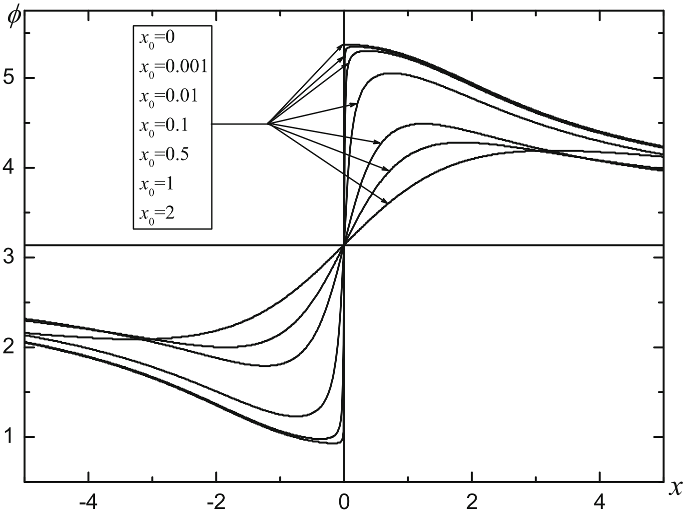

Fig. 1 The behavior of the scalar field ϕ depending on the throat radius x_0. $\beta = 1$ for all lines. Asymptotically $\phi \to \pi$

where a prime denotes differentiation with respect to x. Using Eq. (8), one can rewrite Eq. (9) as follows

$$\phi'' + \left(-\frac{\beta}{2} \frac{x^2 + x_0^2}{x} \phi'^2 + \frac{A'}{A} + \frac{1}{x} + \frac{x}{x^2 + x_0^2} \right) \phi' = \frac{1}{A} \sin \phi. \tag{10}$$

The boundary conditions follow from the equations above. In the wormhole case, one has

$$A(0) = 1 + 2\beta x_0^2, \quad \phi(0) = \pi, \quad \phi'(0) = \sqrt{\frac{2}{\beta x_0^2}}, \tag{11}$$

and in the spherically symmetric case

$$A(0) = 1, \quad \phi(0) = const, \quad \phi'(0) = 0. \tag{12}$$

We solved the system of two Eqs. (7) and (10) numerically using the NDSolve routine from *Mathematica*. Taking the boundary conditions (11) and (12) at different values of x_0 and with some $\beta = 1$, one can obtain the results presented in Figs. 1–3. One can see that asymptotically (at $x \to \pm\infty$) $\phi \to \pi$.

That is, as it was pointed out in Introduction, we have obtained such a sequence of solutions with $x_0 \to 0$ that the solution at $x_0 = 0$ also exists. The last means that the spherically symmetric non-wormhole solution has been found. This limit solution corresponds to two different spaces glued in one point. If we divide them in this point then we will obtain two separate spaces. Both these spaces contain spherically symmetric

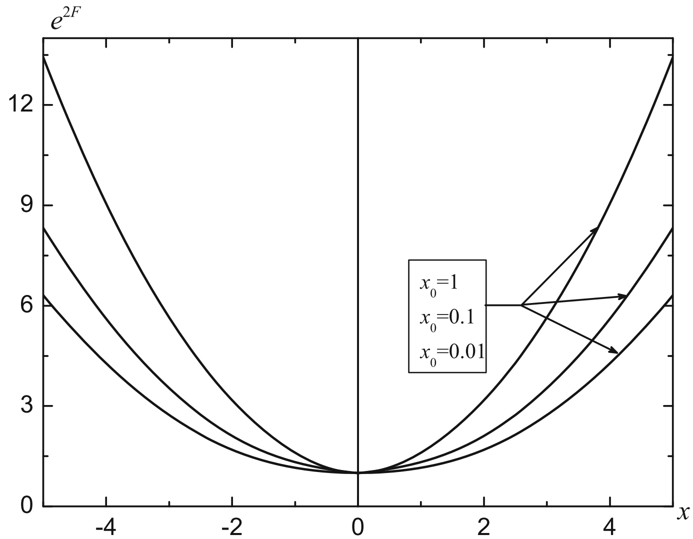

Fig. 2 The behavior of the metric function e^{2F} depending on the throat radius x_0. $\beta = 1$ for all lines. By corresponding rescaling of the time variable t, one can obtain the following asymptotic behavior of the solutions: $e^{2F} \to \frac{2}{3}x^2$ (the anti-de Sitter spacetime)

solutions of the corresponding Eqs. (3)–(5). From the physical point of view, it is clear that solutions with different x_0 differ from each other by different amount of the phantom matter in the wormhole. That is why our interpretation of this result consists in assumption that the increase/decrease of phantom matter decreases/increases the throat radius that results in the brake-up of the throat and appearance of two separate spaces. One can assume that the inverse process, leading to creation of a wormhole, also exists.

Note that all wormhole solutions obtained above are not asymptotically flat (the spacetime is anti-de Sitter one, see Figs. 2, 3). One can try to find a similar sequence of solutions with an asymptotically flat spacetime. It will be done in the next section with use of the Mexican hat potential. Let us note right now that such a sequence of asymptotically flat solutions does not exist.

3 Wormholes supported by a phantom Mexican hat

In this section we follow to Refs. [11,12]. Let us consider a model with the Mexican hat potential

$$V = \frac{1}{2}\left(\frac{\mu}{f}\right)^2 \left[\left(1 - f^2\varphi^2\right)^2 - 1\right]. \tag{13}$$

Here μ and f are some constants. Using (3), (4) and (5), one can obtain the $\binom{t}{t}$ and $\left[\binom{t}{t} - \binom{x}{x}\right]$ components of the Einstein equations (4), respectively

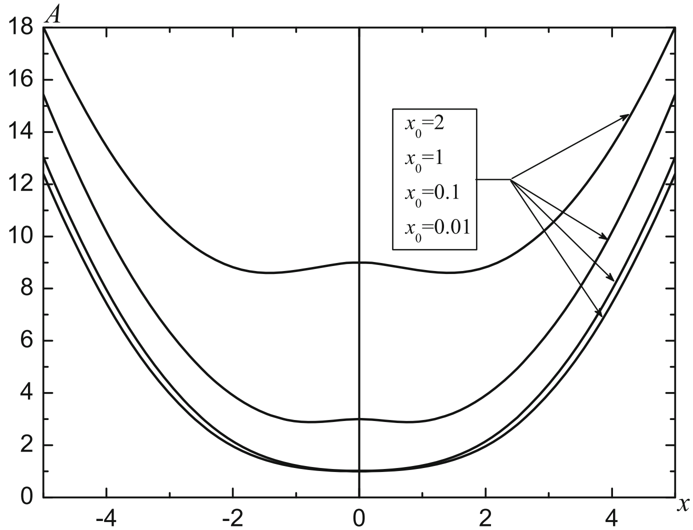

Fig. 3 The behavior of the metric function A depending on the throat radius x_0. $\beta = 1$ for all lines. Asymptotically $A \to \frac{2}{3}x^2$ for all cases that corresponds to the anti-de Sitter behavior of the solutions (the scalar curvature is positive)

$$-\frac{A'}{A}x + \frac{1}{A} + \frac{x^2}{x^2 + x_0^2} - 2 = \frac{x^2 + x_0^2}{f^2 A}\left\{-\frac{A}{2}\phi'^2 + \frac{1}{2}\left[\left(1 - \phi^2\right)^2 - 1\right]\right\}, \tag{14}$$

$$\frac{2x^2}{x^2 + x_0^2} - 2 - \frac{A'}{A}x + 2xF' = -\frac{x^2 + x_0^2}{f^2}\phi'^2, \tag{15}$$

and the scalar field equation

$$\phi'' + \left(F' + \frac{2x}{x^2 + x_0^2} + \frac{1}{2}\frac{A'}{A}\right)\phi' = \frac{2}{A}\phi\left(1 - \phi^2\right), \tag{16}$$

where a prime denotes differentiation with respect to x. Here and further we will use the dimensionless variables $x = \mu r$, $\phi = f\varphi$, and $8\pi G = c = 1$.

Using Eq. (15), one can rewrite Eq. (16) as follows

$$\phi'' + \left(-\frac{1}{2f^2}\frac{x^2 + x_0^2}{x}\phi'^2 + \frac{A'}{A} + \frac{1}{x} + \frac{x}{x^2 + x_0^2}\right)\phi' = \frac{2}{A}\phi\left(1 - \phi^2\right). \tag{17}$$

We solve the system of equations (14) and (17) with the following boundary conditions at $x = 0$:

$$\phi(0) = const, \quad \phi'(0) = \sqrt{2}\frac{f}{x_0}, \quad A(0) = 1 - \frac{x_0^2}{2f^2}\left[\left(1 - \phi(0)^2\right)^2 - 1\right]. \tag{18}$$

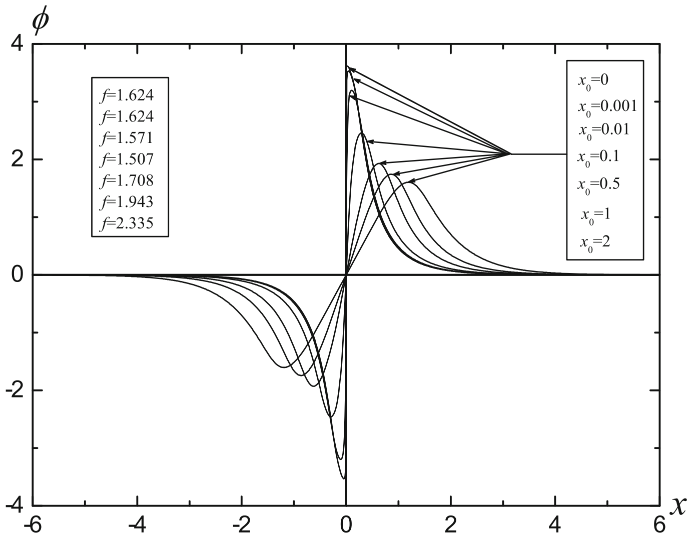

Fig. 4 The behavior of the scalar field ϕ depending on the throat radius x_0. On the *left*, the corresponding values of the parameter f from the potential (13) are presented

If one wants to start with $A(0) = 1$ and to get anti-symmetric solutions with respect to $x = 0$, it is necessary to take $\phi(0) = 0$.

The obtained results are presented in Fig. 4. As one can see, the regular wormhole ($x_0 \neq 0$) and spherically symmetric ($x_0 = 0$) solutions only exist at different values of the parameter f from the potential (13). Every solution for a given x_0 exists only at definite f. In this sense, f is an eigenvalue of the problem. This means that the sequence of solutions necessary for us does not exist since for each f only one solution with a special value of x_0 does exist.

Summarizing, we have shown that there exists some engineering possibility to create a sequence of wormholes with the vanishing throat radius in the asymptotically AdS spacetime. It is shown that such a set of static regular wormhole-like solutions only exists at some special choice of a phantom scalar field potential energy—the Sine-Gordon potential. This sequence of static solutions can be interpreted as a set of "snapshots" of the dynamical formation process of the wormhole. Physically, this means that it is possible to break up the wormhole into two separate spherically symmetric solutions by changing the phantom mass of the wormhole. It seems to be physically sensible that the inverse process is also possible: by changing mass in two regions of a space containing spherically symmetric distribution of the phantom matter created by a scalar field with the Sine-Gordon potential, it is possible to create a wormhole between these two regions. Note that the obtained solutions are nonsingular, and cover all the spacetime.

In conclusion we note that the classical (i.e. non-quantum) solutions obtained here can be applied to the description of only macroscopic objects. However, as the wormhole throat radius will be decreasing up to the microscopic Planckian sizes, the role of quantum effects becomes larger. Then, as one can see from the boundary conditions

(11), when x_0 tends to the Planckian values then the derivative $\phi'(0)$ and accordingly the scalar curvature diverge. Of course, in this case the classical model under consideration turns out to be inapplicable, and one should take into account quantum gravity effects. One of these effects is the possible topology change when a throat radius becomes comparable with the Planckian sizes. Modeling these effects can be performed by a consideration of spacetime as the Wheeler spacetime foam. As an example of such a process, in the recent paper [13] a possible mechanism of a conversion of a dark energy star into a wormhole is considered. In that case the topological change is provided by the presence of the negative radial pressure which implies the appearance of a tunnel. In the above paper, the theoretical difficulties associated with changes in topology are discussed as well.

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